3.0. VARIABILITY INDEX $\frac{R_{\text{max}}}{R_a}$ DURING TURNING

3.1. INTRODUCTION

The tool geometry plays a very important role in curving the basic surface followed by its further deterioration or improvement due to other factors such as (a) machine tool rigidity and performance characteristic, (b) work material and metallurgy, (c) Cutting Variables and environment, (d) type of machining and other unknown factors.

Though the theoretical surface roughness relation for $R_{\text{max}}$ due to turning and plain milling can be easily derived from geometry, difficulty arises in the verification of those relations through experiments, as most of the widely used instruments measure the so called CLA or $R_a$ value as per the BS (120) or ISO recommendation. The literature show wide variations in the value of $\frac{R_{\text{max}}}{R_a}$ ratio. An attempt is made here to find out analytically and experimentally the values of $\frac{R_{\text{max}}}{R_a}$ ratio. Applying simulation technique the value of $\frac{R_{\text{max}}}{R_a}$ ratio is also found based on such classical experimental results of Olsen (18) and Brewer (122). Though the analytical results show an invariant relationship between $R_a$ and $R_{\text{max}}$, the experimental results show wide variation. The experimental results obtained at large number of speeds, feeds and nose radii clearly indicates that (a) one set of factors play dominant role at very fine turning level and produces surface finish lesser.
than the ideally expected level, (b) at some medium range of finish turning the ideal and actual value of roughness almost tally each other; (c) where as at a feed higher than (b), the surface produced is actually finer than the theoretically expected level, so here, another set of factors are operating to produce a finer surface. It may therefore be assumed that the value, $\frac{R_{\text{max}}}{R_a}$ is a real indicator of the variability in surface roughness and can be used to identify the variability factors operating at various ranges.

3.1.1. Previous work-

The fact that the actual $R_{\text{max}}$ differs greatly from the theoretical value given by the geometric relation $\frac{s^2}{8r}$ (where $s =$ feed/rev, $r =$ nose radius) was reported earlier by Peck (19). Olsen (18) found experimentally the $\frac{R_{\text{max}}}{R_a}$ ratio to be around 6, however he observed that both $R_a$ and $R_{\text{max}}$ follow a normal Gaussian distribution curve. Solaja (121) and Brewer et al (122) had derived empirical relations for $R_a$ and showed that the ratio $\frac{R_{\text{max}}}{R_a}$ must be about 10 (by Solaja) or 4 (by Brewer). These widely different predictions though well founded on mathematical models led to the present investigation. Chattacharyaya (23) further analysed the Olsen data to derive empirical relations for $R_a$ which clearly indicate that $R_a$ follows an altogether different characteristics at three different cutting zones based on feed and speeds and so does not have a constant relation with the theoretical,

$$R_{\text{th}} = \frac{s^2}{8r}.$$
3.1.2. **Theoretical Background**

The well known geometrical relation for $R_{\text{max}}$ in turning is given by (123):

$$R_{\text{max}} = R_{\text{th}} = \frac{s^2}{8r}$$

for a tool having nose radius, $r$ mm and feed, $s$ mm/rev.

For turning with a sharp tool having principal and auxiliary cutting edge angles respectively as $\phi$ and $\phi_1$, the theoretical relation for roughness height is given by:

$$R_{\text{max}} = R_{\text{th}} = \frac{s}{\cot \phi + \cot \phi_1}$$

For plain milling with cutter diameter, $D$ in mm and feed $s_t$ in mm/tooth, the relation for $R_{\text{max}}$ is

$$R_{\text{max}} = R_{\text{th}} = \frac{s_t^2}{4D}$$

Now as per B.S. 1134 : 1950, the centre line average, $C_L$ value can be determined as follows:

The centre line is defined as a line conforming to the prescribed geometrical form of the profile and parallel to the general direction of the profile throughout the sampling length, such that the sums of the areas contained between it and those parts of the profile which lie on either side of it are equal.

