

## CHAPTER IV

### MATHEMATICAL MODEL FOR AUSTENITIZATION KINETICS OF DUCTILE IRON

The two steps heat treatment process involving austenitization and austempering of ductile iron results in austempered ductile iron. Ductile iron has graphite nodules embedded in the matrix of ferrite / ferrite + pearlite / pearlite, and is austenitized at a temperature in the range between 850 to 950<sup>0</sup>C for time period generally extending from 1 to 1.5 hrs. Austenitization is followed by quenching the iron to the austempering temperature between 250 to 425<sup>0</sup>C in a salt bath, where it is held for predetermined time, before cooling it immediately in water or air to room temperature. Previous studies by Rouns (1984, 1987) and Darwish (1993) noted the importance of homogeneous austempered microstructure to achieve the best mechanical properties of ADI. Homogeneous austenitization is an important prerequisite for producing homogeneous austempered microstructure. Thus, the selection of appropriate austenitization parameters, i.e., temperature and time, becomes important.

In the present chapter, austenitization process has been analyzed qualitatively and quantitatively. A model has been developed to study austenitization kinetics in ductile iron to find the optimum time required to obtain homogeneous austenite with carbon content in equilibrium with graphite nodules.

#### 4.1 Analysis of austenitization

The ductile iron may be considered to consist of large number of cells. Each cell has a graphite nodule at its center and it is surrounded by the matrix structure. The matrix structure surrounding the graphite nodule in a cell may be ferrite / pearlite / ferrite and

pearlite. In case of ductile iron with ferrite + pearlite matrix, a thin ferrite region surrounds the graphite nodule and the rest of the cell consists of ferrite as well as pearlite. The ratio of ferrite to pearlite areas depends upon the initial carbon content of the matrix. Figure 4.1 gives schematic representation of the different forms of ductile irons. It is assumed that all the cells and the graphite nodules are spherical and are of the same size. Let  $r_g$  be the radius of a graphite nodule,  $N_A$  be the nodule count and  $N_v$  be the number of nodules per unit volume. Since cells are around the graphite nodules, the number of cells in unit volume is also  $N_v$ . Let  $r_{cell}$  be the radius of each cell with its center coinciding with the center of the graphite nodule, and then one may write,

$$N_v \times \frac{4\pi}{3} r_{cell}^3 = 1 \quad (4.1)$$

$$r_{cell} = \left( \frac{3}{4\pi N_v} \right)^{1/3} \quad (4.2)$$

One may correlate  $N_v$  with the nodule count,  $N_A$ , using standard relation of quantitative metallography [Rhines and D. Hoff, 1968]

$$N_v = \frac{N_A}{2\pi r_g} \quad (4.3)$$

Therefore,

$$r_{cell} = \left( \frac{3 \times r_g}{2 \times N_A} \right)^{1/3} \quad (4.4)$$

$$\text{Also, } \frac{4\pi}{3} \times r_g^3 \times N_v = V_g \quad (4.5)$$

Using Eq.(4.1) one may write,

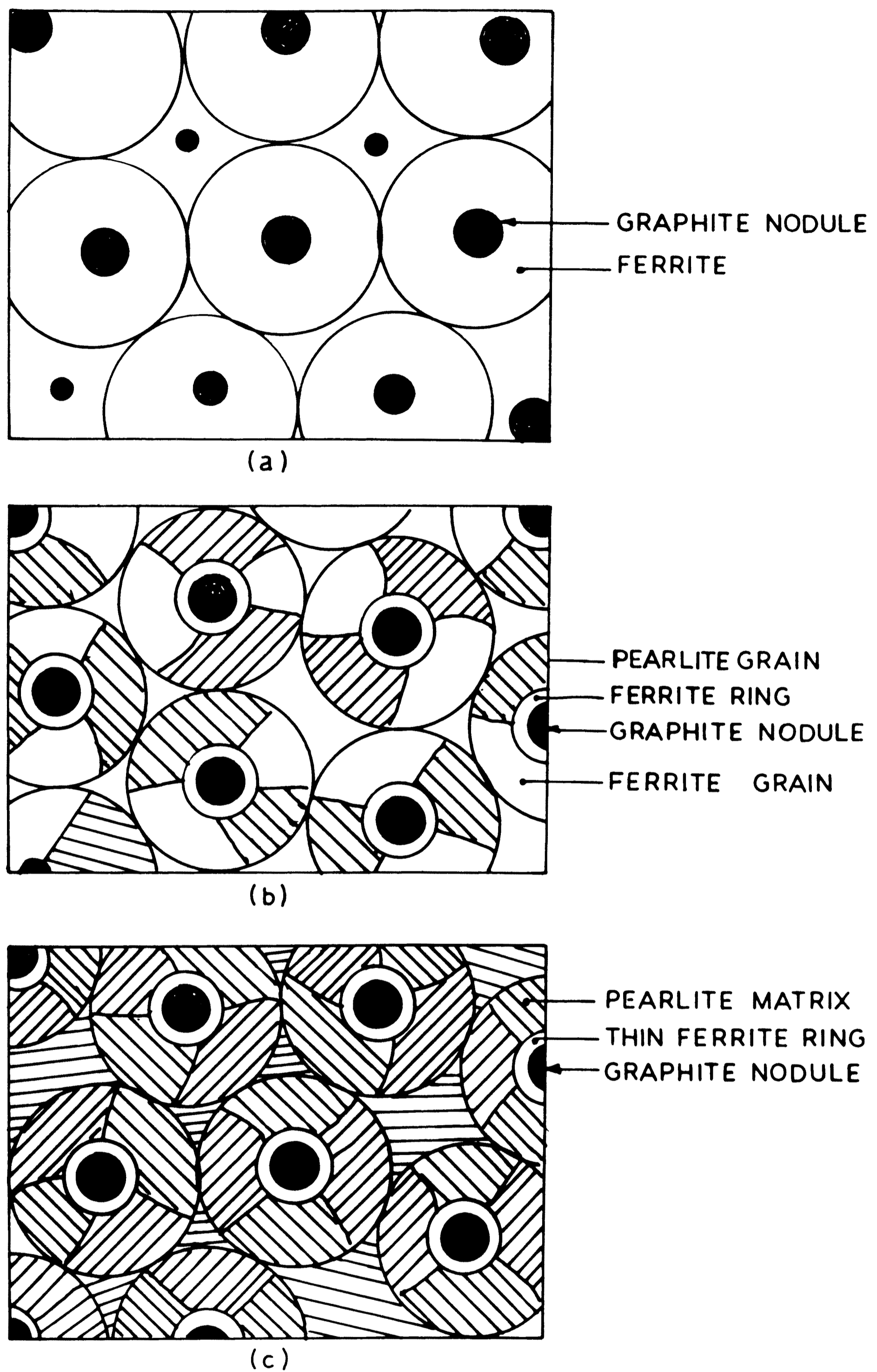


Fig. 4.1 : Schematic representation of different forms of ductile iron.

