Chapter 3

Reflection and transmission of plane elastic waves at a plane interface between two distinct porous elastic half-spaces

3.1 Introduction

In the case of two dissimilar porous elastic half-spaces in contact, the pores at their common interface may be totally connected, partially connected or not connected at all. All these three possibilities may happen in real situation. If the porous half-spaces are separated by a very thin layer of impermeable elastic membrane, then mathematically this layer may be taken as the interface between these half-spaces. In this very situation, the pores may be taken as closed pores or sealed pores. This situation may also arise if the cross sections of pores of two media do not match at the interface. Sharma (2008) has studied the problem of reflection and refraction of plane harmonic waves at a welded contact plane interface between two dissimilar poroelastic half-spaces. He introduced pore alignment parameter to study the effect of connection between the interstices at the common interface of the two media, on amplitudes of reflected and refracted waves. He derived a set of boundary conditions to represent the partial connection of pores at the interface. Such a partial connection is considered as a basis for an imperfect bonding between two saturated porous solids. At the plane interface, the imperfect bonding is represented by tangential slipping and hence, results in the

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dissipation of a part of strain energy.

In the present chapter, we shall study the phenomena of reflection and transmission of plane elastic waves striking at the interface between two dissimilar porous media saturated by two immiscible fluids. The pores at the common interface of these two media are considered to be sealed and the boundary conditions suggested by Deresiewicz and Skalak (1963) for the closed pores are employed. The cases of plane $P-$ wave and plane $SV-$ wave propagating through the upper half-space and striking obliquely at the interface, are considered. We consider two-dimensional $x - z$ plane to discuss the reflection and transmission phenomena at a plane interface.

### 3.2 Reflection and refraction phenomena

Consider two different porous elastic solid half-spaces saturated by two immiscible fluids and separated by a plane interface $z = 0$, but in perfect contact with each other. The upper porous medium is occupying the region $\mathcal{R}_1$ defined by

$$\mathcal{R}_1 = \{(x, z) : -\infty < z \leq 0; -\infty < x < \infty\}$$

and the lower porous medium is occupying the region $\mathcal{R}_2$ defined by

$$\mathcal{R}_2 = \{(x, z) : 0 \leq z < \infty; -\infty < x < \infty\}.$$

The $x$-axis is taken along the interface between $\mathcal{R}_1$ and $\mathcal{R}_2$ and the $z$-axis is taken vertically downward pointing into the region $\mathcal{R}_2$. The quantities concerning the upper half-space will be denoted with superscript $(l)$ and the quantities corresponding to the lower half-space without superscripts.

Following Tuncay and Corapcioğlu (1997), the field equations and constitutive relations for the porous media containing two immiscible fluids are given by the equations (1.121)-(1.123) and (1.104)-(1.106), respectively. We shall use the equation (2.1) to decompose the displacement vector in the upper medium into scalar and vector potentials denoted by $\phi^l$ and $\Psi^l$ and that in the lower medium by the scalar and vector potentials denoted by $\phi^l$ and $\Psi^l$ respectively. We follow the relation given by (2.11) between $\Psi_i$ and $\Psi_s$. Similar relation holds between $\Psi^l_j$ and $\Psi^l_s$ by putting the superscript $(l)$ at appropriate places.
3.2.1 Boundary conditions

We assume that the pores on the interface are closed, that is, there is no fluid flow from one half-space to another half-space. Therefore, the appropriate boundary conditions at the plane interface \( z = 0 \) can be obtained from the consideration discussed earlier by Deresiewicz and Skalak (1963) and are given as follows

(a) Continuity of total normal stress across the interface

\[
\langle \tau' \rangle_{zz} + \langle \tau_1 \rangle_{zz} + \langle \tau_2 \rangle_{zz} = \langle \tau \rangle_{zz} + \langle \tau_1 \rangle_{zz} + \langle \tau_2 \rangle_{zz},
\]

(b) Continuity of total shear stress across the interface

\[
\langle \tau' \rangle_{zx} = \langle \tau \rangle_{zx}.
\]

(c) Continuity of tangential velocities of the solids across the interface

\[
\dot{u}_s'_{xx} = \dot{u}_s_{xx},
\]

(d) Continuity of normal velocities of the solids across the interface

\[
\dot{u}_s'_{zz} = \dot{u}_s_{zz}.
\]

(e) Since the liquid can not flow from one half-space to another half-space, therefore, the normal velocity of each of the fluid relative to the solid skeleton in the porous material vanishes at the interface

\[
\dot{u}_s_{zz} - \dot{u}_1_{zz} = 0, \quad \dot{u}_s_{zz} - \dot{u}_2_{zz} = 0, \quad \dot{u}_s'_{zz} - \dot{u}_1'_{zz} = 0, \quad \dot{u}_s'_{zz} - \dot{u}_2'_{zz} = 0.
\]

Using (2.1) and (2.8)-(2.10), the above boundary conditions can be written in terms of potentials, respectively, as follows

\[
\phi_1' \nabla^2 \phi_1' + \phi_2' \nabla^2 \phi_2' + \phi_3' \nabla^2 \phi_2' = \phi_1 \nabla^2 \phi_1 + \phi_2 \nabla^2 \phi_1 + \phi_3 \nabla^2 \phi_2,
\]

\[
G_f \left( \frac{\partial^2 \psi'_s}{\partial x^2} - \frac{\partial^2 \psi'_s}{\partial z^2} + 2 \frac{\partial^2 \psi'_s}{\partial x \partial z} \right) = G_f \left( \frac{\partial^2 \psi_s}{\partial x^2} - \frac{\partial^2 \psi_s}{\partial z^2} + 2 \frac{\partial^2 \psi_s}{\partial x \partial z} \right),
\]

\[
\frac{\partial}{\partial t} \left( \frac{\partial \psi'_s}{\partial x} - \frac{\partial \psi'_s}{\partial z} \right) = \frac{\partial}{\partial t} \left( \frac{\partial \psi_s}{\partial x} - \frac{\partial \psi_s}{\partial z} \right),
\]
\[
\begin{align*}
\frac{\partial}{\partial t} \left( \frac{\partial \phi_s}{\partial z} + \frac{\partial \psi_s}{\partial x} \right) &= \frac{\partial}{\partial t} \left( \frac{\partial \phi_s}{\partial z} + \frac{\partial \psi_s}{\partial x} \right), \\
\frac{\partial}{\partial t} \left( \frac{\partial \phi_s}{\partial z} + \frac{\partial \psi_s}{\partial x} \right) &= \frac{\partial}{\partial t} \left( \frac{\partial \phi_1}{\partial z} + \frac{\partial \psi_1}{\partial x} \right), \\
\frac{\partial}{\partial t} \left( \frac{\partial \phi_s}{\partial z} + \frac{\partial \psi_s}{\partial x} \right) &= \frac{\partial}{\partial t} \left( \frac{\partial \phi_2}{\partial z} + \frac{\partial \psi_2}{\partial x} \right), \\
\frac{\partial}{\partial t} \left( \frac{\partial \phi_s}{\partial z} + \frac{\partial \psi_s}{\partial x} \right) &= \frac{\partial}{\partial t} \left( \frac{\partial \phi_3}{\partial z} + \frac{\partial \psi_3}{\partial x} \right)
\end{align*}
\]

where

\[
\phi_1 = a_{11} + a_{12} + a_{13} - \frac{2}{3} \sigma_{11}, \quad \phi_2 = a_{21} + a_{22} + a_{23}, \quad \phi_3 = a_{31} + a_{32} + a_{33}
\]