The $C_L$ or $R_a$ is mathematically defined as:
\[ R_a = \frac{1}{L} \int_0^L |y| \, dx \]

\[ = (\text{sum of areas above} + \text{sum of areas below the centre line}) \times M \times \frac{1}{L} \]

3.2. ANALYSIS

3.2.1. Round Nose Tool:

The generated surface by turning a nose radius is shown in Fig. 3.1. The areas above the centre line if located properly can be given as \( A_1 = A_3 = A_5 = \ldots \) etc., whereas areas below will be \( A_2 = A_4 = A_6 = \ldots \) etc. If there are \( n \) cycles of repetition in the sampling length then:

\[ R_a = \frac{nA_1 + nA_2}{nL} = \frac{A_1 + A_2}{L} \]

For correct location of Centre line,

\[ \sum A_{15} = \sum A_{25} \quad \text{therefore, } A_1 = A_2 \]

As,

\[ A_1 = \int_0^{\phi_1} 2r^2 \sin^2 \theta \, d\theta \]

\[ = 2r^2 \left( \frac{\phi_1}{2} - \frac{\sin 2\phi_1}{4} \right) \ldots \quad (3.1) \]

Further if, \( x_1 = r \cos \phi_1 \)

The approximated area

\[ A_1 = \frac{3}{2} s_1 (r-x_1) = \frac{3}{2} s_1 (r-r \cos \phi_1) \]

\[ = \frac{3}{2} s_1 r (1- \cos \phi_1) \]

\[ = \frac{3}{2} s_1 r \left( 1-\sqrt{1-\frac{3}{2}x_1^2/4r^2} \right) \ldots \quad (3.2) \]
The area $A_2$ can be approximated as a triangle and given by

$$A_2 = \frac{1}{2} (s-3t) \left\{ x_1 - (r - s^2/8r) \right\}$$

$$= \frac{1}{2} (s-3t) \left\{ r \sqrt{1 - \frac{s^2}{4r^2}} - (r - \frac{s^2}{8r}) \right\} \quad \text{... (3.3)}$$

The analysis will be valid only if the cutting is restricted within the round nose. From the geometry of a triangular carbide throwaway tip, the condition for validity is found as

$$S_{\text{max}} = r \quad \text{or} \quad (S/r)_{\text{max}} = 1$$

(For a particular value of $r$).

Also, it is found that the angle $\Theta_{\text{max}}$ varies depending on the value of $(S/r)$. Some values of $(S/r)$ and $\Theta_{\text{max}}$ are given in table 3.1.

Using the relation,

$$\theta = \cos^{-1} \left[ 1 - \frac{1}{8} \left( \frac{S}{r} \right)^2 \right]$$

**TABLE - 3.1**

<table>
<thead>
<tr>
<th>$(S/r)$</th>
<th>$\Theta_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1°</td>
</tr>
<tr>
<td>0.2</td>
<td>5.75°</td>
</tr>
<tr>
<td>0.3</td>
<td>8.55°</td>
</tr>
<tr>
<td>0.4</td>
<td>11.5°</td>
</tr>
<tr>
<td>0.5</td>
<td>14.45°</td>
</tr>
<tr>
<td>0.6</td>
<td>17.25°</td>
</tr>
<tr>
<td>0.7</td>
<td>20.15°</td>
</tr>
<tr>
<td>0.8</td>
<td>23.1°</td>
</tr>
<tr>
<td>0.9</td>
<td>26°</td>
</tr>
<tr>
<td>1.0</td>
<td>28.95°</td>
</tr>
</tbody>
</table>
For the problem, the actual validity region is bounded by

\[ s_{\text{min}} = 0.079 \text{ mm/rev} \text{ to } s_{\text{max}} = 0.4 \text{ mm/rev} \text{ and } r_{\text{min}} = 0.4 \text{ mm to } r_{\text{max}} = 4 \text{ mm.} \]

\[ \Rightarrow (\frac{s}{r})_{\text{min}} = \frac{0.079}{0.4} = 0.02 \]

and

\[ (\frac{s}{r})_{\text{max}} = \frac{0.4}{0.4} = 1.0 \]

To calculate values of \( A_1 \) and \( A_2 \), first the approximate expressions given by equations (3.2) and (3.3) were used assuming values of nose radii \( r's \) and feeds \( s's \) and calculating at near level of centre line with values of \( s_1 = y \), where \( y \) is a decimal fraction. It is found that the values of \( y \) may be anything between 0.5 and 0.6. By plotting all the values of \( A_1 \) and \( A_2 \) as in Fig 3.2, the exact value of \( y \), also \( \theta_1 \) and \( s_1 \) for the correct location of centre line where, \( A_1 = A_2 \), can be found in all cases. With the correct value of \( \theta \) as \( \theta_0 \), the value of \( A_1 \) is calculated again by using equation (3.1) and also a slight lower value of \( y \) is accepted considering that the approximated area \( A_2 \) of the triangle is slightly higher at that level.

