THEORETICAL ANALYSIS

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Chapter 2

THEORETICAL ANALYSIS

2.1 GENERAL

This chapter deals with the generalized formulation of problems concerning circular (Fig. 2.1) and non circular (Fig. 2.2 - 2.3) bearings with micropolar lubricants. Elastohydrodynamic (EHD) studies require simultaneous solutions of the following:

1. Two dimensional modified Reynold's equation derived from Navier Stoke's, continuity and Fick's second law equations.
2. The three dimensional elasticity equations to determine the deformation of the bearing liner.

The representative models for the fluid flow field and the displacement field with boundary conditions to be adopted are described in this chapter. To achieve the analysis the two dimensional modified Reynold's equation and the three dimensional elasticity equations are to be solved to obtain the pressure distributions in the fluid flow field and the deformation of the bearing liner. In the present work, it is proposed to use the powerful technique, finite element method, to obtain the numerical solution of the elastohydrodynamic problem.

2.2 HYDRODYNAMIC ANALYSIS

To obtain the hydrodynamic pressure distribution in the fluid flow field in the clearance space of journal bearing, Reynold's equation is used. To consider micropolar effect in the pressure field, Reynold's equation is modified by incorporating Fick's second law of diffusion in the equation.

2.2.1 Governing Equations

Navier-Stoke's equation for incompressible fluid is written as

$$\rho\left(\frac{\partial \vec{q}}{\partial t} + \vec{q}.\nabla\vec{q}\right) = \rho \vec{f} - \nabla p + \nabla.\mu\nabla\vec{q} \tag{2.1}$$