- a) Ferritic ductile iron.
- b) Ferrite + Pearlite ductile iron.
- c) Pearlitic ductile iron.

$$\frac{r_g}{r_{cell}} = (V_g)^{1/3} \quad (4.6)$$

Now to correlate  $V_g$  with other parameters of ductile iron, let the volume fraction of pearlite in the matrix be  $V_p$  and the volume fraction of ferrite in the matrix be  $V_f$ , so that

$$V_p + V_f = 1$$

Total carbon in the ductile iron may be written as:

$$\begin{aligned} C &= (100 \times V_g) + \{0.68 \times (1 - V_g) \times V_p\} + \{0.02 \times (1 - V_g) \times V_f\} \\ &= [\{(100 - 0.68) \times V_g\} + \{(0.68 - 0.02) \times V_g \times V_f\} - \{(0.68 - 0.02) \times V_f\} + 0.68] \end{aligned} \quad (4.7)$$

Therefore,

$$V_g = \frac{[C + (0.66 \times V_f) - 0.68]}{[99.32 + (0.66 \times V_f)]} \quad (4.8)$$

On substituting the value of ' $V_g$ ' from Eq. (4.8) into Eq. (4.6) one gets,

$$\frac{r_g}{r_{cell}} = [\{C + (0.66 \times V_f) - 0.68\} / \{99.32 + (0.66 \times V_f)\}]^{1/3} \quad (4.9)$$

If the volume fraction of ferrite in the matrix,  $V_f$ , the size of the graphite nodule,  $r_g$ , and the total carbon present in the ductile iron,  $C$ , are known,  $r_{cell}$  can be calculated by using Eq. (4.9).

One may consider a ductile iron with ferrite + pearlite matrix with composition  $C_o$ , austenitized at temperature  $T_\gamma$ , between  $Ae_3$  and  $912^\circ\text{C}$ , as shown in Fig. 4.2. Since at this temperature there is no equilibrium between ferrite and graphite it may be presumed that a layer of austenite will develop surrounding the graphite nodule. The austenitization process will progress to achieve finally austenite of uniform composition  $C_{\gamma g}$ , which is the

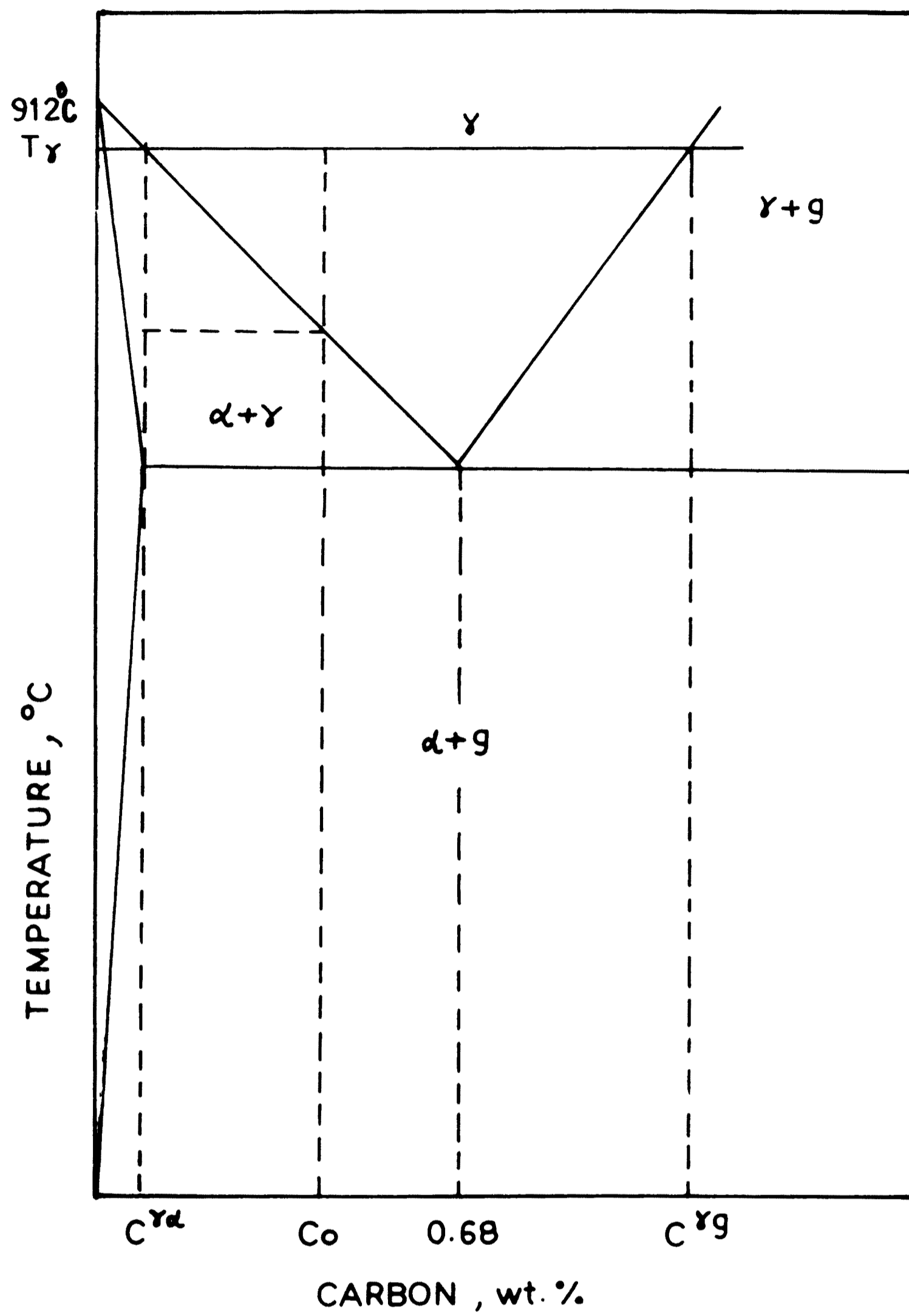


Fig. 4.2 : Local equilibrium concentrations during austenitization of ductile iron with ferrite-pearlite matrix with composition  $C_0$  at temperature,  $T_{\gamma}$  [Sharma, 1996].

austenite composition in equilibrium with graphite nodules. The process of austenitization will be accompanied by two dissolution processes (i) dissolution of matrix carbide if it contains pearlite and (ii) dissolution of a part of graphite nodule since overall matrix composition is lower than  $C_{\gamma g}$ . Since the carbide lamellae in pearlite have larger surface area and are relatively thin, it may be presumed that this process may be relatively faster. The completion of homogenization will therefore, be controlled by dissolution of carbon from the graphite nodules.