Quantities \(\phi_1', \phi_2'\) and \(\phi_3'\) can be obtained by putting superscript \(t\) appropriately on the quantities appearing in \(\phi_1, \phi_2\) and \(\phi_3\).

**3.2.2 Incidence of a plane \(P\)–wave**

When a train of plane \(P\)–wave with amplitude \(a'_r\) traveling through the upper half-space, strikes the interface \(z = 0\) making an angle \(\theta_0\) with the normal to the interface, then the following reflection and transmission phenomena takes place.

![Figure 3.1: Schematic diagram](image)

*Reflected waves:* Three reflected \(P\)-waves and one reflected \(SV\)-wave, with amplitudes...
and \(a_3\) making angles \(\theta_1, \theta_2, \theta_3\) and \(\theta_4\), respectively, with the normal to the interface.

**Transmitted waves:** Three transmitted \(P\)-waves and one transmitted \(SV\)-wave, with amplitudes \(a_s, a_1, a_2\) and \(a_3\) making angles \(\gamma_s, \gamma_1, \gamma_2\) and \(\gamma_3\), respectively, with the normal to the interface.

The schematic diagram of the model is given in Figure 3.1.

We take the following form of appropriate potentials

In the upper porous half-space \(\Re_1\), we have

\[
\phi_j' = a_j' \exp \left\{ i\omega \left( \frac{V_j' \sin \theta_j + z \cos \theta_j}{V_j' - t} \right) \right\}, \quad j = 1, 2, 3 (3.6)
\]

\[
\phi_i' = a_i' \exp \left\{ i\omega \left( \frac{V_i' \sin \theta_i + z \cos \theta_i}{V_i' - t} \right) \right\}, \quad i = 1, 2 (3.7)
\]

\[
\phi_i' = a_i' \exp \left\{ i\omega \left( \frac{V_i' \sin \theta_i + z \cos \theta_i}{V_i' - t} \right) \right\}, \quad i = 1, 2 (3.8)
\]

The quantities \(V_1', V_2'\) and \(V_3'\) are the phase speeds of the three reflected \(P\)-waves and the quantity \(V_4'\) is the phase speed of the reflected \(SV\)-wave.

In the lower porous half-space \(\Re_2\), the potentials corresponding to the refracted waves are given by (2.24) and (2.25). These are re-written as follows

\[
\phi_j = a_j \exp \left\{ i\omega \left( \frac{V_j \sin \gamma_j + z \cos \gamma_j}{V_j} - t \right) \right\}, \quad j = s, 1, 2 (3.9)
\]

\[
\psi_s = a_s \exp \left\{ i\omega \left( \frac{V_s \sin \gamma_s + z \cos \gamma_s}{V_s} - t \right) \right\}, (3.10)
\]

where the quantities \(V_1, V_2, V_3\) and \(V_4\) are the phase speeds of three refracted \(P\)-waves, respectively, and the quantity \(V_4\) is phase speed of the refracted \(SV\)-wave. The phase speeds of the three \(P\)-waves and one \(SV\)-wave in the porous media \(\Re_1\) and \(\Re_2\) can be obtained from (1.139) and (1.151), respectively.

Substituting the potentials given by (3.6)-(3.10) into the boundary conditions (3.1)-(3.5) and making use of Snell’s law given by

\[
\frac{\sin \theta_0}{V_1'} = \frac{\sin \theta_1}{V_1'} = \frac{\sin \theta_2}{V_2'} = \frac{\sin \theta_3}{V_3'} = \frac{\sin \gamma_4}{V_4'} = \frac{\sin \gamma_1}{V_1} = \frac{\sin \gamma_2}{V_2} = \frac{\sin \gamma_3}{V_3} = \frac{\sin \gamma_4}{V_3},
\]

96
we obtain a non-homogeneous system of eight equations in eight unknowns. These equations can be written as

\[ \sum_{j=1}^{8} b_{ij} N_j = R_i, \quad i = 1, 2, 3, \ldots, 8. \]  

(3.11)

Here, \( N_1 = \frac{a'_s}{a'_0} \) and \( N_{k+1} = \frac{a'_s}{a'_0} \) \((k = 1, 2, 3)\) are the reflection coefficients, while \( N_5 = \frac{a_s}{a_0} \) and \( N_{k+5} = \frac{a_k}{a_0} \) \((k = 1, 2, 3)\) are the refraction coefficients due to an incident \( P \)-wave. And

\[
R_1 = \frac{(a^*)' \gamma'}{G'_{fr}} \cos^2 \theta_0, \quad R_2 = \sin \theta_0 \cos \theta_0, \quad R_3 = -\sin \theta_0,
\]

\[
R_4 = R_7 = R_8 = -\cos \theta_0, \quad R_5 = R_6 = 0, \quad a^* = a_{11} + a_{21} + a_{31} - \frac{2}{3} G_{fr}.
\]

The non-zero expressions for the coefficients \( b_{ij} \) of equation (3.11) are given as follows

\[
b_{11} = -\left( \frac{(a^*)' \gamma'}{G'_{fr}} + \cos^2 \theta_0 \right),
\]

\[
b_{12} = -\frac{(b^*)' V_{12}^2}{G'_{fr} V_{2}^2},
\]

\[
b_{13} = -\frac{(c^*)' V_{12}^2}{G'_{fr} V_{3}^2},
\]

\[
b_{14} = \sin \theta_0 \sqrt{\frac{V_{12}^2}{V_{4}^2} - \sin^2 \theta_0},
\]

\[
b_{15} = \left( \frac{a^*}{G_{fr}} + \frac{G_{fr}}{G'_{fr}} \right) \frac{V_{12}^2}{V_{1}^2} - \frac{G_{fr}}{G'_{fr}} \sin^2 \theta_0,
\]

\[
b_{16} = \frac{b^* V_{12}^2}{G'_{fr} V_{2}^2},
\]

\[
b_{17} = \frac{c^* V_{12}^2}{G'_{fr} V_{3}^2},
\]

\[
b_{18} = \frac{G_{fr}}{G'_{fr}} \sin \theta_0 \sqrt{\frac{V_{12}^2}{V_{4}^2} - \sin^2 \theta_0},
\]

\[
b_{21} = \sin \theta_0 \cos \theta_0,
\]

\[
b_{24} = \frac{V_{12}^2}{2 V_{4}^2} - \sin^2 \theta_0,
\]