Then \( R_a \) is calculated by using equation

\[ R_a = (A_1 + A_2)/3 \]

It is found that for all combinations of \( r \) and \( s \), the value of
\[
\frac{R_{\text{max}}}{R_a} = \frac{s^2/8r}{(A_1 + A_2)/s} = 3.7
\]

is constant.

Some results are given in Table 3.2.

**TABLE 3.2**

<table>
<thead>
<tr>
<th>(r_{\text{mm}})</th>
<th>(s_{\text{mm/rev}})</th>
<th>(A_1 = A_2\times 10^5 \text{mm}^2)</th>
<th>(R_a\times 10^{-3} \text{mm})</th>
<th>(R_{\text{max}}\times 10^{-3} \text{mm})</th>
<th>(\frac{R_{\text{max}}}{R_a}) (approx.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.1</td>
<td>4.1</td>
<td>0.84</td>
<td>3.1</td>
<td>3.7</td>
</tr>
<tr>
<td>0.4</td>
<td>0.2</td>
<td>30.4</td>
<td>3.37</td>
<td>12.5</td>
<td>3.7</td>
</tr>
<tr>
<td>0.4</td>
<td>0.3</td>
<td>107.1</td>
<td>7.51</td>
<td>28.1</td>
<td>3.7</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>271.5</td>
<td>13.5</td>
<td>50.3</td>
<td>3.7</td>
</tr>
<tr>
<td>0.8</td>
<td>0.1</td>
<td>2.1</td>
<td>0.42</td>
<td>1.96</td>
<td>3.7</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>16.8</td>
<td>1.68</td>
<td>6.3</td>
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</tr>
<tr>
<td>0.8</td>
<td>0.4</td>
<td>135</td>
<td>6.75</td>
<td>25</td>
<td>3.7</td>
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<tr>
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<td>0.2</td>
<td>11.25</td>
<td>1.12</td>
<td>4.16</td>
<td>3.7</td>
</tr>
<tr>
<td>1.2</td>
<td>0.4</td>
<td>9.0</td>
<td>4.5</td>
<td>16.7</td>
<td>3.7</td>
</tr>
<tr>
<td>1.6</td>
<td>0.3</td>
<td>27</td>
<td>1.9</td>
<td>7.1</td>
<td>3.7</td>
</tr>
<tr>
<td>2.4</td>
<td>0.4</td>
<td>44.6</td>
<td>2.20</td>
<td>8.2</td>
<td>3.7</td>
</tr>
</tbody>
</table>

\(31\) at centre line for all is 0.585

3.2.2. GROUND TOOL:

The generated surface is as shown in Fig. 3.3

Here

\[
R_{\text{max}} = R_{\text{th}} = \frac{s}{\cot \phi + \cot \phi_1}
\]
and

\[ R_a = \frac{A_1 + A_2}{3} = \frac{2 \times \frac{c}{2} \left( \frac{R_{\text{max}}}{2} \right)}{3} = \frac{R_{\text{max}}}{4} \]

Hence \[ \frac{R_{\text{max}}}{R_a} = 4 \]

Therefore, we see that though for round nose tool the \( R_{\text{max}}/R_a \) ratio is centred around 3.7, for sharp tool its value is 4, both are invariant. But the experimental results show wide variation in the value of \( R_a \) measured. This was recognised by many a researchers.