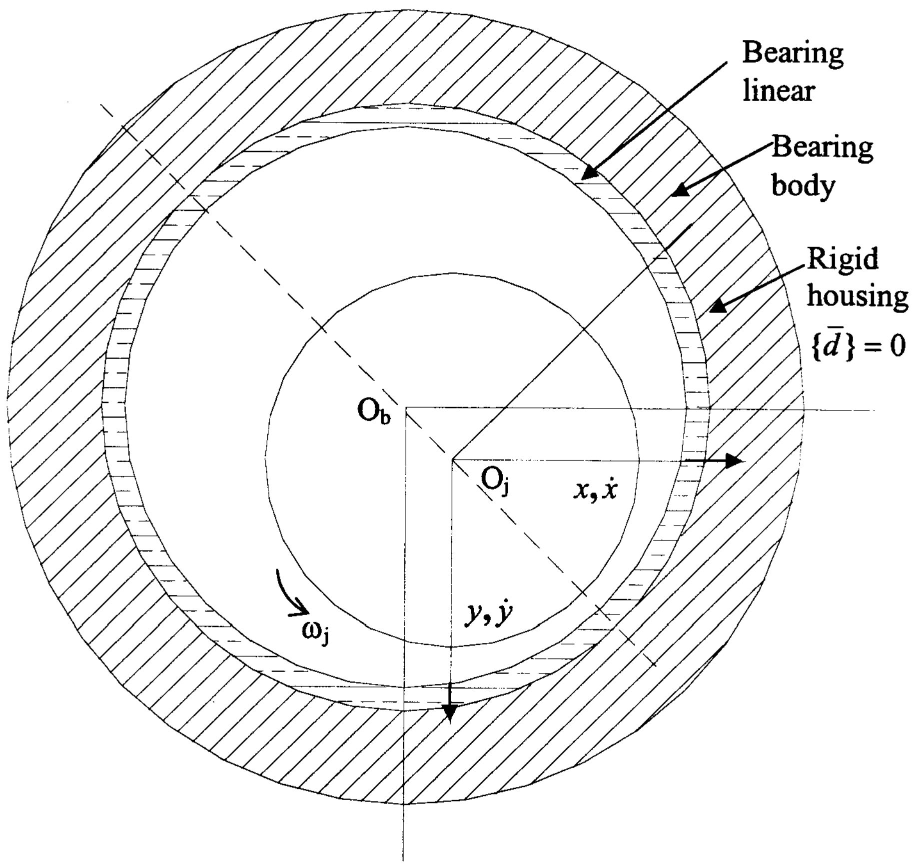

Fig. 2.1 Circular Bearing Geometry

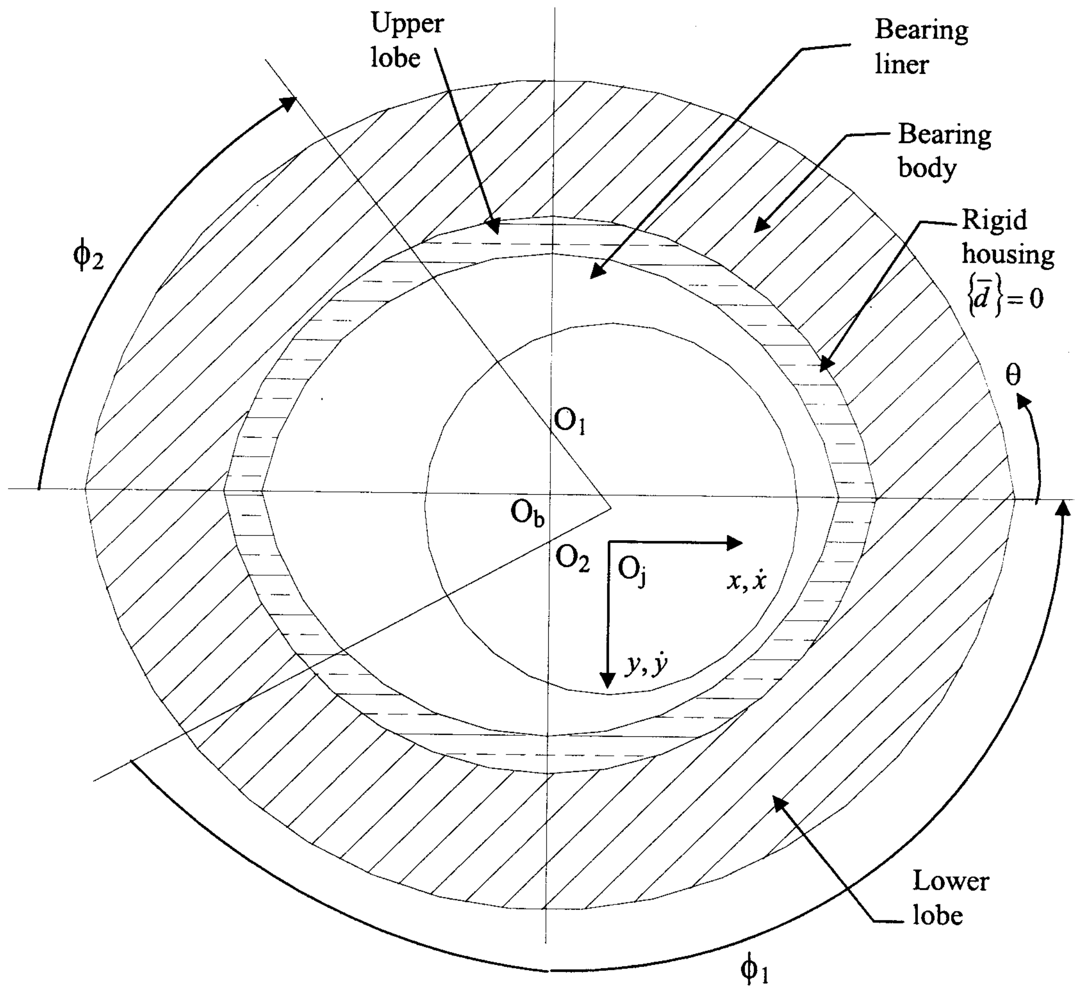

Fig. 2.2 Two Lobe Bearing Geometry

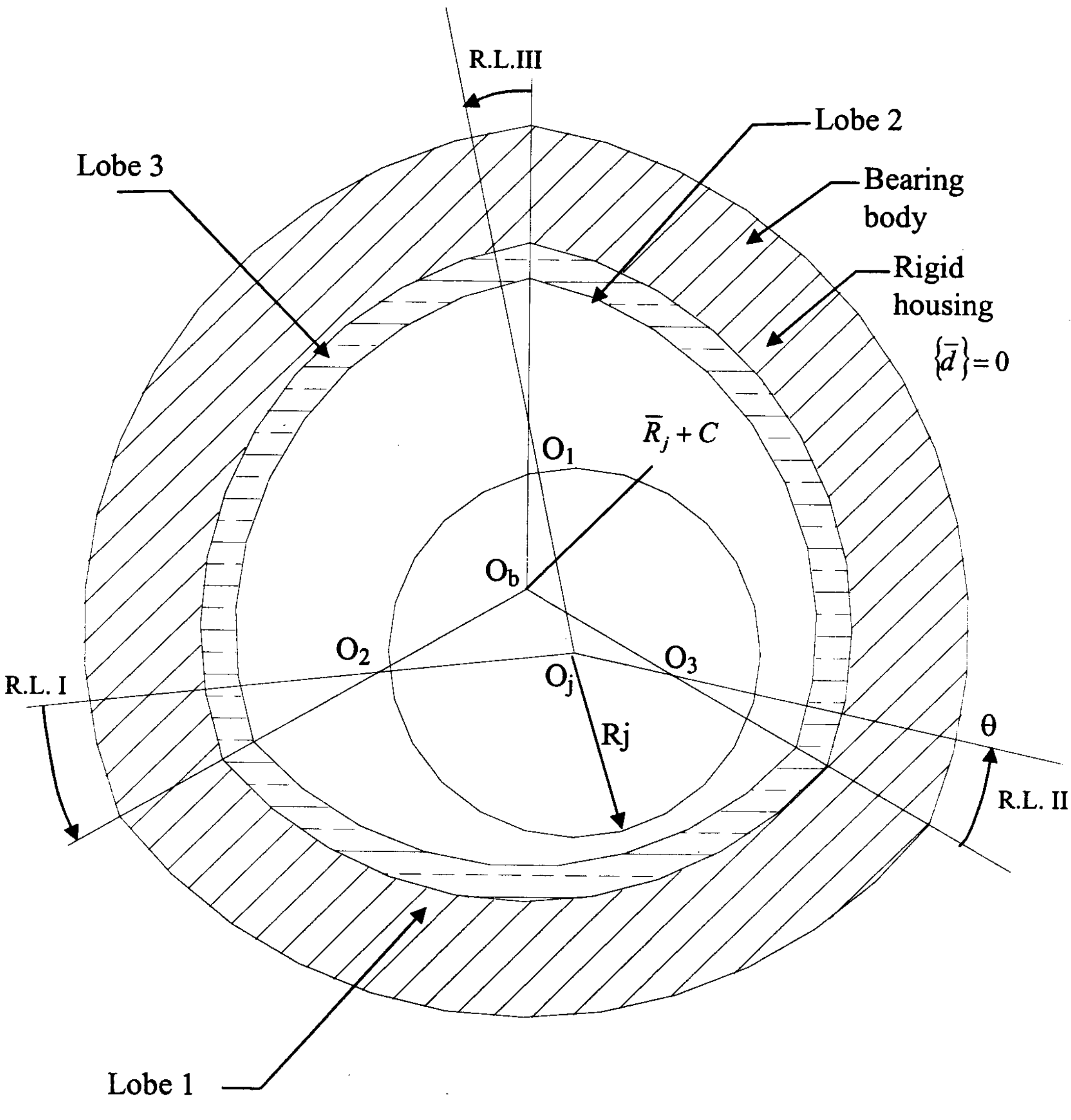

Fig. 2.3 Three Lobe Bearing Geometry

Continuity equation for steady flow of an incompressible fluid is given by

$$\nabla.\vec{q} = 0 \tag{2.2}$$

Fick's Second law of mass diffusion is written as

$$\frac{\partial c}{\partial t} + \bar{q}.\nabla c = K_d \nabla^2 c \tag{2.3}$$

For the case of a steady, uniform flow [92] Eq.(2.3) reduces to

$$K_d \left\{ \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2} \right\} = w \frac{\partial c}{\partial z} \tag{2.4}$$