97
\[
b_{25} = \frac{G_{fr}}{G_{fr}^2} \sin \theta_0 \sqrt{\frac{V_1^2}{V_1^2} - \sin^2 \theta_0},
\]
\[
b_{28} = -\frac{G_{fr}}{G_{fr}^2} \left( \frac{V_1^2}{2V_4^2} - \sin^2 \theta_0 \right).
\]
\[
b_{31} = \sin \theta_0,
\]
\[
b_{34} = \sqrt{\frac{V_1^2}{V_4^2} - \sin^2 \theta_0},
\]
\[
b_{35} = -b_{31},
\]
\[
b_{38} = \sqrt{\frac{V_1^2}{V_4^2} - \sin^2 \theta_0},
\]
\[
b_{41} = -\cos \theta_0,
\]
\[
b_{44} = b_{31},
\]
\[
b_{45} = -\sqrt{\frac{V_1^2}{V_4^2} - \sin^2 \theta_0},
\]
\[
b_{48} = b_{35},
\]
\[
b_{55} = -b_{45},
\]
\[
b_{56} = -\sqrt{\frac{V_1^2}{V_3^2} - \sin^2 \theta_0},
\]
\[
b_{68} = \left( \frac{(\rho_1)\omega}{C_{11} + (\rho_1)\omega} \right) \sin \theta_0,
\]
\[
b_{65} = b_{55},
\]
\[
b_{67} = -\sqrt{\frac{V_1^2}{V_3^2} - \sin^2 \theta_0},
\]
\[
b_{68} = \left( \frac{(\rho_2)\omega}{C_{22} + (\rho_2)\omega} \right) \sin \theta_0,
\]
\[
b_{71} = -b_{41},
\]
\[ b_{72} = \sqrt{\frac{V_1'^2}{V_2'^2} - \sin^2 \theta_0}, \]

\[ b_{74} = \left( \frac{\langle \rho_1' \rangle \omega}{C_1't + \langle \rho_1' \rangle \omega} \right) \sin \theta_0, \]

\[ b_{81} = b_{41}, \]

\[ b_{83} = \sqrt{\frac{V_1'^2}{V_3'^2} - \sin^2 \theta_0}, \]

\[ b_{84} = \left( \frac{\langle \rho_2' \rangle \omega}{C_2't + \langle \rho_2' \rangle \omega} \right) \sin \theta_0, \]

where

\[ b^* = a_{12} + a_{22} + a_{32}, \quad c^* = a_{13} + a_{23} + a_{33}. \]

The expressions of the quantities \((a^*)', (b^*)'\) and \((c^*)'\) for the upper medium can be obtained by putting the superscript \(t\) appropriately in the expressions of \(a^*, b^*\) and \(c^*\).

### 3.2.3 Incidence of a plane SV-wave

Assuming the same geometry as in the case of incident P-wave, we consider a train of plane SV-wave striking at the interface \(z = 0\) making an angle \(\theta_0\) with the normal to the interface. This will give rise to the same reflected and transmitted waves as considered in the case of incident P-wave. In this case, we shall take the following potentials in the upper porous half-space \(H_1\) as

\[ \phi_s' = a_s' \exp \left\{ \imath \omega \left( \frac{x \sin \theta_1 - z \cos \theta_1}{V_1'} - t \right) \right\}, \quad (3.12) \]

\[ \Psi' = a_{so}' \exp \left\{ \imath \omega \left( \frac{x \sin \theta_0 + z \cos \theta_0}{V_4'} - t \right) \right\} + a_3' \exp \left\{ \imath \omega \left( \frac{x \sin \theta_4 - z \cos \theta_4}{V_4'} - t \right) \right\}, \quad (3.13) \]

\[ \phi_i' = a_i' \exp \left\{ \imath \omega \left( \frac{x \sin \theta_{i+1} - z \cos \theta_{i+1}}{V_{i+1}'} - t \right) \right\}, \quad i = 1, 2, \quad (3.14) \]

where \(a_s', a_3'\) and \(a_{so}'\) are the amplitudes of incident and reflected SV-waves, respectively, while \(a_s', a_1'\) and \(a_2'\) are the amplitudes of three reflected P-waves at angles \(\theta_1, \theta_2\) and \(\theta_3\), respectively. The potentials in the lower porous half-space will be the same as given by (3.9) and (3.10). Substituting the expressions of potentials from (3.9), (3.10) and
(3.12)-(3.14) in the boundary conditions (3.1)-(3.5) and using Snell’s law given by

\[
\frac{\sin \theta_0}{V_4'} = \frac{\sin \theta_1}{V_1'} = \frac{\sin \theta_2}{V_2'} = \frac{\sin \theta_3}{V_3'} = \frac{\sin \gamma_1}{V_1} = \frac{\sin \gamma_2}{V_2} = \frac{\sin \gamma_3}{V_3} = \frac{\sin \gamma_4}{V_4},
\]

we find that the amplitude ratios \(\overline{N}_j\) satisfy the equations

\[
\sum_{j=1}^{8} \overline{b}_{ij} \overline{N}_j = \overline{R}_i, \quad i = 1, 2, 3, \ldots, 8. \tag{3.15}
\]

Here, the amplitude ratios given by \(\overline{N}_1 = \frac{a'_1}{a'_{so}}\) and \(\overline{N}_{k+1} = \frac{a'_k}{a'_{so}}\) \((k = 1, 2, 3)\) are the reflection coefficients, while the amplitude ratios given by \(\overline{N}_5 = \frac{a_2}{a'_{so}}\) and \(\overline{N}_{k+5} = \frac{a_k}{a'_{so}}\) are the refraction coefficients due to an incident SV-wave. And

\[
\overline{R}_1 = \sin \theta_0 \cos \theta_0, \quad \overline{R}_2 = -\cos 2\theta_0, \quad \overline{R}_3 = \cos \theta_0,
\]

\[
\overline{R}_4 = -\sin \theta_0, \quad \overline{R}_5 = 0, \quad \overline{R}_6 = 0, \quad \overline{R}_7 = \sin \theta_0, \quad \overline{R}_8 = -\sin \theta_0.
\]