If  $C_o$  is the carbon content in terms of weight fraction of the ductile iron, then the total amount of the carbon in the cell,  $TC_{cell}$ , is

$$TC_{cell} = \frac{4\pi}{3} r_{cell}^3 \rho_o C_o \quad (4.10)$$

Where  $\rho_o$  is the density of the ductile iron. Out of this total carbon, the nodule has total carbon of,  $TC_{nodule}$ , given by:

$$TC_{nodule} = \frac{4\pi}{3} r_g^3 \rho_g \quad (4.11)$$

The remaining carbon is in the matrix of the cell as  $C_{matrix}$  given by

$$TC_{matrix} = \frac{4\pi}{4} r_{cell}^3 \left[ (\rho_o C_o) - \rho_g \left( \frac{r_g}{r_{cell}} \right)^3 \right] \quad (4.12)$$

When the equilibrium is attained at the austenitization temperature  $T_\gamma$ , the austenite should contain  $C_{\gamma g}$  amount of carbon, the total additional carbon that will be required by the matrix to attain this state is

$$TC_{Add} = \frac{4\pi}{3} (r_{cell}^3 - r_g^3) \rho_m C_{\gamma g} - TC_{matrix}$$

$$\frac{4\pi}{3} r_{\text{cell}}^3 [\{(\rho_m C_{\gamma g}) - (\rho_o C_o)\} + \{(\rho_g - (\rho_m C_{\gamma g})) (r_g / r_{\text{cell}})^3\}] \quad (4.13)$$

Where,  $\rho_m$  is the density of the matrix. This additional carbon  $TC_{\text{Add}}$ , will have to come by dissolution of carbon for the graphite nodule located at the center of the cell, through a thickening of the layer of austenite. If  $l_\gamma$  is the thickness of  $\gamma$  layer at time  $t$  and one assumes that steady state is established quickly within the  $\gamma$ -layer, the concentration profile may be written as,

$$C_\gamma = C_{\gamma g} - \frac{(r_g + l_\gamma)}{r \times l_\gamma} \times (r - r_g) \times (C_{\gamma g} - C_{\gamma \alpha}) \quad \text{for } r_g \leq r \leq r_g + l_\gamma \quad (4.14)$$

Figure 4.3 gives the schematic representation of the concentration profile in a cell at any time,  $t$  during this process. The total carbon content in the  $\gamma$ -layer of thickness  $l_\gamma$ , is  $TC_\gamma$  and it is given by,

$$TC_\gamma = \rho_\gamma \int_{r_g}^{r_g + l_\gamma} 4\pi r^2 dr C_\gamma \quad (4.15)$$

$$= \frac{2\pi}{3} \rho_\gamma l_\gamma [C_{\gamma g} r_g (3r_g + l_\gamma) + C_{\gamma \alpha} (3r_g^2 + 5r_g l_\gamma + 2l_\gamma^2)] \quad (4.16)$$

For sustaining a growth of  $dl_\gamma/dt$ , the rate of carbon accumulation in the layer will be  $dTC_\gamma/dt$  and their relation follows from Eq. (4.16) as,

$$\frac{dTC_\gamma}{dt} = \frac{2\pi}{3} \rho_\gamma [C_{\gamma g} r_g (3r_g + 2l_\gamma) + C_{\gamma \alpha} (3r_g^2 + 10r_g l_\gamma + 6l_\gamma^2)] \frac{dl_\gamma}{dt} \quad (4.17)$$

It may be reasonably presumed that all the carbon that is able to diffuse in  $\gamma$ -layer will first be used up solely for the transformation of  $\alpha$  to  $\gamma$  at the boundary between  $\alpha$  and  $\gamma$ . The total flux of solute through austenite per unit time for the given steady state concentration profile may be estimated following Fick's first law as,

$$J_1 = -D_C^\gamma A \rho_\gamma \frac{\partial C_\gamma}{\partial r} = -D_C^\gamma \rho_\gamma 4\pi r^2 \frac{dC_\gamma}{dr} = 4\pi D_C^\gamma \rho_\gamma \frac{r_g}{l_\gamma} (r_g + l_\gamma)(C_{\gamma g} - C_{\gamma \alpha}) \quad (4.18)$$

As the thickness of the austenite layer around graphite nodule will increase, the flux of solute will come down. However, as it has been already argued that to sustain growth at the rate of  $dl_\gamma/dt$  one has to have  $dTC_\gamma/dt$ , which should satisfy,

$$J_1 = \frac{dTC_\gamma}{dt} \quad (4.19)$$

Therefore,

$$4\pi D_C^\gamma \rho_\gamma \frac{r_g}{l_\gamma} (r_g + l_\gamma)(C_{\gamma g} - C_{\gamma \alpha}) = \frac{2\pi \rho_\gamma}{3} [C_{\gamma g} r_g (3r_g + 2l_\gamma) + C_{\gamma \alpha} (3r_g^2 + 10r_g l_\gamma + 6l_\gamma^2)] \frac{dl_\gamma}{dt} \quad (4.20)$$

Rewriting the above equation and integrating one gets,

$$6D_C^\gamma \int_0^t dt = \frac{C_{\gamma g}}{(C_{\gamma g} - C_{\gamma \alpha})} \int_0^{l_\gamma} \left(3 - \frac{l_\gamma}{r_g + l_\gamma}\right) l_\gamma dl_\gamma + \frac{C_{\gamma \alpha}}{(C_{\gamma g} - C_{\gamma \alpha})} \int_0^{l_\gamma} \left[3l_\gamma + \frac{7l_\gamma^2}{r_g} - \frac{l_\gamma^3}{r_g(r_g + l_\gamma)}\right] dl_\gamma \quad (4.21)$$

For short time  $t$ , when  $l_\gamma \ll r_g$  one may ignore higher order terms in  $l_\gamma$  and arrive at the following approximate equation,

$$6D_C^\gamma t = \frac{4}{4} \times \frac{(C_{\gamma g} + C_{\gamma \alpha})}{(C_{\gamma g} - C_{\gamma \alpha})} \times l_\gamma^2 \quad (4.22)$$

Thus,  $l_\gamma$  may grow with time following parabolic  $t^{1/2}$  behaviour for a short period of austenitization.