Since $\frac{\partial c}{\partial z} << 1$ and $\frac{\partial^2 c}{\partial z^2} << \frac{\partial^2 c}{\partial y^2}$ Eq. (2.4) becomes

$$\frac{\partial^2 c}{\partial y^2} = 0 \tag{2.5}$$

Eq. (2.5) can be solved for 'c' subject to the boundary conditions

$c = c_r$ at $y = 0$ and

$$K_d \frac{\partial c}{\partial y} = -K_s c_r \quad \text{at } y = h \tag{2.6}$$

Thus we get

$$c = c_r \left(1 + \frac{K_s y}{K_d} \right) \tag{2.7}$$

Eq. (2.7) with Eq. (1.1) gives the relation for viscosity in the following form.

$$\mu = \mu_0 \left[1 + \lambda c_r \left\{ 1 + \frac{K_s y}{K_d} \right\} \right] \tag{2.8}$$

For getting modified Reynold's equation from Eqs. (2.1), (2.2) and (2.8), the following assumptions are made.

1. Inertia and body forces are negligible when compared with the pressure and viscous forces.

2. There is no variation of pressure across the fluid film. i.e., $\frac{\partial p}{\partial y} = 0$

3. There is no slip in the fluid-solid boundaries.

4. Flow is laminar and incompressible.

5. Compared with the two velocity gradients $\frac{\partial u}{\partial y}$ and $\frac{\partial w}{\partial y}$, all other velocity gradients are considered to be negligible.

6. The height of the fluid film is very small. This permits us to ignore the curvature of the fluid film.

Using the relevant assumptions given, Eq. (2.1) can be written as

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left[\mu \frac{\partial u}{\partial y}\right] \tag{2.9}$$

and

$$\frac{\partial p}{\partial z} = \frac{\partial}{\partial y}\left[\mu \frac{\partial w}{\partial y}\right] \tag{2.10}$$

Integrating Eq. (2.9) with respect to y twice we get

$$u = \frac{\partial p}{\partial x}\int \frac{y}{\mu}\,dy + \int \frac{C_1}{\mu}\,dy + C_2 \tag{2.11}$$

From Eq. (2.8) and Eq. (2.11) we get the following relation.

$$u = \frac{K_d}{\mu_0 K_s \lambda c_r}\frac{\partial p}{\partial x}\left\{y - \frac{(1+\lambda c_r)K_d}{K_s \lambda c_r}\ln \mu(y)\right\} + \frac{C_1 K_d}{\mu_0 K_s \lambda C_r}\ln \mu(y) + C_2 \tag{2.12}$$

Similarly integrating Eq.(2.10) and using Eq.(2.8) we obtain

$$w = \frac{K_d}{\mu_0 K_s \lambda c_r}\frac{\partial p}{\partial z}\left\{y - \frac{(1+\lambda c_r)K_d}{K_s \lambda c_r}\ln \mu(y)\right\} + \frac{C_3 K_d}{\mu_0 K_s \lambda C_r}\ln \mu(y) + C_4 \tag{2.13}$$

The boundary conditions are given below.

$$u = U,\ \ w = 0 \ \ \text{at } y = 0$$
$$u = 0,\ \ w = 0 \ \ \text{at } y = h \qquad (2.14)$$

Using Eqs. (2.14) in Eq. (2.12) we get

$$u = \left[-U \ln\frac{\mu(y)}{\mu(h)} + \frac{K_d}{\mu_0 K_s \lambda C_r}\frac{\partial p}{\partial x}\left\{ \left[y \ln\frac{\mu(h)}{\mu(0)} \right] + h \ln\left[\frac{\mu(y)}{\mu(0)} \right] \right\} \right] \div \ln\frac{\mu(h)}{\mu(0)} \qquad (2.15)$$

Using boundary conditions in Eq. (2.13) we have

$$w = \frac{ \dfrac{K_d}{\mu_0 K_s \lambda C_r}\dfrac{\partial p}{\partial z}\left[y \ln\dfrac{\mu(h)}{\mu(0)} - h \ln\dfrac{\mu(y)}{\mu(0)} \right] }{ \ln\dfrac{\mu(h)}{\mu(0)} } \qquad (2.16)$$