The non-zero expressions for the coefficients \(\overline{b}_{ij}\) of equation (3.15) are given as follows

\[
\overline{b}_{11} = -\left(\frac{(a^*)' G'_{fr}}{G'_{fr} + 1}\right) \frac{V_4'^2}{V_4'^2} - \sin^2 \theta_0,
\]

\[
\overline{b}_{12} = -\frac{(b^*)' V_4'^2 G'_{fr}}{V_4'^2 G'_{fr}},
\]

\[
\overline{b}_{13} = -\frac{(c^*)' V_4'^2 G'_{fr}}{V_4'^2},
\]

\[
\overline{b}_{14} = \sin \theta_0 \cos \theta_0,
\]

\[
\overline{b}_{15} = \frac{a^*}{G'_{fr}} + \frac{G'_{fr}}{G'_{fr}} \frac{V_4'^2}{V_4'^2} - \frac{G'_{fr}}{G'_{fr}} \sin^2 \theta_0,
\]

\[
\overline{b}_{16} = \frac{(b^*)' V_4'^2}{G'_{fr} V_4'^2}, \quad \overline{b}_{17} = \frac{(c^*)' V_4'^2}{G'_{fr} V_4'^2},
\]

\[
\overline{b}_{18} = \frac{G'_{fr}}{G'_{fr}} \sin \theta_0 \sqrt{\frac{V_4'^2}{V_4'^2} - \sin^2 \theta_0},
\]

\[
\overline{b}_{21} = 2 \sin \theta_0 \sqrt{\frac{V_4'^2}{V_4'^2} - \sin^2 \theta_0},
\]

100
\[ \bar{b}_{14} = \cos 2\theta_0, \]
\[ \bar{b}_{25} = 2 \frac{G_{tr}}{G'_{tr}} \sin \theta_0 \sqrt{\frac{V_4^2}{V_1^2} - \sin^2 \theta_0}, \]
\[ \bar{b}_{28} = -\frac{G_{tr}}{G'_{tr}} \left( \frac{V_4^2}{V_2^2} - 2 \sin^2 \theta_0 \right), \]
\[ \bar{b}_{31} = \sin \theta_0, \]
\[ \bar{b}_{34} = \cos \theta_0, \]
\[ \bar{b}_{35} = -\bar{b}_{31}, \]
\[ \bar{b}_{38} = \sqrt{\frac{V_4^2}{V_2^2} - \sin^2 \theta_0}, \]
\[ \bar{b}_{41} = -\sqrt{\frac{V_4^2}{V_1^2} - \sin^2 \theta_0}, \]
\[ \bar{b}_{44} = \bar{b}_{31}, \]
\[ \bar{b}_{45} = -\sqrt{\frac{V_4^2}{V_2^2} - \sin^2 \theta_0}, \]
\[ \bar{b}_{48} = \bar{b}_{35}, \]
\[ \bar{b}_{55} = -\bar{b}_{45}, \]
\[ \bar{b}_{56} = -\sqrt{\frac{V_4^2}{V_2^2} - \sin^2 \theta_0}, \]
\[ \bar{b}_{98} = \left( \frac{\langle \rho_1 \rangle_\omega}{C_{1t} + \langle \rho_1 \rangle_\omega} \right) \sin \theta_0, \]
\[ \bar{b}_{65} = \bar{b}_{55}, \]
\[ \bar{b}_{17} = -\sqrt{\frac{V_4^2}{V_3^2} - \sin^2 \theta_0}, \]
\[ \bar{b}_{98} = \left( \frac{\langle \rho_2 \rangle_\omega}{C_{2t} + \langle \rho_2 \rangle_\omega} \right) \sin \theta_0, \]
\[ \bar{b}_{71} = \bar{b}_{41}, \]
\[ \bar{b}_{72} = \sqrt{\frac{V_4'^2}{V_2'^2} - \sin^2 \theta_0}, \]
\[ \bar{b}_{74} = \left( \frac{\langle \rho \gamma \rangle \omega}{C_4'^4 + \langle \rho \gamma \rangle \omega} \right) \sin \theta_0, \]
\[ \bar{b}_{81} = \bar{b}_{46}, \]
\[ \bar{b}_{83} = \sqrt{\frac{V_4'^2}{V_3'^2} - \sin^2 \theta_0}, \]
\[ \bar{b}_{84} = \left( \frac{\langle \rho \gamma \rangle \omega}{C_3'^4 + \langle \rho \gamma \rangle \omega} \right) \sin \theta_0. \]

Now, we consider the partitioning of incident energy between different reflected and transmitted waves. The rate of energy communicated per unit area denoted by \( P_p^* \) is the scalar product of the surface traction and the particle velocity. The expression for \( P_p^* \) at the interface is given by equation (2.28). Using the appropriate potentials given by (3.6)-(3.10), we obtain the expressions of energy ratios \( E_i (i = 1, 2, \ldots 8) \) corresponding to various reflected and refracted waves. These energy ratios give the time rate of average energy transmission for the respective wave to that of incident P-wave. The energy ratios \( E_1, E_2, E_3 \) and \( E_4 \) correspond to the reflected three P-waves and one SV-wave, respectively, and the energy ratios \( E_5, E_6, E_7 \) and \( E_8 \) correspond to the refracted three P-waves and one SV-wave, respectively. Their expressions are given as follows

\[ E_1 = -N_1^2, \]
\[ \Delta_1 E_2 = -N_2^2 a_{22} V_2'^2 \sqrt{\frac{V_1'^2}{V_2'^2} - \sin^2 \theta_0}, \]
\[ \Delta_1 E_3 = -N_3^2 a_{30} V_3'^2 \sqrt{\frac{V_1'^2}{V_3'^2} - \sin^2 \theta_0}, \]
\[ \Delta_1 E_4 = -N_4^2 G_f r V_4'^2 \sqrt{\frac{V_1'^2}{V_4'^2} - \sin^2 \theta_0}, \]
\[ \Delta_1 E_5 = N_5^2 \left( a_{11} + \frac{G_f r}{3} \right) \frac{V_1'^2}{V_1'^2} \sqrt{\frac{V_1'^2}{V_1'^2} - \sin^2 \theta_0}, \]
\[ \Delta_1 E_6 = N_6^2 a_{22} \frac{V_2'^2}{V_2'^2} \sqrt{\frac{V_1'^2}{V_2'^2} - \sin^2 \theta_0}. \]
\[ \Delta_1 E_1 = N_1^2 a_{33} \frac{V_3^2}{V_1^2} \sqrt{\frac{V_1^2}{V_3^2} - \sin^2 \theta_0}, \]
\[ \Delta_1 E_8 = N_8^2 G_f \frac{V_4^2}{V_4^2} \sqrt{\frac{V_4^2}{V_4^2} - \sin^2 \theta_0}, \]

where \( \Delta_1 = (a_{11} + \frac{G_{f_r}}{3}) \cos \theta_0. \)