When  $l_\gamma \gg r_g$ ,



$$6D_C^\gamma \int_0^t dt = \frac{C_{\gamma g}}{(C_{\gamma g} - C_{\gamma \alpha})} \int_0^{l_\gamma} \left( 2 + \frac{r_g}{l_\gamma} - \frac{r_g^2}{l_\gamma^2} \right) l_\gamma dl_\gamma + \frac{C_{\gamma \alpha}}{(C_{\gamma g} - C_{\gamma \alpha})} \int_0^{l_\gamma} \left( 4l_\gamma - 7r_g + \frac{6l_\gamma^2}{r_g} \right) dl_\gamma \quad (4.23)$$

Retaining only up to  $(r_g)^0$  terms, one may get

$$6D_C^\gamma t = \frac{C_{\gamma g}}{(C_{\gamma g} - C_{\gamma \alpha})} l_\gamma^2 + \frac{C_{\gamma \alpha}}{(C_{\gamma g} - C_{\gamma \alpha})} + 2 l_\gamma^2 \left( \frac{l_\gamma}{r_g} + 1 \right) \quad (4.24)$$

If the matrix contains ' $f_\alpha$ ' volume fraction of ferrite, initially austenitization will transform this  $f_\alpha$  only. The volume fraction of pearlite,  $(1-f_\alpha)$ , will transform to austenite relatively faster and the ferrite surrounding this austenite may even transform till the carbon content of austenite reaches the limiting composition of  $C_{\gamma \alpha}$ . Thus, the volume fraction of austenite formed through dissolution of carbide in austenite,  $f_\gamma^C$ , is given by

$$f_\gamma^C = (1-f_\alpha) \times \frac{0.68}{C_{\gamma \alpha}} \quad (4.25)$$

The remaining matrix volume in the cell may transform to austenite only by dissolution of carbon from the nodule. The maximum thickness of the  $\gamma$ -region,  $l_\gamma^m$ , forming around the nodule could thus be,

$$(1-f_\gamma^C) (r_{\text{cell}}^3 - r_g^3) = \{(r_g + l_\gamma^m)^3 - r_g^3\} \quad (4.26)$$

$$l_\gamma^m = [r_{\text{cell}}^3 \times (1-f_\gamma^C) + f_\gamma^C \times r_g^3]^{1/3} - r_g \quad (4.27)$$

The time,  $t_\gamma$ , when the entire matrix has transformed to austenite may be estimated approximately as

$$t_\gamma = \frac{C_{\gamma g}}{(C_{\gamma g} - C_{\gamma \alpha})} \times \frac{l_\gamma^{m2}}{6D_C^\gamma} + \frac{C_{\gamma \alpha}}{(C_{\gamma g} - C_{\gamma \alpha})} \times \frac{l_\gamma^{m2}}{3D_C^\gamma} \times \frac{(l_\gamma^m + r_g)}{r_g} \quad (4.28)$$

At this time,  $t_\gamma$ , the concentration profile in the austenite may be given schematically as shown in Fig. 4.4

Once  $l_\gamma$  has reached the maximum thickness of  $l_\gamma^m$ , the further dissolution of carbon from the graphite nodule will enrich and homogenize austenite matrix in the cell. One may presume two distinct phases of homogenization of concentration of carbon in austenite by dissolution of carbon from the graphite nodule located at the center of cell: (a) increasing the diffusion distance from  $r_g$  to  $r_{\text{cell}}$  and (b) increasing the level of carbon at  $r_{\text{cell}}$ , reaching eventually the saturation level when a homogeneous matrix of austenite is achieved.

The schematic profile of carbon concentration at any time during phase (a) is given in Fig. 4.5. The carbon concentration profile is assumed linear and may be written as,

$$C_\gamma = C_{\gamma g} - \frac{C_{\gamma g} - C_{\gamma \alpha}}{d} (r - r_g) \quad \text{for } r_g \leq r \leq r_g + d \quad (4.29)$$

$$\frac{dc}{dr} = - \frac{C_{\gamma g} - C_{\gamma \alpha}}{d} \quad (4.30)$$

Total carbon in the austenite cell,  $TC(d)$ , which is a function of diffusion distance,  $d$ , is given by,

$$TC(d) = \frac{4}{3} \pi \rho_\gamma (r_{\text{cell}}^3 - r_g^3) C_{\gamma \alpha} + 4\pi \rho_\gamma \int_{r_g}^{r_g + d} (C_\gamma - C_{\gamma \alpha}) r^2 dr \quad (4.31)$$

where  $l_\gamma^m \leq d \leq r_{\text{cell}} - r_g$

$$TC(d) = \frac{4}{3} \pi \rho_\gamma (r_{\text{cell}}^3 - r_g^3) C_{\gamma \alpha} + 4\pi \rho_\gamma \int_{r_g}^{r_g + d} (C_{\gamma g} - C_{\gamma \alpha}) \left[ r^2 - \frac{(r - r_g)}{d} r^2 \right] dr \quad (4.32)$$

$$= \frac{4}{3} \pi \rho_\gamma (r_{\text{cell}}^3 - r_g^3) C_{\gamma \alpha} + \frac{\pi \rho_\gamma}{3} (C_{\gamma g} - C_{\gamma \alpha}) (d^3 + 4 r_g d^2 + 6 r_g^2 d) \quad (4.33)$$

