

CHAPTER 4

HANDEL'S THEORY OF 1/f NOISE

Handel's theory of 1/f noise is also called as the quantum 1/f noise theory. This theory is the culmination of a series of efforts to develop a fundamental and universal theory of 1/f noise by Handel and co-workers since 1961.

Applications of quantum Electro-Dynamics(QED) and perturbation theory calculations [1] lead to

$$\text{Elastic Cross-section}_{exp} = \text{Elastic Cross section}_{ela} + \text{Elastic cross-section}_{soft}$$

in turn the soft elastic cross-section was found to be the product of a cross section of lowest order perturbation found proportional to an integral function 'B' defined as

$$B = -1/(2\pi)^2 \int_{0 \leq |K| \leq \Delta E}^{\infty} \delta K / \omega [p^l / p^{lk} - p / p^k]^2 \quad \dots \quad (4.1)$$

where the symbols have their usual meaning as in Ref 1, with K having its upper and lower limits as ΔE and 0 respectively. The integral is related to our discussion. This integrand introduces a logarithmic divergence at the lower limit of integration. This divergence is the infrared divergence valid for photon energy $\omega \leq \Delta E$. According to Bloch-Nordsieck theorem,[2] for all processes in QED, the infrared divergences cancel exactly to all orders of perturbation theory calculations, leaving finite radiative corrections of order α . These corrections are known as infrared radiative corrections. According to Handel, Quantum 1/f effect in a cross-section is a generalisation of infrared radiative corrections[1]. These depend on time and exhibit fluctuations with a 1/f spectral density.

Thus Handel's theory is based on linking charged particle motion to infrared quanta emitted as bremsstrahlung. 1/f fluctuations in particle currents are produced by the self interference between components differing very slightly in their energies. The spectral density of

these fluctuations were found to be [3] $\alpha A / N f^{(1-\alpha A)}$ where αA is the infrared exponent also known as bremsstrahlung coefficient and defined as

$$\alpha A = (2\alpha / 3\pi) (\Delta V/c)^2 \ll 1 \quad \dots \quad (4.2)$$

where ΔV is the change in the velocity vector of the particles during the scattering process considered, α is the fine structure constant, c is the velocity of light and N is the number of carriers defining the current.

Also any current with weak infrared divergent coupling to massless infra quanta such as photons, phonons etc., will exhibit quantum fluctuations in time with a $1/f$ spectral density[3]. Handel's general $1/f$ noise principle as enunciated by him at the 1977 Tokyo conference reads "whenever infrared divergences are introduced into conventional perturbation theory by emission of massless particles, the currents coupled to these particles will exhibit macroscopic quantum fluctuations with a $1/f$ type power spectrum at sufficiently low frequencies".

4.1 ELEMENTARY AND QUANTITATIVE DERIVATION OF QUANTUM $1/f$ EFFECT[12]

The basic principles of the new branch of infra quantum physics being difficult to comprehend, Handel first described the quantum $1/f$ effect based on intuitive pictures such as a beam of charged particles getting scattered under the influence of a potential. Here one should note that during such a scattering, the particles in the beam are very small angle scattered and a bremsstrahlung will be emitted. Bremsstrahlung means emission of massless infra quanta which leads to an infrared divergence. The following derivation of quantum $1/f$ effect is based on Schrodinger's statistical model without using second quantisation.

The incoming beam of electrons is described by a wave function $\exp [(i/n)(p \cdot r - Et)]$. The scattered beam will consist of a bremsstrahlung part with energy loss and amplitude $a b_T(\epsilon)$ and a large non-bremsstrahlung part of amplitude 'a'. The scattered beam can be described by the wave equation

$$\Psi_T = \exp [(i/n)(p \cdot r - Et)] a \left(1 + \int_0^\Lambda b_T(\epsilon) e^{\epsilon \cdot r/n} d\epsilon \right) \quad \dots \quad (4.3)$$

Here $b_T(\epsilon) = |b_T(\epsilon)| e^{i\pi}$

where $|b_r(\varepsilon)|$ is the matrix element for scattering with energy loss ε and γ_ε represents the random phase implying incoherence of all bremsstrahlung parts. The bremsstrahlung energy loss ε ranges from some resolutions threshold to an upper limit $E \geq \Lambda$, where E is the kinetic energy of the electrons. The superscript T indicates that Eq.(4.1) represents only a sample duration $T > f_0^{-1}$ of the Schrodinger field of the scattered wave. Here f is the lowest frequency of the emitted infraquanta measured.