Similarly, the expressions of energy ratios \( E_i, (i = 1, 2, ..., 8) \) corresponding to various reflected and transmitted waves in case of the incident \( SV \)-wave can be obtained and are given as follows

\[ \Delta_2 E_1 = -2N_1^2 (a_{11} + \frac{G_{f_r}}{3}) \frac{V_4^2}{V_1^2} \sqrt{\frac{V_1^2}{V_4^2} - \sin^2 \theta_0}, \]
\[ \Delta_2 E_2 = -2N_2^2 a_{22} \frac{V_4^2}{V_2^2} \sqrt{\frac{V_2^2}{V_4^2} - \sin^2 \theta_0}, \]
\[ \Delta_2 E_3 = -2N_3^2 a_{31} \frac{V_4^2}{V_3^2} \sqrt{\frac{V_3^2}{V_4^2} - \sin^2 \theta_0}, \]
\[ \Delta_2 E_4 = -N_3^2, \]
\[ \Delta_2 E_5 = 2N_5^2 (a_{11} + \frac{G_{f_r}}{3}) \frac{V_4^2}{V_1^2} \sqrt{\frac{V_1^2}{V_4^2} - \sin^2 \theta_0}, \]
\[ \Delta_2 E_6 = 2N_6^2 a_{22} \frac{V_4^2}{V_2^2} \sqrt{\frac{V_2^2}{V_4^2} - \sin^2 \theta_0}, \]
\[ \Delta_2 E_7 = 2N_7^2 a_{33} \frac{V_4^2}{V_3^2} \sqrt{\frac{V_3^2}{V_4^2} - \sin^2 \theta_0}, \]
\[ \Delta_2 E_8 = N_8^2 G_f \frac{V_4^2}{V_4^2} \sqrt{\frac{V_4^2}{V_4^2} - \sin^2 \theta_0}, \]

where \( \Delta_2 = G_{f_r} \cos \theta_0. \)

### 3.3 Special cases

(a) To reduce the problem at a plane interface between two different porous elastic half-spaces, each of them saturated by a single fluid, we shall neglect the presence of one fluid from each half-space. For this, we take \( S_1 = A_2 = 0 \) and \( S_1' = A_2' = 0, \)
which implies $a_{12} = a_{22} = a_{32} = 0$ as well as $a'_{12} = a'_{22} = a'_{32} = 0$ [see Tuncay and Corapcioglu, (1997)]. In this case, the boundary conditions $\hat{u}_{sz} = \hat{u}_{iz}$ and $\hat{u}'_{sz} = \hat{u}'_{iz}$ are meaningless and not required. The remaining six boundary conditions yield a system of six non-homogeneous equations in six unknowns. This system of equations can be written in matrix form as

$$TL = Q,$$

where the non-zero elements $t_{ij}$ and $q_i$ of the coefficient matrix $T$ and the column matrix $Q$, respectively, are given as follows

In the case of incident $P$-wave,

$$t_{11} = -\left(\frac{a'_{11} + a'_{31}}{G'_{fr}} - \frac{2}{3}\right) + \cos^2 \theta_0,$$

$$t_{12} = -\frac{(a'_{13} + a'_{33}) V_{1}^2}{G'_{fr} V_{3}^2},$$

$$t_{13} = \sin \theta_0 \sqrt{\frac{V_{1}^2}{V_{4}^2} - \sin^2 \theta_0},$$

$$t_{14} = \left(\frac{a_{11} + a_{31}}{G'_{fr}} + \frac{G_{fr}}{3G'_{fr}}\right) \frac{V_{1}^2}{V_{3}^2} - \frac{G_{fr}}{G'_{fr}} \sin^2 \theta_0,$$

$$t_{15} = \frac{(a_{13} + a_{33}) V_{1}^2}{G'_{fr} V_{3}^2},$$

$$t_{16} = \frac{G_{fr}}{G'_{fr}} \sin \theta_0 \sqrt{\frac{V_{1}^2}{V_{4}^2} - \sin^2 \theta_0},$$

$$t_{21} = \sin 2\theta_0,$$

$$t_{23} = \frac{V_{1}^2}{V_{4}^2} - 2 \sin^2 \theta_0,$$

$$t_{24} = 2 \frac{G_{fr}}{G'_{fr}} \sin \theta_0 \sqrt{\frac{V_{1}^2}{V_{4}^2} - \sin^2 \theta_0},$$

$$t_{26} = -2 \frac{G_{fr}}{G'_{fr}} \left(\frac{V_{1}^2}{V_{4}^2} - 2 \sin^2 \theta_0\right),$$

$$t_{31} = \sin \theta_0,$$

$$t_{33} = \sqrt{\frac{V_{1}^2}{V_{4}^2} - \sin^2 \theta_0},$$

104
\[ t_{34} = -t_{31}, \]
\[ t_{36} = \sqrt{\frac{V_1^2}{V_2^2}} - \sin^2 \theta_0, \]
\[ t_{41} = -\cos \theta_0, \]
\[ t_{43} = t_{31}, \]
\[ t_{44} = -\sqrt{\frac{V_1^2}{V_2^2}} - \sin^2 \theta_0, \]
\[ t_{46} = t_{34}, \]
\[ t_{54} = -t_{44}, \]
\[ t_{55} = -\sqrt{\frac{V_1^2}{V_3^2}} - \sin^2 \theta_0, \]
\[ t_{56} = \left(\frac{(\rho_2)\omega}{C_{23} + (\rho_2)\omega}\right) \sin \theta_0, \]
\[ t_{61} = t_{41}, \]
\[ t_{62} = \sqrt{\frac{V_2^2}{V_3^2}} - \sin^2 \theta_0, \]
\[ t_{63} = t_{56}, \]
\[ q_i = \frac{a_{11} + a_{31}}{G'_{fr}} - \frac{2}{3} \cos^2 \theta_0, \quad q_2 = \sin 2\theta_0, \quad q_3 = -\sin \theta_0, \quad q_4 = q_6 = -\cos \theta_0, \quad q_5 = 0. \]