3.3. CORRECTION FOR RANDOMNESS BY SIMULATION

The surface roughness as recorded by any roughness measuring instrument shows a distinct random pattern quite different from the ideal geometrical regular pattern based on which the earlier results were obtained. This randomness may be due to several factors such as microchatter, change in hardness of work material and tool nose and flank surface finish (tool flank wear) etc. As these are stochastic in nature and is very difficult to determine quantitatively by any mathematical model or even by experiments, the only course left with is to incorporate this random variation in the model by the SIMULATION technique.
3.3.1. **Simulation**

Simulation is the process of conducting experiments on a model instead of attempting the experiments with the real system. Because sometimes actual phenomenon may be difficult to observe or hard to analyse. Monte Carlo technique is the best known simulation technique for analysing a stochastic behaviour of certain function.

To develop a simulation model it is the prerequisite to gather statistics or frequency distribution of the major data required for simulation.

In this model the variability data about the surface finish was taken from the wide range of experimental data obtained by Olsen (18) and Brewer (122). From their experimental results, the mean, variance and standard deviation were calculated. It is found that there is always random variation within a limit of 20%. With the statistical analysis the distribution of $R_{max}$ in terms of $R_{th}$ i.e. $xR_{th}$ where $x$ is a fraction. The following probability and cumulative Probability is assigned as shown in bracket.
The areas $A_s$ are given by

$$A_s = \frac{S_s}{2} \left( \frac{S_s}{R_s} \right)$$

where $R_s$ and $S_s$ are only fractional.

The simulated values of $R_s$ and $S_s$ are then calculated on the basis of the last two digits of randomly selected numbers. The areas are given by

$$A_s = \frac{S_s}{2} \left( \frac{S_s}{R_s} \right)$$

where $R_s$ and $S_s$ are only fractional.

<table>
<thead>
<tr>
<th>Height (ft)</th>
<th>Area (sq ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>0.6</td>
<td>1.2</td>
</tr>
<tr>
<td>0.7</td>
<td>1.5</td>
</tr>
<tr>
<td>0.8</td>
<td>1.3</td>
</tr>
<tr>
<td>0.9</td>
<td>1.3</td>
</tr>
<tr>
<td>1.0</td>
<td>1.1</td>
</tr>
<tr>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td>1.3</td>
<td>1.1</td>
</tr>
<tr>
<td>1.4</td>
<td>1.1</td>
</tr>
<tr>
<td>1.5</td>
<td>1.1</td>
</tr>
<tr>
<td>1.6</td>
<td>1.1</td>
</tr>
</tbody>
</table>

The surface roughness pattern being produced is assumed to be triangular so that the areas can be calculated as functions of simulated values of $R_s$ and $S_s$ as height and length. The simulated values of $R_s$ and $S_s$ are then calculated on the basis of the last two digits of randomly selected numbers. The areas are given by

$$A_s = \frac{S_s}{2} \left( \frac{S_s}{R_s} \right)$$

where $R_s$ and $S_s$ are only fractional.
\[
\sum s_{is} = \left[ \sum_{i=1}^{20} \frac{1}{n} (s_{si} R_{si}) \right] s_{R_{th}}
\]

\[L = \sum l_{is} = \frac{1}{2} \sum_{i=1}^{20} S_{si} \cdot S = \frac{S}{2} \sum_{i=1}^{20} S_{si}\]

\[R_{a, \text{simulated}} = \frac{s_{R_{th}}}{\frac{S}{2}} \left[ \sum_{i=1}^{20} \frac{1}{n} (s_{si} R_{si}) \right]
\]

\[R_{th} \cdot \frac{1}{4} = \frac{\sum_{i=1}^{20} \frac{1}{n} (s_{si} R_{si})}{\sum_{i=1}^{20} (s_{si})}\]

Similarly, \(R_{\text{max}}\) is picked up from the random numbers in the form \(R_{\text{max}} = x R_{th}\), where value of \(x\) is the maximum depending on the probability of the area to which the random numbers are drawn. Lastly, the ratio of the simulated value of \(R_{\text{max}}\) and \(R_{a}\) is calculated. The simulated values were analysed in this manner for ten samples of twenty points each. The ratio of \(R_{\text{max}}/R_{a}\) found thus are respectively:

5.58, 4.64, 5.03, 5.64, 5.83,
5.28, 5.47, 5.45, 4.78, 5.05 with an average of 5.28.