Differentiating Eq. (2.15) with respect to x and Eq. (2.16) with respect to y we get

$$\frac{\partial u}{\partial x} = -\frac{\partial}{\partial x}\left[\frac{U \ln\dfrac{\mu(y)}{\mu(h)}}{\ln\dfrac{\mu(h)}{\mu(0)}} \right] + \frac{\partial}{\partial x}\left[\frac{\dfrac{\partial p}{\partial x}\dfrac{K_D}{\mu_0 K_s \lambda C_r}}{\ln\dfrac{\mu(h)}{\mu(0)}} \right]\left\{ y \ln\frac{\mu(h)}{\mu(0)} + h \ln\frac{\mu(y)}{\mu(0)} \right\} \qquad (2.17)$$

$$\frac{\partial w}{\partial z} = \frac{K_d}{\mu_0 K_s \lambda C_r}\frac{\partial}{\partial z}\left[\frac{ \dfrac{\partial p}{\partial z}\left[y.\ln\dfrac{\mu(h)}{\mu(0)} - h \ln\dfrac{\mu(y)}{\mu(0)} \right] }{ \ln\dfrac{\mu(h)}{\mu(0)} } \right] \qquad (2.18)$$

Integrating continuity equation, Eq (2.2) we get

$$v = -\int \frac{\partial u}{\partial x} dy - \int \frac{\partial w}{\partial z} dy + C_5 \qquad (2.19)$$

The boundary conditions are

$$y = 0 \qquad \text{at } v = V$$
$$y = h \qquad \text{at } v = 0 \qquad (2.20)$$

Using Eqs. (2.17), (2.18) and boundary conditions given by Eq. (2.20) in Eq. (2.19) and neglecting higher order terms we get the following relation.

$$V=-\int_0^h \frac{\partial}{\partial x}\left\{\frac{U\ln\frac{\mu(y)}{\mu(h)}dy}{\ln\frac{\mu(h)}{\mu(0)}}\right\}+\int_0^h \frac{\partial}{\partial x}\left\{\frac{\frac{K_d}{\mu_0 K_s \lambda C_r}\frac{\partial p}{\partial x}}{\ln\frac{\mu(h)}{\mu(0)}}y\ln\frac{\mu(h)}{\mu(0)}dy\right\}$$

$$+\int_0^h \frac{\partial}{\partial z}\left\{\frac{\frac{K_d}{\mu_0 K_s \lambda C_r}\frac{\partial p}{\partial z}}{\ln\frac{\mu(h)}{\mu(0)}}y\ln\frac{\mu(h)}{\mu(0)}dy\right\} \tag{2.21}$$

Simplifying equation (2.21) we get

$$\frac{\partial}{\partial x}\left[\frac{h^3}{12\mu_0}\left\{1-\lambda c_r\left(1+\frac{K_s h}{2K_d}\right)\right\}\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial z}\left[\frac{h^3}{12\mu_0}\left\{1-\lambda c_r\left(1+\frac{K_s h}{2K_d}\right)\right\}\frac{\partial p}{\partial z}\right]$$

$$=\frac{U\partial}{\partial x}\left[\frac{h}{2}\left\{1-\frac{\lambda c_r K_s h}{6K_d}\right\}\right]+V \tag{2.22}$$

Changing in to polar co ordinates and non dimensionalising

$$\frac{\partial}{\partial \theta}\left[\frac{\bar{h}^3}{12\bar{\mu}}\left\{1-\lambda c_r\left(1+\frac{\bar{K}_s \bar{h}}{2}\right)\right\}\frac{\partial \bar{p}}{\partial \theta}\right]+\frac{\partial}{\partial \bar{z}}\left[\frac{\bar{h}^3}{12\bar{\mu}}\left\{1-\lambda c_r\left(1+\frac{\bar{K}_s \bar{h}}{2}\right)\right\}\frac{\partial \bar{p}}{\partial \bar{z}}\right]$$

$$=\frac{\bar{U}\partial}{\partial \theta}\left[\frac{\bar{h}}{2}\left\{1-\frac{\lambda c_r \bar{K}_s \bar{h}}{6}\right\}\right]+\bar{R}\bar{V} \tag{2.23}$$

The following relations are used for the velocity components in the tangential and radial directions.