In the case of an incident SV-wave, the non-zero elements of matrices \( t_{ij} \) and \( q_i \) are given by
\[
\begin{align*}
t_{11} &= -\left(\frac{a'_{11} + a'_{31}}{G'_{fr}} + \frac{1}{3}\right) \frac{V_4^2}{V_1^2} - \sin^2 \theta_0, \\
t_{12} &= -\left(\frac{a'_{13} + a'_{33}}{G'_{fr}}\right) \frac{V_4^2}{V_3^2}, \\
t_{13} &= \sin \theta_0 \cos \theta_0,
\end{align*}
\]
$$\begin{align*}
t_{14} &= \left( \frac{a_{11} + a_{31}}{G'_{fr}} + \frac{G_{fr}}{3G'_{fr}} \right) \frac{V'_{1}^{2}}{V_{1}^{2}} - \frac{G_{fr}}{G'_{fr}} \sin^2 \theta_0, \\
t_{15} &= \frac{(a_{11} + a_{33}) V'_{4}^{2}}{G'_{fr} V_{3}^{2}}, \\
t_{16} &= \frac{G_{fr} \sin \theta_0}{G'_{fr}} \sqrt{\frac{V'_{4}^{2}}{V_{4}^{2}} - \sin^2 \theta_0}, \\
t_{21} &= 2 \sin \theta_0 \sqrt{\frac{V'_{4}^{2}}{V_{4}^{2}} - \sin^2 \theta_0}, \\
t_{23} &= \cos 2 \theta_0, \\
t_{24} &= 2 \frac{G_{fr}}{G'_{fr}} \sin \theta_0 \sqrt{\frac{V'_{4}^{2}}{V_{4}^{2}} - \sin^2 \theta_0}, \\
t_{26} &= \frac{G_{fr}}{G'_{fr}} \left( \frac{V'_{4}^{2}}{V_{4}^{2}} - 2 \sin^2 \theta_0 \right), \\
t_{31} &= \sin \theta_0, \\
t_{33} &= \cos \theta_0, \\
t_{34} &= -t_{31}, \\
t_{36} &= \sqrt{\frac{V'_{4}^{2}}{V_{4}^{2}} - \sin^2 \theta_0}, \\
t_{41} &= -\sqrt{\frac{V'_{4}^{2}}{V_{4}^{2}} - \sin^2 \theta_0}, \\
t_{43} &= t_{31}, \\
t_{44} &= -\sqrt{\frac{V'_{4}^{2}}{V_{4}^{2}} - \sin^2 \theta_0}, \\
t_{46} &= t_{34}, \\
t_{54} &= \sqrt{\frac{V'_{4}^{2}}{V_{4}^{2}} - \sin^2 \theta_0}, \\
\end{align*}$$
(b) In the absence of both the fluids in the upper porous half-space, the problem reduces to the problem of reflection and transmission of elastic waves at a plane interface between an elastic half-space and a porous half-space saturated by two immiscible fluids. To neglect the presence of both the fluids from the upper half-space, we set $S_1 = S_2 = A_2 = 0$. This implies $a_{12} = a_{22} = a_{13} = a_{33} = 0$. In this case, both the fluids are evacuated from the media and hence, the boundary conditions $\dot{u}_{1z} - \dot{u}_{2z} = 0$ and $\ddot{u}_{1z} - \ddot{u}_{2z} = 0$ are meaningless. The remaining six boundary conditions yield a system of six non homogeneous equations in six unknowns. These equations can be written in matrix form as

$$\bar{T} \bar{L} = \bar{Q}.$$ 

The elements $\bar{t}_{ij}$ and $\bar{q}_{ij}$ of the coefficient matrix $\bar{T}$ and column matrix $\bar{Q}$ are given as follows.

For incident $P$-wave:

$$\bar{t}_{11} = - \left( \frac{a_{11}'}{G'_{fr}} - \frac{2}{3} \right) + 2 \cos^2 \theta_0,$$

$$\bar{t}_{12} = \sin \theta_0 \sqrt{\frac{V_1'^2}{V_1^2} - \sin^2 \theta_0},$$

$$\bar{t}_{13} = \left( \frac{a^*}{G'_{fr}} + \frac{G_{fr}}{G'_{fr}} \right) \frac{V_1'^2}{V_1^2} - \frac{G_{fr}}{G'_{fr}} \sin^2 \theta_0,$$

$$\bar{t}_{14} = \bar{q}_{1z} \frac{V_1'^2}{G'_{fr} V_1^2}.$$
\[ \dot{r}_{15} = \frac{G_{fr} V_0^2}{C_{fr} V_0^2}, \]
\[ \dot{r}_{16} = \frac{G_{fr}}{C_{fr}} \sin \theta_0 \sqrt{\frac{V_0^2}{V_0^2} - \sin^2 \theta_0}, \]
\[ \dot{r}_{21} = \sin 2\theta_0, \]
\[ \dot{r}_{22} = \frac{V_1^2}{V_4^2} - 2 \sin^2 \theta_0, \]
\[ \dot{r}_{23} = 2G_{fr} \sin \theta_0 \sqrt{\frac{V_1^2}{V_4^2} - \sin^2 \theta_0}, \]
\[ \dot{r}_{26} = -\frac{G_{fr}}{C_{fr}} \left( \frac{V_1^2}{V_4^2} - 2 \sin^2 \theta_0 \right), \]
\[ \dot{r}_{31} = \sin \theta_0, \]
\[ \dot{r}_{32} = \sqrt{\frac{V_1^2}{V_4^2} - \sin^2 \theta_0}, \]
\[ \dot{r}_{33} = -\sin \theta_0, \]
\[ \dot{r}_{36} = \sqrt{\frac{V_1^2}{V_4^2} - \sin^2 \theta_0}, \]
\[ \dot{r}_{41} = \cos \theta_0, \]
\[ \dot{r}_{42} = -\sin \theta_0, \]
\[ \dot{r}_{43} = \sqrt{\frac{V_1^2}{V_4^2} - \sin^2 \theta_0}, \]
\[ \dot{r}_{46} = \sin \theta_0, \]
\[ \dot{r}_{53} = \sqrt{\frac{V_1^2}{V_4^2} - \sin^2 \theta_0}, \]
\[ \dot{r}_{54} = -\sqrt{\frac{V_1^2}{V_4^2} - \sin^2 \theta_0}, \]
\[ \dot{r}_{56} = \left( \frac{< \rho_1 > \omega}{iC_1 + < \rho_1 > \omega} \right) \sin \theta_0, \]
\[ \tilde{t}_{63} = \sqrt{\frac{V_f^2}{V_i^2} - \sin^2 \theta_0}, \]

\[ \tilde{t}_{65} = -\sqrt{\frac{V_f^2}{V_i^3} - \sin^2 \theta_0}, \]

\[ \tilde{t}_{66} = \left( \frac{<\rho_2 > \omega}{C_f + <\rho_2 > \omega} \right) \sin \theta_0, \]

\[ \tilde{q}_1 = \frac{\alpha t_{11}}{G_{fr}} - \frac{2}{3} + \cos^2 \theta_0, \quad \tilde{q}_2 = \sin 2\theta_0, \quad \tilde{q}_3 = -\sin \theta_0, \quad \tilde{q}_4 = \cos \theta_0, \quad \tilde{q}_5 = \tilde{q}_6 = 0. \]