Since the ratio of \(R_{\text{max}}/R_{a}\) for round nose tool is found as 3.7 and sharp tool as 4 (Theoretically), the actual values of
\( \frac{R_{\text{max}}}{R_a} \) is expected to be 5.2 as per the results of simulation.

**Example-1.**

For the 1st sample as per the random numbers the simulated values of \( R_a \) as multiplier of \( R_{th} \) are:

0.9, 0.9, 1.0, 1.0, 1.1, 1.0, 1.3, 0.9, 1.2, 1.0, 1.2, 0.6,
0.9, 1.0, 1.0, 1.0, 0.8, 1.0, 1.3, 0.7.

And simulated values of \( S_{a_i} \) as multiplier of \( '3' \) are:

0.6, 1.0, 1.0, 1.0, 0.8, 1.0, 1.0, 0.7, 0.7, 1.0, 1.0, 0.9, 0.8, 0.7, 1.2, 1.0, 1.0, 1.0, 1.5.

Then

\[
\sum_{i=1}^{20} S_{a_i} = 18.9, \quad \therefore L = 18.9 \times \frac{3}{2}
\]

\[
\sum_{i=1}^{20} S_{a_i} R_{a_i} = 18.52, \quad \therefore \sum a_i = \frac{18.52}{3} S_{a_i} R_{th}.
\]

\[
R_a = \frac{1}{4} \times \frac{18.52}{18.9} \times R_{th} = \frac{R_{th}}{4.06}
\]

However, the searching for the maximum amplitude, it is found to be 1.3 \( R_{th} \) for \( R_{\text{max}} \).

\[
R_{\text{max}} = 1.3 \times R_{th} \quad \text{and} \quad R_a = \frac{R_{th}}{4.06}
\]

\[
\frac{R_{\text{max}}}{R_a} = \frac{1.3}{R_{th}/4.06} = 4.06 \times 1.3 = 5.28.
\]
3.3.2. Standard Error in Mean

The mean value of $\frac{R_{\text{max}}}{R_a}$ as found from simulation is 5.2827 with standard deviation = 0.395 out of ten samples of twenty points each. Standard error depends on the sample size or in other words, justification for a correct sample size will be proved by the confidence level of the estimate.

For small sample size where, sample size $N$ is less than 30, the standard error is calculated thus, here $N = 10$

$$s^2 = \frac{N \sigma^2}{N-1} = \frac{10 \times 0.395^2}{9} = 0.173361$$

$$\therefore s = 0.416, \text{ where } \sigma = \text{standard deviation}$$

Then, $s_x = \frac{s}{\sqrt{N}} = \frac{0.416}{\sqrt{10}} = 0.13167$

and, $\frac{s}{s_x} = \frac{0.416}{0.13167} = 3.16$,

From the standard 't' table of book of statistics, the value of confidence level is found as 98.8% corresponding to the value of $\frac{s}{s_x}$, therefore the sample size is correct.

The first five samples yielded a mean of 5.224 with standard deviation of 0.3126. The value of $\frac{s}{s_x}$ is found to be 2.236 which gives only 92% confidence level, therefore, the sample size was increased to ten. The satisfactory level of the estimate is atleast to attain 95% confidence with value of $s \geq 2.776 s_x$. 
3.4. EXPERIMENTAL OBSERVATION AND RESULTS:

Experiments were carried out on a mild steel bar of 40 mm original diameter with (BHN 120, C-14 grade), a Sandvik T-max tool holder and inserts of S_4/P_30 grade, TPUN 160304 and TPUN 160308, which gives tool geometry: Orthogonal Rake = +6°, Inclination Angle = 0°, Principal Cutting edge Angle = 90°, and Orthogonal Clearance Angle = 11°.

Two nose radii were used namely 0.4 and 0.8 mm. The tool is rotated slightly to make the Principal cutting edge Angle = 60°, as it was found that with PCEA= 90°, the surface finish was deteriorating due to ring formation ahead of the cutting edge. Though the experiments were carried out for a large numbers of speed and feed combinations, only a few representative data are plotted and shown in Fig. 3.4 and 3.5. A set of cut was also given by another brazed tip tool of same geometry but with nose radius equal to 0.3 mm. Fig. 3.2 is also plotted for nose radius, \( r = 0.3 \) mm.