$$\bar{U}=1+\frac{\dot{\eta}}{\bar{R}}\sin\theta-\frac{\varepsilon\dot{\xi}}{\bar{R}}\cos\theta \tag{2.24}$$

$$\bar{V}=\frac{\dot{\eta}}{\bar{R}}\cos\theta-\frac{\varepsilon\dot{\xi}}{\bar{R}}\sin\theta \tag{2.25}$$

$$\frac{\partial \bar{U}}{\partial \theta}=\frac{\dot{\eta}}{\bar{R}}\cos\theta+\frac{\varepsilon\dot{\xi}}{\bar{R}}\sin\theta \tag{2.26}$$

Using Eqs. (2.24), (2.25) and (2.26) in Eq. (2.23) we get

$$\frac{\partial}{\partial \theta}\left[\frac{\bar{h}^3}{12\bar{\mu}}\left[1-\lambda c_r\left[1+\frac{\bar{K}_s \bar{h}}{2}\right]\right]\frac{\partial \bar{p}}{\partial \theta}\right]+\frac{\partial}{\partial \bar{z}}\left[\frac{\bar{h}^3}{12\bar{\mu}}\left[1-\lambda c_r\left[1+\frac{\bar{K}_s \bar{h}}{2}\right]\right]\frac{\partial \bar{p}}{\partial \bar{z}}\right]$$

$$= \frac{1}{2}\frac{\partial \bar{h}}{\partial \theta}\left[1 - \frac{\lambda c_r \bar{K}_s \bar{h}}{6}\right] + \dot{\eta}\cos\theta - \varepsilon\dot{\xi}\sin\theta \qquad (2.27)$$

2.2.2 Finite Element Formulation

The flow field in the clearance space of circular bearing and in each lobe of non circular bearing has been descretized into four noded isoparametric elements. The total region has been divided into 14 elements in circumferential direction and 4 elements in axial direction. It has been descretized by decreasing the size of the elements in circumferential direction, so that the steep variation of pressure gradient at the trailing edge of the fluid film is accurately calculated. The element numbering is done in such a way that the bandwidth is a minimum. The descretization of the flow field is given in Fig 2.4.

The Langrangian interpolation function for the four nodded isoperimetric elements are given below.

$N_1 = 0.25\,(1 - \xi)\,(1 - \eta)$

$N_2 = 0.25\,(1 - \xi)\,(1 + \eta)$

$N_3 = 0.25\,(1 + \xi)\,(1 + \eta)$

$N_4 = 0.25\,(1 + \xi)\,(1 - \eta)$

Where ξ and η are the local co-ordinates as shown in Fig. 2.5.

Applying the orthogonality condition of Galerkin's method for Eq(2.27) we have

$$\iint_{\Omega^e}\left\{\frac{\partial}{\partial\theta}\left[\frac{\bar{h}^3}{12\bar{\mu}}\left\{1-\lambda c_r\left(1+\frac{\bar{K}_s\bar{h}}{2}\right)\frac{\partial\bar{p}}{\partial\theta}\right\}+\frac{\partial}{\partial\bar{z}}\frac{\bar{h}^3}{12\bar{\mu}}\left\{1-\lambda c_r\left(1+\frac{\bar{K}_s\bar{h}}{2}\right)\frac{\partial\bar{p}}{\partial\bar{z}}\right\}\right]\right.$$

$$\left.-\frac{\partial}{\partial\theta}\left[\frac{\bar{h}}{2}\left[1-\frac{\lambda c_r\bar{K}_s\bar{h}}{6}\right]\right]-\dot{\eta}\cos\theta+\varepsilon\dot{\xi}\sin\theta\right\}N_i\,d\theta d\bar{z}=0 \qquad (2.28)$$

where N_i (i = 1, 2, 3, 4) are the Langrangian interpolation functions. Since $\bar{p} = N_j\bar{p}_j$ and $\bar{h} = N_j\bar{h}_j$ (j = 1, 2, 3, 4).