For incident SV-wave:

\[ \tilde{t}_{11} = -\left( \frac{\alpha t_{11}}{G_{fr}} + \frac{1}{3} \right) \frac{V_{i2}^2}{V_i^2} + \sin^2 \theta_0, \]

\[ \tilde{t}_{12} = \sin \theta_0 \cos \theta_0, \]

\[ \tilde{t}_{13} = \left( \frac{\alpha^*}{G_{fr}} + \frac{G_{fr}}{C_{fr}^2} \right) \frac{V_{i4}^2}{V_i^2} - \frac{G_{fr}}{C_{fr}^2} \sin^2 \theta_0, \]

\[ \tilde{t}_{14} = \frac{b V_{i4}^2}{C_{fr} V_i^2}, \]

\[ \tilde{t}_{15} = \frac{c V_{i4}^2}{C_{fr} V_i^2}, \]

\[ \tilde{t}_{16} = \frac{G_{fr} \sin \theta_0}{C_{fr}} \sqrt{\frac{V_{i4}^2}{V_i^2} - \sin^2 \theta_0}, \]

\[ \tilde{t}_{21} = 2 \sin \theta_0 \sqrt{\frac{V_{i4}^2}{V_i^2} - \sin^2 \theta_0}, \]

\[ \tilde{t}_{22} = \cos 2\theta_0, \]

\[ \tilde{t}_{23} = \frac{2 G_{fr} \sin \theta_0}{C_{fr}} \sqrt{\frac{V_{i4}^2}{V_i^2} - \sin^2 \theta_0}, \]

\[ \tilde{t}_{26} = -\frac{G_{fr}}{C_{fr}} \left( \frac{V_{i4}^2}{V_i^2} - 2 \sin^2 \theta_0 \right), \]

\[ \tilde{t}_{31} = \sin \theta_0, \]

\[ \tilde{t}_{32} = \cos \theta_0, \]
\[ \begin{align*}
\tilde{t}_{33} &= -\sin \theta_0, \\
\tilde{t}_{36} &= \sqrt{\frac{V_4^2}{V_1^2}} - \sin^2 \theta_0, \\
\tilde{t}_{41} &= -\sqrt{\frac{V_4^2}{V_1^2}} - \sin^2 \theta_0, \\
\tilde{t}_{42} &= \sin \theta_0, \\
\tilde{t}_{43} &= -\sqrt{\frac{V_4^2}{V_1^2}} - \sin^2 \theta_0, \\
\tilde{t}_{46} &= -\sin \theta_0, \\
\tilde{t}_{53} &= \sqrt{\frac{V_4^2}{V_1^2}} - \sin^2 \theta_0, \\
\tilde{t}_{54} &= -\sqrt{\frac{V_4^2}{V_1^2}} - \sin^2 \theta_0, \\
\tilde{t}_{56} &= \frac{\langle \rho_1 > \omega}{iC_1 + < \rho_1 > \omega} \sin \theta_0, \\
\tilde{t}_{63} &= \sqrt{\frac{V_4^2}{V_1^2}} - \sin^2 \theta_0, \\
\tilde{t}_{65} &= -\sqrt{\frac{V_4^2}{V_1^2}} - \sin^2 \theta_0, \\
\tilde{t}_{66} &= \frac{\langle \rho_2 > \omega}{iC_2 + < \rho_2 > \omega} \sin \theta_0.
\end{align*} \]

and
\[ q_1 = \sin \theta_0 \cos \theta_0, \quad q_2 = \cos 2\theta_0, \quad q_3 = \cos \theta_0, \quad q_4 = -\sin \theta_0, \quad q_5 = q_6 = 0. \]

One can see that these coefficients match with the coefficients obtained for the corresponding problem in chapter 2.
### 3.4 Numerical results and discussion

In order to study the dependence of amplitude and energy ratios on the angle of incidence, we perform numerical computations by taking the following values of relevant elastic parameters.

In the upper porous medium:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk modulus of the upper porous medium</td>
<td>$K_{fr}$</td>
<td>$0.75 \times 10^{11}$ dyne/cm$^2$</td>
</tr>
<tr>
<td>Bulk modulus of the lower porous medium</td>
<td>$K_{fr}$</td>
<td>$0.40 \times 10^{11}$ dyne/cm$^2$</td>
</tr>
<tr>
<td>Bulk modulus of solid phase in upper medium</td>
<td>$K_s$</td>
<td>$2.0 \times 10^{11}$ dyne/cm$^2$</td>
</tr>
<tr>
<td>Bulk modulus of solid phase in lower medium</td>
<td>$K_s$</td>
<td>$0.3 \times 10^{11}$ dyne/cm$^2$</td>
</tr>
<tr>
<td>Bulk modulus of fluid phase 1 in upper medium</td>
<td>$K_1'$</td>
<td>$0.2475 \times 10^{11}$ dyne/cm$^2$</td>
</tr>
<tr>
<td>Bulk modulus of fluid phase 1 in lower medium</td>
<td>$K_1$</td>
<td>$0.1375 \times 10^{11}$ dyne/cm$^2$</td>
</tr>
<tr>
<td>Bulk modulus of fluid phase 2 in upper medium</td>
<td>$K_2'$</td>
<td>$0.2256 \times 10^{11}$ dyne/cm$^2$</td>
</tr>
<tr>
<td>Bulk modulus of fluid phase 2 in lower medium</td>
<td>$K_2$</td>
<td>$0.1156 \times 10^{11}$ dyne/cm$^2$</td>
</tr>
<tr>
<td>Shear modulus of solid matrix in upper medium</td>
<td>$G_{fr}$</td>
<td>$2.0 \times 10^{11}$ dyne/cm$^2$</td>
</tr>
<tr>
<td>Shear modulus of solid matrix in lower medium</td>
<td>$G_{fr}$</td>
<td>$0.35 \times 10^{11}$ dyne/cm$^2$</td>
</tr>
<tr>
<td>Density of solid phase in upper medium</td>
<td>$\rho_s$</td>
<td>2.34 gm/cm$^3$</td>
</tr>
<tr>
<td>Density of solid phase in lower medium</td>
<td>$\rho_s$</td>
<td>2.6 gm/cm$^3$</td>
</tr>
<tr>
<td>Density of fluid phase 1 in upper medium</td>
<td>$\rho_1$</td>
<td>1.25 gm/cm$^3$</td>
</tr>
<tr>
<td>Density of fluid phase 1 in lower medium</td>
<td>$\rho_1$</td>
<td>0.82 gm/cm$^3$</td>
</tr>
<tr>
<td>Density of fluid phase 2 in upper medium</td>
<td>$\rho_2$</td>
<td>1.5 gm/cm$^3$</td>
</tr>
<tr>
<td>Density of fluid phase 2 in lower medium</td>
<td>$\rho_2$</td>
<td>0.92 gm/cm$^3$</td>
</tr>
<tr>
<td>Parameter in upper medium</td>
<td>$\alpha'_s$</td>
<td>0.06</td>
</tr>
<tr>
<td>Parameter in lower medium</td>
<td>$\alpha'_s$</td>
<td>0.08</td>
</tr>
<tr>
<td>Parameter in upper medium</td>
<td>$\alpha'_t$</td>
<td>0.03</td>
</tr>
<tr>
<td>Parameter in lower medium</td>
<td>$\alpha'_t$</td>
<td>0.04</td>
</tr>
</tbody>
</table>