The particle density in the scattered Schrodinger field is and is $|\Psi_r|^2$ ultimately simplified as

$$|\Psi_r|^2 = a^2 \left(1 + \int_{\varepsilon_0}^{\Lambda} |b_r(\varepsilon)|^2 d\varepsilon \right) \quad \dots \quad (4.4)$$

Thus the autocorrelation function for the probability density can be derived [4]. The particle concentration fluctuation, its correlation function, the current density and its fluctuations, and the spectral density of fractional probability density fluctuations are deduced.

$$\delta j(t) \delta j(t+T) = 2 |a|^4 (p/m)^2 \int_{\varepsilon_0}^{\Lambda} |b_r(\varepsilon)|^2 \cos(\varepsilon t / T) d\varepsilon \quad \dots \quad (4.5)$$

From Weinner Khintchine's theorem, the Fourier transform of the autocorrelation function of the particle current density gives the spectral density of the current density fluctuation S_j ,

$$S_j = 2h (p/m)^2 |a|^4 |b_r(\varepsilon)|^2 \quad \text{for } \varepsilon_0 < \varepsilon = hf < \Lambda \quad \dots \quad (4.6)$$

the spectral density of the current density fluctuations is

$$S_j = 2h (p/m)^2 a^4 \propto A / f \quad \dots \quad (4.7)$$

The resulting spectral density of fractional probability density fluctuations is obtained by dividing with $|\Psi|^4$

$$|\Psi|^4 = S_{|\Psi|^2}(f)^2 \cong 2 \propto A / f \quad \dots \quad (4.8)$$

where again "noise of noise" being very small has been neglected. The quantum $1/f$ noise contribution of each carrier is independent and therefore the quantum $1/f$ noise from N carriers N times larger and so is the current j . For the case in which the cross-section fluctuation is observed on N carriers simultaneously, the spectral density of the fractional current density fluctuations is,

$$j^2 S_j = 2 \propto A / fN \quad \dots \quad (4.9)$$

4.2 QUANTUM 1/f EFFECT AND DIFFRACTION [5]

The mechanism proposed by Handel to prove the quantum 1/f effect was the interference between the part of the scattered beam which has suffered bremsstrahlung losses with the main non-bremsstrahlung part resulting in beats. These beats according to Handel will be present in the probability density along the direction of the scattered beam and will manifest themselves as low frequency fluctuations in the current. Such low frequency fluctuations will also be exhibited in the scattering cross-sections. It has been stressed by Handel that 1/f noise cannot be observed with a single carrier. If there is a detecting system which can detect such low frequency fluctuations, the effect of a single carrier demonstrating quantum 1/f effect will only be a pulse in the detector. To demonstrate the quantum 1/f effect completely, many carriers are necessary. This is quite akin to the diffraction phenomenon wherein a single point of impact on the photographic plate whereas diffraction of many particles yields a diffraction pattern. Therefore it has been said that quantum 1/f effect is a collective effect or at least a two particle effect which has to be described by a two particle wave function and a two particle correlation function.

The total one particle Schrodinger field can be written as

$$\Psi_1 = a e^{i/m} (p \cdot r - E t) \left(1 + \int_{\omega}^{\Lambda} b_r(\epsilon) e^{i \epsilon v_n} d\epsilon \right) \quad \dots \quad (4.10)$$

where all the terms have the usual meanings and have been described in section (a) Using Eq.(4.11) as the

basis, the two particle Schrodinger equation can be written as

$$\begin{aligned} |\Psi|^2 = a^4 & \left[\left(1 + \int_{\omega}^{\Lambda} |b_r(\epsilon)|^2 ds \right)^2 + 2 \int_{\omega}^{\Lambda} |b_r(\epsilon)|^2 \cos(\epsilon/h) (t_1 - t_2) ds \right. \\ & \left. + \int_{\omega}^{\Lambda} \int_{\omega}^{\Lambda} |b_r(\epsilon)|^2 |b_r(\epsilon^1)|^2 e^{i(\epsilon^1 - \epsilon)} (t_1 - t_2) ds dt \right] \quad \dots \quad (4.11) \end{aligned}$$

Eq.(4.12) is seen to be consistent with Eq.(4.6) which is the equation for the autocorrelation function by using a one particle wave function in the Schrodinger field. Sherif and Handel [5] have treated the quantum mechanical diffraction problem in terms of autocorrelation functions and have

established[5] the similarity between diffraction and 1/f noise. Also by using density matrix language, they have shown[3] that the wave function is bilinear and in accordance with the principles of quantum mechanics.