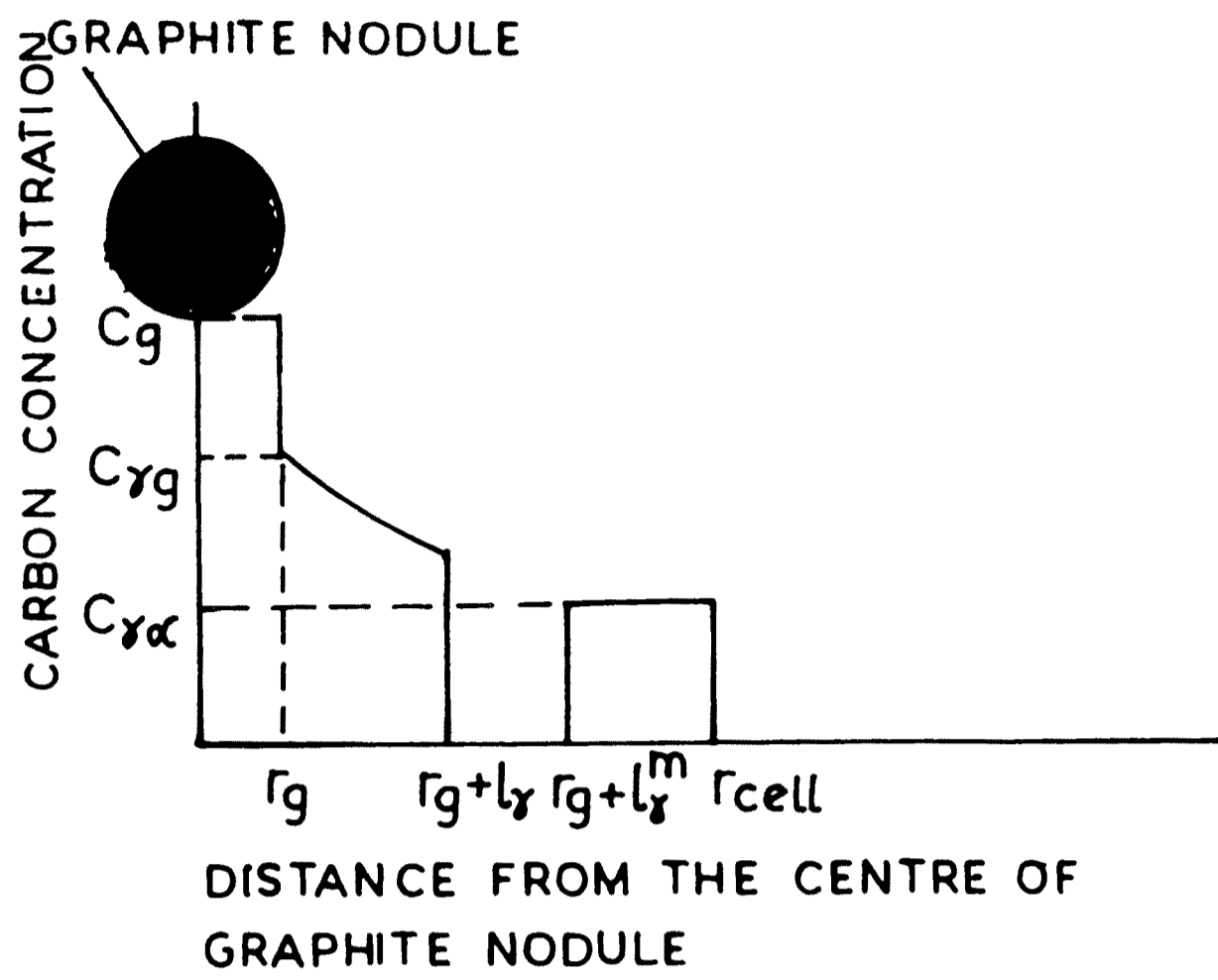


Fig. 4.3 : Schematic representation of carbon concentration profile at any time,  $t_\gamma$ , when austenite layer has grown to a thickness,  $l_\gamma$ , surrounding graphite nodule.

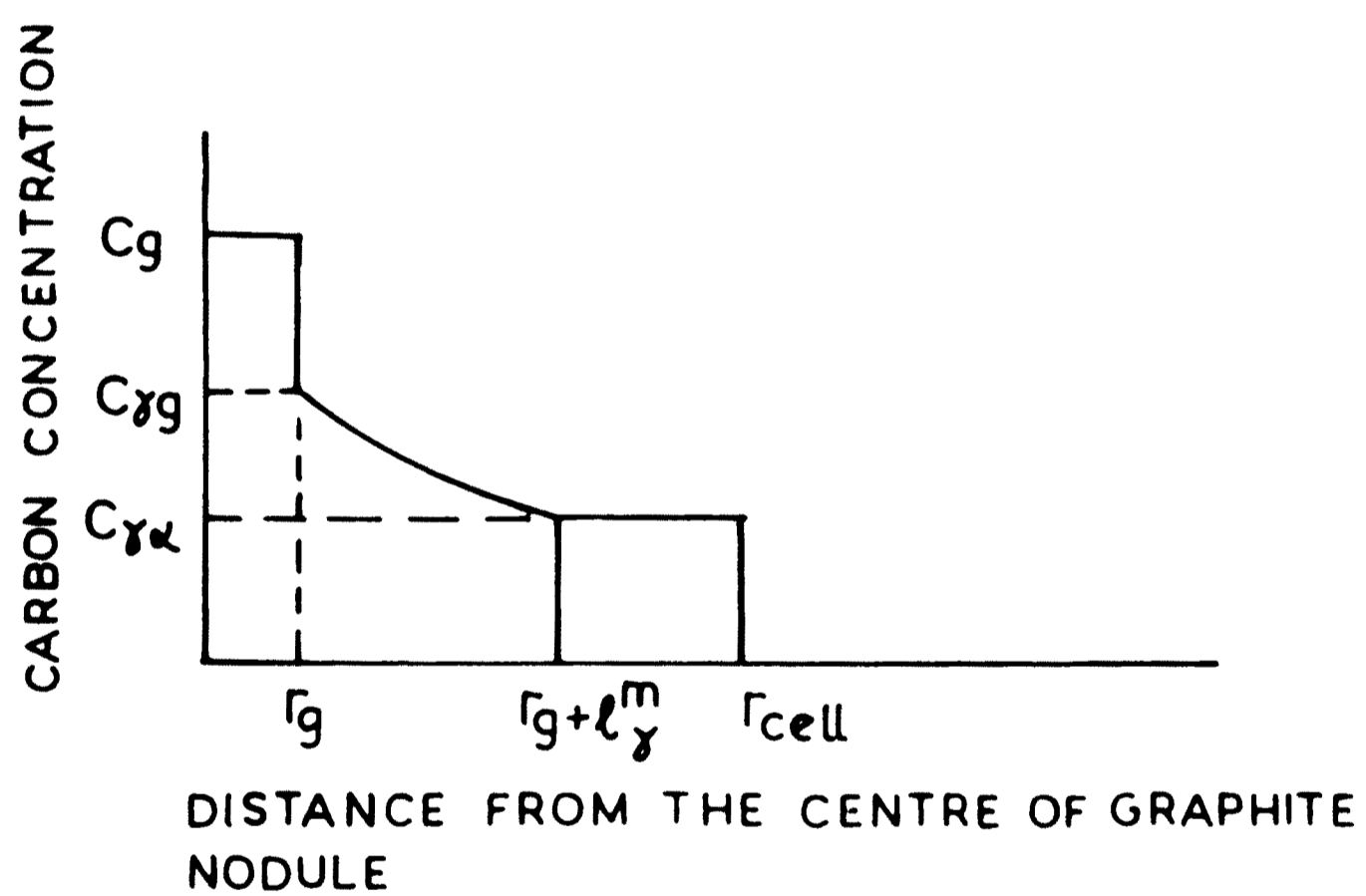


Fig. 4.4: Schematic representation of carbon concentration profile at any time  $t_\gamma$ , when austenite layer has grown to a thickness  $l_\gamma^m$ , surrounding graphite nodule.