and \(d\rho/dS = 0.3\). The relevant phase speeds of longitudinal and transverse waves in the two porous half-spaces are computed as
Moreover, we shall assume that both the porous half-spaces contain two immiscible non-viscous fluids. This leads to $C_1 = C_2 = C'_1 = C'_2 = 0$. Equations (3.11) and (3.15) are solved using Gauss elimination method through FORTRAN programming. The absolute values of reflection and transmission coefficients for an incident $P-$ wave and for an incident $SV-$ wave are computed against the angle of incidence. Figures 3.2 and 3.3 depict the variation of the modulus of amplitude ratios in the range $0^\circ < \theta_0 < 90^\circ$, when a $P$-wave is made incident.

We observe that the nature of dependence of different amplitude ratios is different
at different angles of incidence. The value of amplitude ratio $N_1$ has value 0.66 near normal incidence, which decreases with $\theta_0$ and approaches towards zero as $\theta_0 \rightarrow 75^\circ$. Beyond this, its value increases sharply and approaches to the value 1 as $\theta_0 \rightarrow 90^\circ$. The value of the amplitude ratios $N_2$ and $N_3$ are found to be very small near normal incidence, they decrease with further increase of the angle of incidence and remain almost zero at and after $\theta_0 = 67^\circ$. The amplitude ratio of reflected $SV$-wave is zero at $\theta_0 = 0^\circ$ and thereafter, it increases with angle of incidence up to a certain angle, beyond which its value starts decreasing and becomes zero at $\theta_0 = 90^\circ$. It is clear from Figure 3.3 that no transmission of waves occurs at grazing incidence. The amplitude ratios corresponding to all the transmitted waves are found to decrease with the angle of incidence, in general, except the amplitude ratio corresponding to the transmitted transverse wave. The amplitude ratio of transmitted transverse wave is zero at normal incidence, it then increases with $\theta_0$ achieving a certain maximum value, thereafter, it decreases and approaches to zero as $\theta_0$ approaches to $90^\circ$.

The variations of energy ratios of various reflected and transmitted waves with angle of incidence are shown in Figures 3.4 and 3.5, when a $P$-wave is made incident.

![Figure 3.4: Variations of modulus of energy ratios with angle of incidence of longitudinal wave](image1)

![Figure 3.5: Variations of modulus of energy ratios with angle of incidence of longitudinal wave](image2)

We notice that the energy ratio corresponding to transmitted $SV$-wave dominates in the range $55^\circ < \theta_0 < 85^\circ$. Outside this range, the energy ratio corresponding to reflected $P$-wave dominates.
Figure 3.6-3.7 depict the variation in amplitude ratios of various reflected and transmitted waves when SV-wave is made incident. In this case, the angle $\theta_0 = 60^\circ$ is found to be the critical angle.

Figure 3.6 reveals that the value of amplitude ratio $\bar{N}_1$ is almost zero near normal incidence and it goes on increasing with the increase in $\theta_0$. At the critical angle $\theta_0 = 60^\circ$, the value of the amplitude ratio $\bar{N}_1$ is found to be nearly unity. The values of the amplitude ratios $\bar{N}_2$ and $\bar{N}_3$ are almost zero near normal incidence, i.e., at an angle $\theta_0 = 0^\circ$. Their values increase very slightly with increase of the angle $\theta_0$. The value of the amplitude ratio $\bar{N}_4$ is found to be 0.39 near the normal incidence, it then decreases very slightly with increase of $\theta_0$ till 35° angle of incidence, thereafter, its value increases and goes to the value 0.70 at $\theta = 60^\circ$ angle of incidence. It can be observed that the value of the amplitude ratio $\bar{N}_4$ is greater than that of the amplitude ratio $\bar{N}_1$ up to $\theta_0 = 47^\circ$ angle of incidence, after which the value of the amplitude ratio $\bar{N}_1$ becomes greater than that of the amplitude ratio $\bar{N}_4$. Figure 3.7 reveals that the values of the amplitude ratios $\bar{N}_5, \bar{N}_6$ and $\bar{N}_7$ corresponding to the transmitted longitudinal waves are very small in the entire range of $\theta_0$. The value of the amplitude ratio $\bar{N}_8$ is maximum near normal incidence and its value decreases monotonically with increase in the angle of incidence.

Figures 3.8 and 3.9 show the variation of the energy ratios $E_i$ of various reflected
and transmitted waves with the angle of incidence.

**Figure 3.8**: Variations of modulus of energy ratios with angle of incidence of transverse wave

**Figure 3.9**: Variations of modulus of energy ratios with angle of incidence of transverse wave

The energy conversion in different ranges of the angle of incidence is clearly noticed. In these figures, some variations are shown after magnifying their original values as they were very small in comparison to other variations. It is verified that algebraic sum of energy ratios is unity at each angle of incidence. This means that there is no dissipation of energy during transmission of waves at the interface.

### 3.5 Conclusions

Phenomena of reflection and transmission of waves at plane interface between two dissimilar porous half-spaces is studied. It is assumed that two porous half-spaces are in perfect contact and the pores of two half-spaces are sealed, i.e., no fluid moves from one half-space to another half-space. A plane elastic wave (P/SV) is assumed to strike at the interface between the two half-spaces after propagating through an upper porous half-space. It is concluded that

(I) Amplitude and energy ratios of all the reflected and transmitted waves are found to depend upon the angle of incidence.

(II) When a P-wave is made incident at 90°, no reflected or transmitted wave appears except one reflected longitudinal wave.
(III) In case of incidence of $SV$–wave, $\theta_0 = 60^0$ is found to be the critical angle.

(IV) At normal incidence of $SV$–wave, reflection and transmission of only $SV$–wave takes place.

(V) No dissipation of energy takes place at the interface during transmission.