4.3 SECOND QUANTISATION FORMULATION OF QUANTUM 1/f NOISE [6,7]

The techniques of second quantisation is a very useful one for a system of identical particles, since, it automatically includes the symmetry required by the identity of particles in quantum mechanics. This representation is used here in order to demonstrate the quantum 1/f effect.

Handel has started with the expression in the Heisenberg representation state $|S\rangle$ of N bosons which are identical with mass m and coming from some scattering process at an angle θ . The emerging beam of bosons have suffered bremsstrahlung energy losses. These are reflected in their one particle waves $\varphi_1(\xi_1)$

$$|S\rangle = (N!)^{-1/2} \pi_1 \int d^3 \xi_1 \varphi_1(\xi_1) \Psi^\dagger(\xi_1) |O\rangle \quad \dots \quad (4.12)$$

$$= \pi_1 \int d^3 \xi_1 \varphi_1(\xi_1) |S^0\rangle \quad \dots \quad (4.13)$$

In Eq.(4.12) $\Psi^\dagger(\xi_1)$ is a field operator creating a boson with position vectors ξ_1 and $|O\rangle$ is the vacuum state. In Eq.(4.13) $|S^0\rangle$ refers to a state with N bosons of position vectors ξ_i with $i = 1$ to N . The value of the matrix element $N! \langle S^0 | O | S^0 \rangle$ has been evaluated by using the commutation properties of the boson field operators and found to be

$$N! \langle S^0 | O | S^0 \rangle = \sum_{\mu \nu} \sum_{m n} \delta(n_\nu - x_1) \delta(n_\mu - x_1) \delta(\xi_m - x_2) \delta(\xi_n - x_1) \sum_{(i,j)} \prod_{i,j} \delta(\eta_b - \xi_i) \quad \dots (4.14)$$

where the prime indicates that the possibilities of $\mu = \nu$ and $m = n$ have been excluded in the summations and the possibilities $i = m$, $i = n$, $j = \nu$, $j = \mu$ have been excluded in the product. The summation $\sum_{(i,j)}$ runs over all permutations of the remaining $(N-2)$ values of i and j . The result of

Eq.(4.14) is used to calculate the matrix element $\langle S^0 | O | S^0 \rangle$.

$$\langle S^0 | O | S^0 \rangle = [1/N(N-1)] \sum_{\mu \nu} \sum_{m n} \phi_m^*(x_2) \phi_\nu^*(x_1) \phi_m(x_2) \phi_n(x_1) \quad \dots \quad (4.15)$$

The one particle states $\phi(x)$ which are spherical waves emerging from the scattering centre located at $x=0$ have been represented as

$$\phi(x) = (c/x) e^{ikx} \left[1 + \sum_{k,l} b(k,l) e^{-i\varphi} a_{kl}^\dagger \right] \quad \dots \quad (4.16)$$

where c is an amplitude factor, K is the magnitude of the boson wave factor, $b(k,l)$ is the bremsstrahlung amplitude for photons of wave vector k and polarisation l , a_{kl}^\dagger is the corresponding photon creation operator. The emitted photon state can be created from vacuum if Eq.(4.17) is inserted in Eq.(4.14). The magnitude of the momentum loss necessary for energy conservation in the bremsstrahlung process has been calculated to be $\hbar v = Mck/K = 2\pi m f/K$. The matrix element $\langle S^0 | O | S^0 \rangle$ is obtained by substituting Eq.(4.17) into Eq.(4.16)

$$\langle S^0 | O | S^0 \rangle = [c/x]^4 \left\{ N(N-1) + 2(N-1) \sum_{k,l} |b(k,l)|^2 [1 + \cos q(x_1 - x_2)] \right\} \quad \dots \quad (4.17)$$

In Eq.(4.12) the higher orders of $|b(k,l)|$ have been neglected. To determine the angular part of the summation in Eq.(4.18), Handel has calculated the expectation value of the state in Eq.(4.17) and has compared it with the cross-section without and with bremsstrahlung.

$$j = (\hbar k / mx^2) \left[1 + \sum_{k,l} |b(k,l)|^2 \right] = j_0 \left[1 + \alpha A \int df/f \right] \quad \dots \quad (4.18)$$