The experimentally observed values of \( \frac{R_{\text{max}}}{R_a} \) are plotted in Fig. 3.6 for \( r = 0.3, 0.4, \) and 0.8 mm respectively and cutting speed \( V_C = 46 \) m/min. These log-log plots show straight line pattern but with different slopes. As the slopes
are different so no attempt is made to develop any generalised equation. Table 3.3 shows some of the results obtained. The surface roughness is measured with Talyurf-10.

**TABLE 3.3**

**WORK-DATA**: M3 (G-14, BHN-120), Diameter 31.5 mm Length 325 mm (divided into 14 different lengths of 20 mm each separated by 2.3 mm grooves).

**TOOL-DATA**: Carbide 0-6-11-10-30-90-0.4 mm ORS (1)
(3^/P^ grade) 0-6-11-10-30-90-0.8 mm ORS (2)
Sandvik T-max tool holder TPUN 160308. and 160304

**DEPTH OF CUT**: 0.5 mm fixed, **ENVIRONMENT**: Dry.

<table>
<thead>
<tr>
<th>Radius (mm)</th>
<th>Feed (mm/rev)</th>
<th>Speed (m/min)</th>
<th>( R_a ) (micron)</th>
<th>( R_{max} ) (Theoretical)</th>
<th>( R_{max}/R_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.079</td>
<td>52.5</td>
<td>1.6</td>
<td>1.95 \times 10^{-3}</td>
<td>1.22</td>
</tr>
<tr>
<td>0.101</td>
<td></td>
<td>2.2</td>
<td></td>
<td>3.19 \times 10^{-3}</td>
<td>1.45</td>
</tr>
<tr>
<td>0.202</td>
<td></td>
<td>3.8</td>
<td></td>
<td>0.01275</td>
<td>3.36</td>
</tr>
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<td>0.317</td>
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<td></td>
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</tr>
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<td></td>
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<td>8.50</td>
</tr>
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<td>1.2</td>
<td>9.75 \times 10^{-4}</td>
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<td>1.59 \times 10^{-3}</td>
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<td>6.38 \times 10^{-3}</td>
<td>3.75</td>
</tr>
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<td>0.0357</td>
<td>8.72</td>
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<td></td>
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*Table contd.*
### Table

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<thead>
<tr>
<th>r</th>
<th>S</th>
<th>Vc</th>
<th>Ra</th>
<th>R_{max}</th>
<th>R_{max}/R_a</th>
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</thead>
<tbody>
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<td>0.202</td>
<td>32.8</td>
<td>4.2</td>
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<td>&quot;</td>
<td>52.5</td>
<td>3.6</td>
<td>&quot;</td>
<td>3.54</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>64.0</td>
<td>3.5</td>
<td>&quot;</td>
<td>3.64</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>77.8</td>
<td>3.1</td>
<td>&quot;</td>
<td>4.11</td>
</tr>
</tbody>
</table>

| 0.8mm | 0.202mm | 21.0 | 2.0 | 6.38x10^{-3} | 3.19 |
| "    | "       | 32.8 | 2.4 | "          | 2.66 |
| "    | "       | 41.9 | 1.8 | "          | 3.54 |
| "    | "       | 64.0 | 1.7 | "          | 3.75 |
| "    | "       | 77.8 | 1.5 | "          | 4.26 |

#### 3.5. CONCLUSION:

(a) Though theoretically the value of the ratio $\frac{R_{max}}{R_a}$ is found to be constant and is about 3.70 for a round nose turning tool where cutting feed is restricted to allow the generation of the finished surface to within the rounded nose only. The same ratio for the sharp tool is found to be '4' independent of the values of Principal cutting edge angle $\phi$ and the auxiliary cutting edge angle $\phi_1$, though the $R_{max}$ and $R_a$ relations contains the values of $\phi$ and $\phi_1$.

By considering the random variations of the $R_a$ values as observed by Olsen and Brewer, the simulation technique is applied to determine the expected value of $\frac{R_{max}}{R_a}$. It is found to be
Therefore, it is expected that $R_t/R_a$ ratio if both measured by Talysurf should be of the same order.