For sustaining the rate of increase of diffusion distance with time,  $dd/dt$ , the rate of accumulation of carbon in austenite,  $dTC(d)/dt$ , will be required and their relation follows from Eq.(4.33).

$$\frac{dTC(d)}{dd} = \frac{\pi\rho_\gamma}{3} (C_{\gamma g} - C_{\gamma\alpha}) [3d^2 + 8 r_g d + 6r_g^2] \quad (4.34)$$

The rate of change in total carbon with time,  $t$ , i.e.,  $dTC(d)/dt$  is equal to the flux of carbon entering austenite from graphite nodule at the center of the cell. The total flux of carbon through austenite per unit time for the given concentration profile may be estimated following Fick's first law as,

$$J_2 = -D_{C^\gamma} A \rho_\gamma \frac{dc}{dr} = D_{C^\gamma} 4\pi r_g^2 \rho_\gamma \frac{(C_{\gamma g} - C_{\gamma\alpha})}{d} \quad (4.35)$$

As the carbon diffuses through longer distance in the austenite matrix around graphite nodule, the slope of concentration profile around the graphite nodule will decrease and consequently the flux of solute will decrease. In order to sustain increase in diffusion distance at the rate of  $dd/dt$  one has to have  $dTC(C)/dt$ , which should satisfy,

$$J_2 = \frac{dTC}{dt} = \frac{dTC}{dd} \times \frac{dd}{dt} = \frac{\pi\rho_\gamma}{3} (C_{\gamma g} - C_{\gamma\alpha}) [3d^2 + 8 r_g d + 6r_g^2] \frac{dd}{dt} \quad (4.36)$$

The time required for carbon to diffuse into austenite from the graphite/ austenite interface to result in a diffusion distance,  $d$ , is given as,

$$\Delta t_1 = \frac{3d^3 + 8 r_g d^2 + 6r_g^2 d}{12 D_{C^\gamma} r_g^2} \times \Delta d, \quad \text{where } \Delta d = r - r_g - l_\gamma^m \quad (4.37)$$

The schematic profile of carbon concentration at any time,  $t$ , during phase (b) of homogenization of austenite is given in Fig. 4.6. The carbon concentration profile in austenite may be written as,

$$C_{\gamma} = C_{\gamma g} - \frac{C_{\gamma g} - C}{r_{\text{cell}} - r_g} (r - r_g) \quad \text{for } r_g \leq r \leq r_{\text{cell}} \quad (4.38)$$

$$\frac{dc}{dr} = - \frac{C_{\gamma g} - C}{r_{\text{cell}} - r_g} \quad (4.39)$$

The total carbon in the austenite cell of radius,  $r_{\text{cell}}$ ,  $TC(C)$  is a function of carbon level,  $C$ , at the cell boundary and is given by,

$$TC(C) = \frac{4}{3} \pi \rho_{\gamma} (r_{\text{cell}}^3 - r_g^3) C + 4\pi \rho_{\gamma} \int_{r_g}^{r_{\text{cell}}} (C_{\gamma g} - C) \left[ 1 - \frac{(r-r_g)}{(r_{\text{cell}} - r_g)} \times r^2 \right] dr \quad (4.40)$$

where,  $C_{\gamma\alpha} \leq C \leq C_{\gamma g}$

Therefore,

$$\begin{aligned} TC(C) &= \frac{4}{3} \pi \rho_{\gamma} (r_{\text{cell}}^3 - r_g^3) C + \frac{\pi \rho_{\gamma}}{3} \times \frac{(C_{\gamma g} - C)}{(r_{\text{cell}} - r_g)} [r_{\text{cell}}^4 - 4r_g^3 r_{\text{cell}} + 3r_g^4] \\ &= \frac{4}{3} \pi \rho_{\gamma} (r_{\text{cell}}^3 - r_g^3) C + \frac{\pi \rho_{\gamma}}{3} \times (C_{\gamma g} - C) [r_{\text{cell}}^3 + r_{\text{cell}}^2 r_g + r_{\text{cell}} r_g^2 - 3r_g^3] \quad (4.41) \end{aligned}$$

For sustaining a carbon enrichment rate of  $dc/dt$  at the cell boundary, the rate of accumulation of carbon in the austenite matrix will be  $dTC(C)/dt$  and their relation follows from Eq. (4.41).

$$\begin{aligned} \frac{dTC(C)}{dC} &= \frac{4}{3} \pi \rho_{\gamma} (r_{\text{cell}}^3 - r_g^3) - \frac{\pi \rho_{\gamma}}{3} (r_{\text{cell}}^3 + r_{\text{cell}}^2 r_g + r_{\text{cell}} r_g^2 - 3r_g^3) \\ &= \frac{\pi \rho_{\gamma}}{3} (3r_{\text{cell}}^3 - r_{\text{cell}}^2 r_g - r_{\text{cell}} r_g^2 - r_g^3) \quad (4.42) \end{aligned}$$