$e^2/\hbar c$ is the fine structure constant and A is the infrared exponent given by. The $1/f$ dependence is well known since the number of photons emitted per unit frequency range is inversely proportional to the frequency. The photon frequency is given by $f = ck/2$. Thus Eq.(4.18) now becomes

$$\langle S^0 | O | S^0 \rangle = [c/x]^4 \left\{ N(N-1) + 2(N-1) \alpha A \int [1 + \cos q(x_1 - x_2)] df/f \right\} \quad \dots \quad (4.19)$$

which is known as the pair correlation function or the autocorrelation of the scattered beam. The spectral density of fractional scattered particle density ρ , (or current density j or cross-section σ), fluctuations in frequency f of wave number q is obtained by dividing the e by the term $N(N-1)$. Therefore,

$$\rho^2 = s_\rho(f) = j^2 s_j(f) = \sigma^2 s_\sigma(f) = 2 \alpha A / fN \quad \dots \quad (4.20)$$

where N , the number of particles or current carriers used to define the current j whose fluctuations we are interested in. It has also been shown that the exponent of f in Eq.(4.21) is $(\alpha A - 1)$ by taking

into consideration the contributions from all real and virtual multi-photon processes of any order. For fermions, the calculations were repeated by Handel with the replacement of the commutators of Eq.(4.15) by anti commutators. These calculations have resulted in the following spectral density fluctuations,

$$\rho^2 = s_p \quad (f) = j^{-2} s_j (f) = \sigma^{-2} s_\sigma (f) = 2 \alpha A / f(N - 1) \quad \dots \quad (4.21)$$

Thus we see that even with refined techniques of second quantisation, we arrive at the same expression for the spectral density of fractional particle density (or current or cross section) fluctuations.

4.4 COHERENT STATES AND QUANTUM 1/f EFFECT [8,9,10]

This is an equivalent formulations of the quantum 1/f noise theory. It has been shown that the electromagnetic field of a free electron is in a coherent state and is therefore not an Eigen state of the Hamiltonian. Thus the charged particle is not stationary. It has been mathematically shown that the fluctuations resulting from this non stationarity have a 1/f spectral density. Such fluctuations act upon the ordered, collective translational motion of the current carriers. Handel has connected the coherent quantum 1/f noise with the usual quantum 1/f noise is present along with quantum 1/f effect just like the magnetic energy of a biased semiconductor sample co-exists with the kinetic energy of the individual, randomly moving charge carriers.

4.5 KELDYSH SCHWINGER FORMALISM [11]

This form of perturbation theory has also been used to explain the bremsstrahlung model of 1/f noise. The average current is evaluated with Handel's approximation by the Keldysh method and it was found that this agreed with those found by Handel. The current autocorrelation function evaluated from 210 diagrams. These calculations showed that Handel's equations are correct. All the above formulations of Handel have been subject to severe criticisms.

CHAPTER 4 - REFERENCES

- 1 P.H.Handel, Phys. Rev. Lett., 34, p.1492 (1975).
- 2 F.Bloch and A. Nordsieck, Phys. Rev., 52, p.54 (1937).
- 3 P.H.Handel, Proc. of the IV Sysmp. on Quantum 1/f noise and other low frequency fluctuations in electronic Devices Minneapolis, May 10-11, (1990) Ed. by P.H.Handel (Univ. of Missouri, publication office, St.Louis, MO 63121) p.107.
- 4 "Studies on Quantum Noise and Burst Noise", Ph.D Thesis (1992) submitted by M. ATHIBA AZHAR, Dept of Studies in Physics, University of Mysore.
- 5 T.S.Sheriff and P.H.Handel, Proc. of the 2nd Int. Symp. on 1/f noise Ed. by E.R.Chenette, Univ. of Florida, Gainesville, Orlando, p.525 (1980).
- 6 P.H.Handel, Proc. of 4th Symp. on quantum 1/f noise and other low, frequency fluctuations in electronic devices, minneapolis, May 10-11, (1990) Ed. by P.H.Handel (Univ.of Missouri, publication office, St.Louis, MO 63121) p.55.
- 7 P.H.Handel, Proc. (th Int. Conf. on Noise in Physical Systems, Montreal (Ed. by C.M.Van Vliet, World Scientific, Singapore) p.365 (1987).
- 8 P.H.Handel and T.Musha, Z.Phys. B-Condensed Matter, 70, P.515 (1988).
- 9 P.H.Handel, Noise in Physical Systems and 1/f noise, Ed. by M.Savelli, G.Lecoy and J.P. Nougier (Elsevier Science Publishers B.V) p 97 (1983).
- 10 P.H. Handel, Noise in Physical Systems and 1/fNoise, Ed by A.D' Amico and P. Mazetti (Elsevier Science Publishers B.V), p 465 (1986).
- 11 P.H. Handel, "Keldysh-Schwinger method calculations of 1/f low frequency current fluctuations", Unpublished Manuscript.
- 12 P.H. Handel, Phys. Rev. A 22, p.745 (1980).