Chapter 8

Conclusions and discussions

The first problem considered here studied the geometry of dissipative and non-dissipative single-qubit channels, essentially by representing the geometry of a family of channels by means of the geometry of the states obtained under the Choi isomorphism. We showed that the condition of dissipativeness corresponds to the non-commutation of energy eigenstates of the system Hamiltonian and at least one of the Kraus operators for a communication channel. We further pointed out that on account of the unitary freedom (i.e., non-uniqueness) of the Kraus representation, any quantification of the channel performance, such as the gate fidelity or a bound on accessible information, must be an operator norm of some function of the Kraus operators. We noted that the rank of amplitude damping channels is even-valued (2 or 4), unlike for Pauli channels, and it will interesting to study the conditions under which this phenomenon extends beyond the single-qubit case considered here. In particular, in Chapter 4, we study a rank-9 two-qubit amplitude damping channel.

Channel noise is a fundamental issue in quantum cryptography, since legitimate end users estimate an eavesdropper's knowledge based on the observed noise. In Chapter 3, we considered a controlled key distribution task in which the effect of the squeezed generalized amplitude damping (SGAD) channel was studied. In Chapter 2, we noted how channel squeezing counteracts the thermal effects in certain regimes. It would be interesting to explore whether legitimate communicators Alice and Bob can exploit this, possibly using suitable decoy states, to distinguish genuine channel noise from a possible eavesdropper's intervention.

In Chapter 4, we showed that channels that are intractable in the sense that the Choi matrix is not analytically diagonalizable (on account of the Abel-Galois no-go theorem) can still be given a Kraus-like representation– OSDR (operator sum-difference representation)– possibly involving negative Kraus terms. We note that the above theorem does not imply that specific matrices of rank above 4 will be non-diagonalizable, but only that no generic method using radicals exists to solve quintic and higher-order polynomials. An interesting question for future study is to look into the optimization aspect of our method, i.e., to develop an algorithm that would require the fewest number of OSDR elements beyond the rank of the channel. In our worked-out example for the two-qubit amplitude damping (2AD) channel, which is of rank 9, we showed how 13 analytical OSDR elements suffice for representation. A specific instance of our open question is of whether this number can be reduced to any smaller number.

In Chapter 5, we studied the geometric properties of the channel induced by the Unruh effect on one of two Dirac modes, and showed its similarity to and difference from the amplitude damping (AD) channel. We found that non-classicality, as quantified by entanglement (concurrence [85]) or teleportation fidelity [138], does not attain the separable value, whereas the amount of nonlocality, as quantified by the Horodecki quantity [136], does vanish, just like channel capacity [123], which vanishes asymptotically for both fermionic and bosonic Unruh effects. This is consistent with other similar findings by other researchers, and suggests that channel capacity correlates better with nonlocality, rather than with entanglement or teleportation fidelity. It is an interesting question how well this correlation extends beyond the restricted scenario considered here.

In Chapter 6, we proposed "quantum error correction based characterization of quantum dynamics" (QECCD), a method of direct characterization of quantum dynamics (CQD), that exploits techniques from quantum error correcting codes. Unlike earlier CQD techniques, the QECCD protocol is not restricted to a fixed set of initial states, but accepts as input any logical state that protects a code from noise in the given noisy system. As a result, it can be implemented concurrently along with quantum computation.

Apart from its application to CQD, the reasoning behind our method suggests a new channel-state isomorphism, which is similar to the Choi isomorphism. Our method effectively exploits this isomorphism to estimate the channel by measurements on the erroneous state. Along the way, we introduced the mathematical trick of toggling, which exchanges the real and imaginary parts of the off-diagonal terms in the density operator that correspond to a two-colorable cycle.

For QECCD, we introduced a new class of quantum error correcting codes (QECCs), which have the property that the set of allowed Pauli errors forms a group. Codes (6.4) and (6.14) are examples of such QECCs suitable for QECCD.

Finally, in chapter 7, we generalize the concept of degenerate quantum error correcting codes to ambiguous stabilizer codes (ASCs). While ASCs cannot be used for quantum error correction or even error detection, they can be used for ambiguous error detection and characterization of an unknown quantum dynamics. To this end, we proposed a quantum tomography procedure, called 'quantum ASC based characterization of dynamics' (QASCD). This may be considered as QECCD extended to accommodate error ambiguity. The experimental advantage of using ASCs over unambiguous (conventional) QECCs for noise characterization is the reduction in the size of codes, for a price in terms of a larger number of operations, state preparations and codes that must be used.