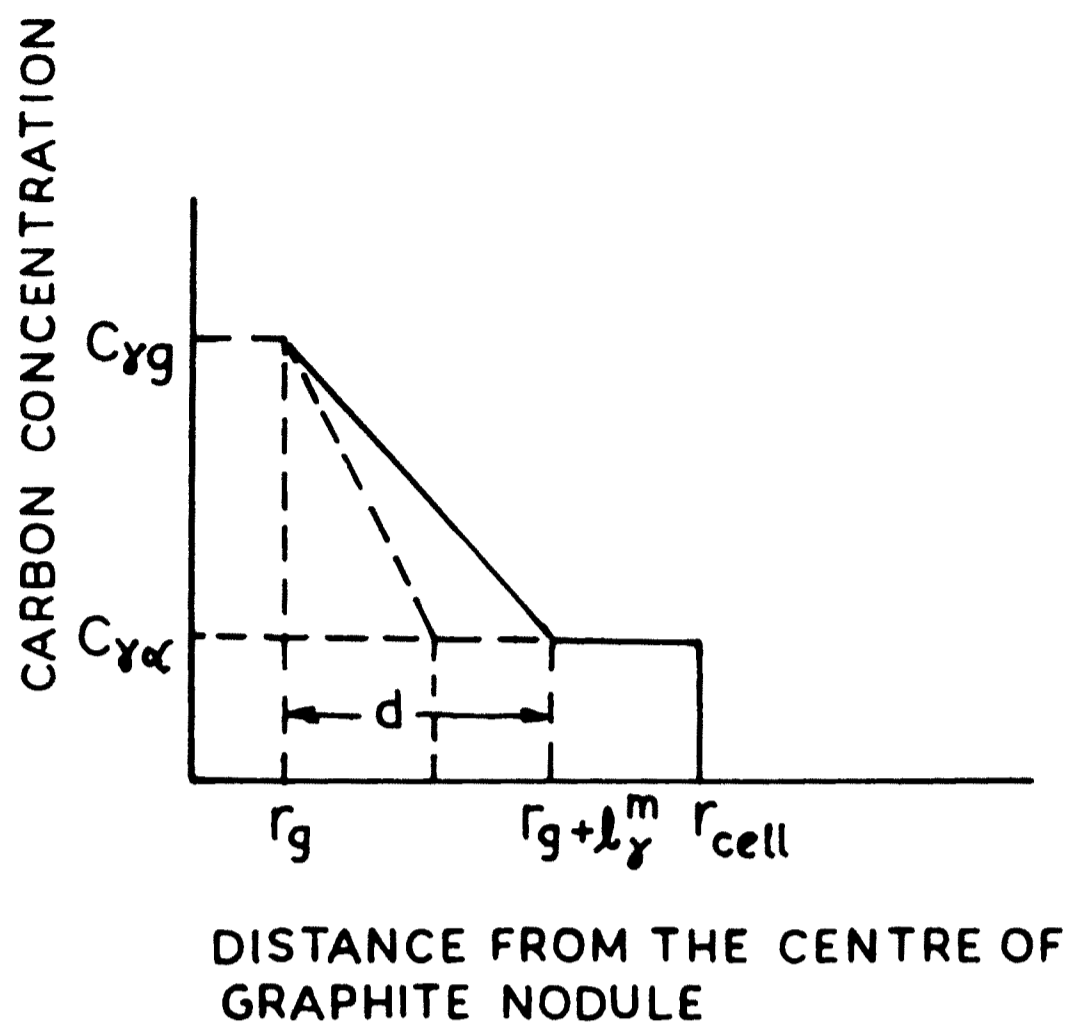


Fig. 4.5 : Schematic representation of concentration profile of carbon in austenite during phase (a) of homogenisation of austenite.

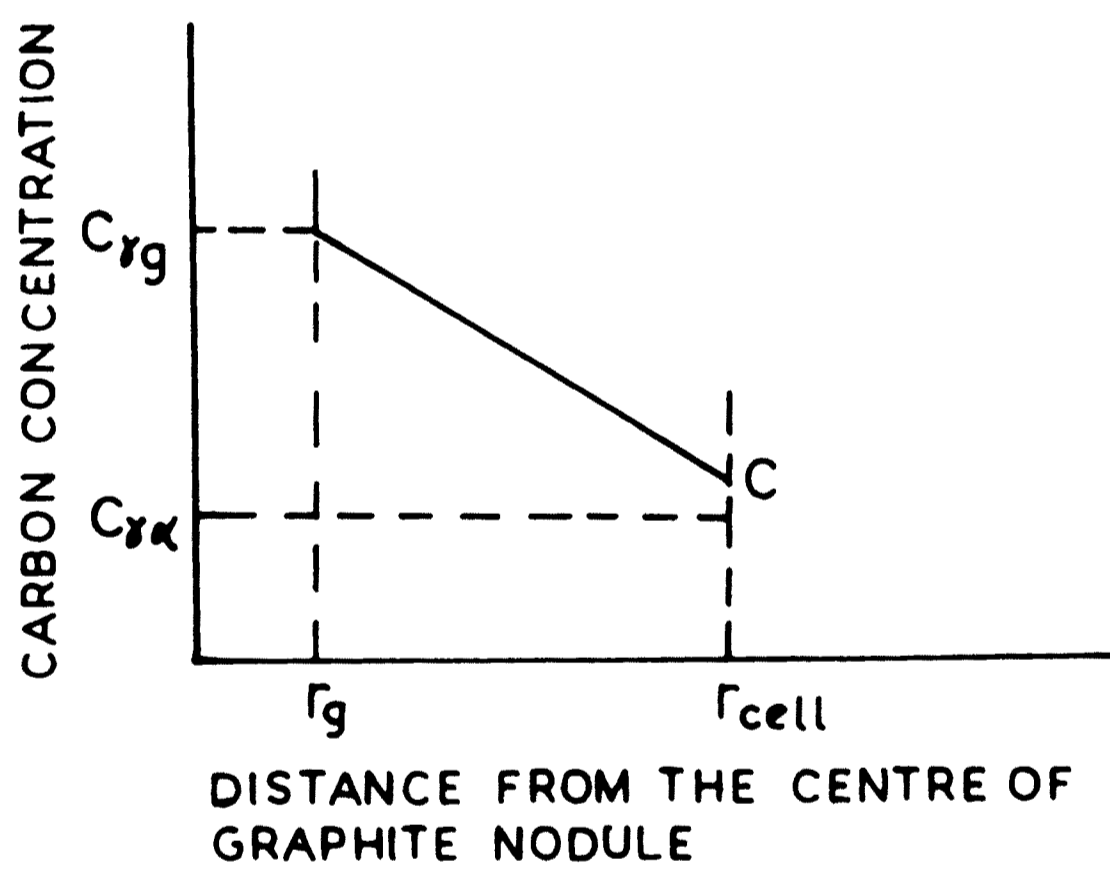


Fig. 4.6 : Schematic representation of concentration profile of carbon in austenite during phase (b) of homogenisation of austenite.

The flux of carbon from the graphite nodule to the austenite matrix,  $J_3$ , is given by,

$$J_3 = -D_C^\gamma A \rho_\gamma \frac{dc}{dr} = D_C^\gamma 4\pi r_g^2 \rho_\gamma \frac{(C_{\gamma g} - C)}{r_{\text{cell}} - r_g} \quad (4.43)$$

The rate of change of total carbon with time,  $dTC(C)/dt$ , is equal to the flux of carbon atoms entering austenite across graphite/ austenite interface; therefore,

$$J_3 = \frac{dTC(C)}{dt} = \frac{dTC(C)}{dc} \times \frac{dc}{dt} = \frac{\pi \rho_\gamma}{3} (3r_{\text{cell}}^3 - r_{\text{cell}}^2 r_g - r_{\text{cell}} r_g^2 - r_g^3) \times \frac{\Delta C}{\Delta t_2} \quad (4.44)$$

The time required for austenite to achieve carbon concentration of 'C' is given by,

$$\Delta t_2 = \frac{3r_{\text{cell}}^4 - 4r_{\text{cell}}^3 r_g + r_g^4}{12 D_C^\gamma r_g^2} \times \Delta C \quad \text{where } \Delta C = C - C_{\gamma\alpha} \text{ for } C > C_{\gamma\alpha} \quad (4.45)$$

Thus the total time required for complete austenitization process may be given as

$$\text{Total time, } t = t_\gamma + \Delta t_1 \theta(d - l_\gamma^m) + \Delta t_2 \theta(C - C_{\gamma\alpha}) \quad (4.46)$$

Where,  $\theta(x)$  is a step function and is defined as,

$$\theta(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$$

where,  $t_\gamma$  is the austenitization time,  $\Delta t_1$  is the time for first phase of homogenization in the cell and  $\Delta t_2$  is the time for the second phase of homogenization when  $C = C_{\gamma g}$ .