(b) On the other hand, the experimental results obtained through a large number of speed, feed and nose radii variation, showed a wide variation in the values of $R_{\text{max}}/R_a$ ratio from around 1 to 15. Fixing a normal range of 3 to 6, we can conclude that a surface worse than expected is produced at very low range of feed and a surface better than expected at moderately high range of feed.

(c) It is obvious that $R_{\text{max}}/R_a$ ratio is working as an indicator to identify the areas and factors that cause the variation in the surface roughness in a particular manner. One set of factors are operating at low range of feed to deteriorate the surface from the ideal, whereas another set of factors are operating at high range of feed (of course within finish feed) to produce a better than the ideal surface, as schematically shown in Fig. 3.7.

\[ R_{\text{max}} > R_{\text{th}} \text{ at } S < S_c \text{ and } R_{\text{max}} < R_{\text{th}} \text{ at } S > S_c, \text{ } R_{\text{max}} = R_{\text{th}} \text{ at } S = S_c. \]

(d) It is therefore necessary to identify the variability factors operating at the three distinctive zones of feed, in order to explain the surface roughness generation to a holistic level.
(e) It is observed that depending on the work material size and machines performance characteristics, there are four zones of cutting speeds spectrum as shown in Fig. 3.5. The zones are one that produces discontinuous chips at a very low speeds resulting in higher than expected values of $R_a$, the second zone where jointed or partially continuous chips are produced at low speeds. Then there is the best zone where continuous ribbon like chips are produced at an intermediate or moderately high speeds under very stable and smooth cutting conditions. In this zone the best surface finish is also observed. This zone is followed by the fifth zone where again the continuous chips are produced but accompanied by a little chatter at a still higher speeds. As a result the surface finish instead of further improvement practically deteriorate.

Therefore, it is recommended that for good surface finish the cutting speeds should be so selected that the cutting is restricted to the 3rd zone of stable cutting only.

The surface finish is bad in the first zone due to mechanism of brittle chip formation which leads to tear marks all over the machined surface. This is contrary to the common belief that the high roughness value in this zone is due to built up edge. The latter phenomenon if accompanied will definitely deteriorate the surface further. The roughness value $R_a$ also
tends to increase at very high speeds that leads to a little chatter. This tendency was observed by earlier investigators like Olsen (13) but was neglected.

(f) It may be necessary to investigate if this variability in $R_a$ has anything to do with the chip formation mechanism. The chip reduction coefficient ($\xi$) or cutting ratio ($\eta$) can be a good index to follow in this context. Correlation analysis between the $R_a$ and chip reduction coefficient ($\xi$) can be a good exercise to explain the variability in $R_a$ at different cutting conditions.
Fig. 3.1 Theoretically Generated Surface in Turning with Round nose Tool.

Fig. 3.2 Areas $A_1$ and $A_2$ for Exact Location of Centre Line

Fig. 3.3 Generated Surface with Sharp Tool
Fig. 3.4 $R_{\text{max}}$ and $R_a$ Versus Feed
CONDITION - DRY
NOSE RADIUS = 0.4 mm
DISCONTINUOUS CHIPS
JOINTED CHIPS
CONTINUOUS CHIPS
UNSTABLE CUTTING
STABLE CUTTING
S = 0.317
0.202
0.101
0.079
CHATTER

CUTTING SPEEDS, $V_c$ m/min

Fig. 3.5 Surface Finish Vs. Cutting Speed

Fig. $R_a/R$ Versus Feed in Log-Log.

$R_{max}/R_a$ Versus Feed in Log-Log.

$R_{max}/R_a$ versus feed in log-log. $R_{max}/R_a$ versus feed in log-log.
Fig. 3.7 Schematic diagram to show VARIABILITY in $R_{\text{max}}$ and $R_a$ causing DETERIORATION and IMPROVEMENT in Surface finish.

$R_{\text{max}}$(theoretical)-----

$R_{\text{max}}$(actual)---\text{HERE} $R_{\text{max}}/R_a = 4$

DETERIORATION ZONE

IMPROVEMENT ZONE

$S_1$ $S_{\text{critical}}$ $S_2$

FEED, $S$ mm/rev. ----->

Fig. 3.7