The total time of austenitization,  $t$ , given in Eq. (4.46) may be written explicitly from Eqs. 4.28, 4.37 and 4.45 as,

$$t = A \frac{1}{D_C^\gamma} \quad (4.47)$$

Where,

$$A = \frac{C_{\gamma g} \times l_{\gamma}^{m2}}{6 (C_{\gamma g} - C_{\gamma \alpha})} + \frac{C_{\gamma \alpha} \times l_{\gamma}^{m2} \times (l_{\gamma}^m + r_g)}{r_g (C_{\gamma g} - C_{\gamma \alpha})} + \frac{3d^3 + 8 r_g d^2 + 6 r_g^2 d}{12 r_g^2} \times \Delta d + \frac{3r_{\text{cell}}^4 - 4 r_{\text{cell}}^3 r_g + r_g^4}{12 r_g^2} \times \Delta C$$

Since  $D_C^{\gamma} = D_o \exp^{-Q/RT}$ , the time,  $t$  may be rewritten as,

$$t = \frac{A}{D_o} \exp^{+Q/RT} \quad (4.48)$$

Taking log on both sides, one may write

$$\ln(t) = \ln\left(\frac{A}{D_o}\right) + \frac{Q}{RT_{\gamma}} \quad (4.49)$$

The logarithm of the time of austenitization,  $t$  is plotted as a function of reciprocal of austenitization temperature,  $T_{\gamma}$ . The straight-line relationship is evident as shown in Fig. 4.7. The slope of the straight line is equal to  $+ Q/R$  and the intercept on the y-axis is logarithm of  $A/D_o$ .

The model developed in this chapter outlines the different stages of austenitization in the presence of graphite nodules which can act as reservoir of carbon for the purpose of establishing equilibrium between graphite nodule and austenite at a given austenitization temperature. An understanding of austenitization is important in the context of austempering since uniform carbon content in austenite helps in developing uniform microstructure in ADI resulting in the best mechanical properties in it.



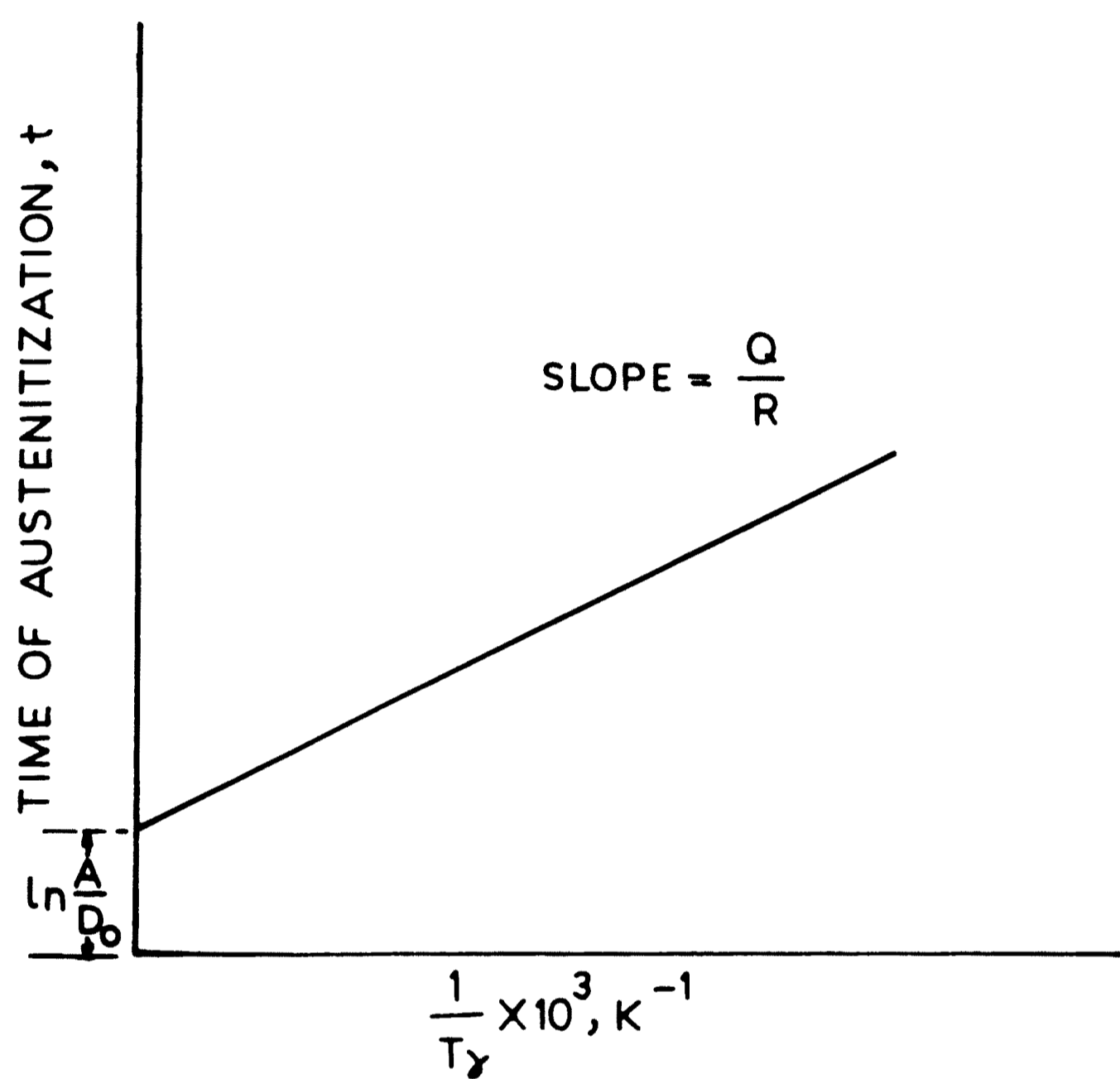


Fig. 4.7 : Plot to show the relationship between the time required for complete austenitization of ductile iron with austenitization temperature as calculated from the present mathematical model.