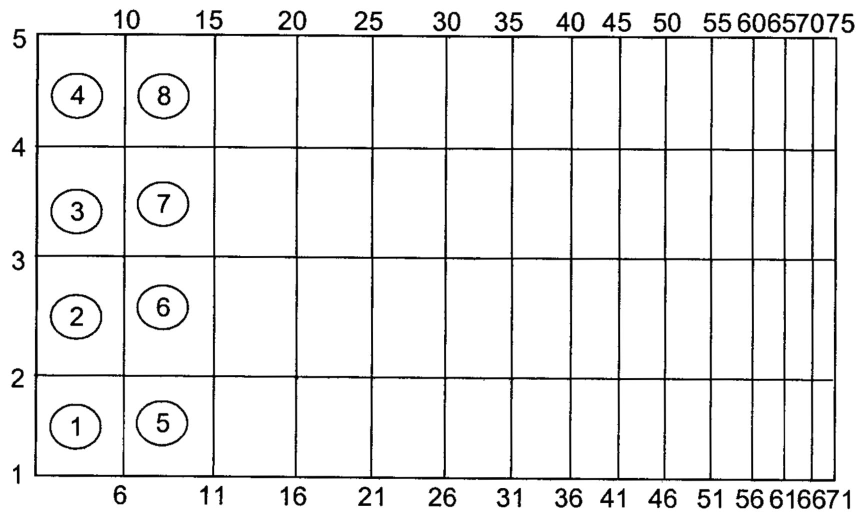

Fig. 2.4 Descretization of Flow Field

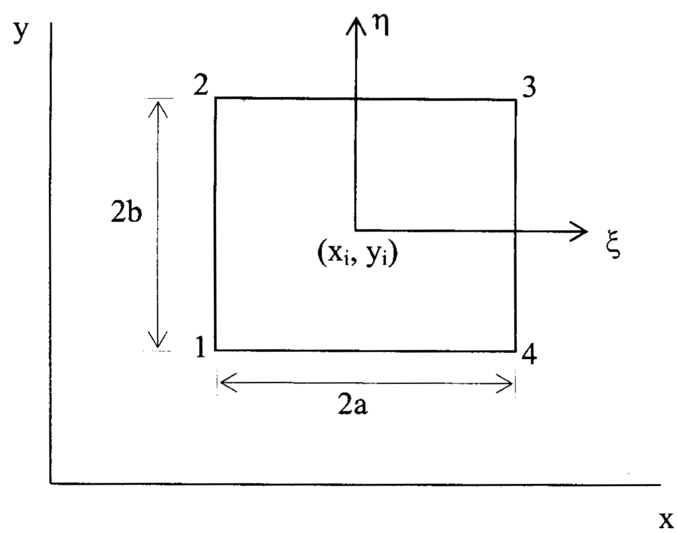

Fig. 2.5 Local Coordinate System

We have on integrating by parts

$$\int_{\Gamma^e} N_i \frac{\bar{h}^3}{12\bar{\mu}}\left\{1-\lambda c_r\left(1+\frac{\bar{K}_s\bar{h}}{2}\right)\frac{\partial\bar{p}}{\partial\theta}\right\}d\bar{z} - \int\int_{\Gamma^e}\left[\frac{\bar{h}^3}{12\bar{\mu}}\left\{1-\lambda c_r\left(1+\frac{\bar{K}_s\bar{h}}{2}\right)\frac{\partial N_i}{\partial\theta}\frac{\partial\bar{p}}{\partial\theta}d\theta\right\}\right]d\bar{z}$$

$$+\int_{\Gamma^e} N_i \frac{\bar{h}^3}{12\bar{\mu}}\left\{1-\lambda\ c_r\left(1+\frac{\bar{K}_s\bar{h}}{2}\right)\frac{\partial\bar{p}}{\partial\bar{z}}\right\}d\theta - \int\int_{\Gamma^e}\left[\frac{\bar{h}^3}{12\bar{\mu}}\left\{1-\lambda\ c_r\left(1+\frac{\bar{K}_s\bar{h}}{2}\right)\frac{\partial N_i}{\partial\bar{z}}\frac{\partial\bar{p}}{\partial\bar{z}}d\bar{z}\right\}\right]d\theta$$

$$= \int_{\Gamma^e} N_i \frac{\bar{h}}{2}\frac{1-\lambda c_r\bar{K}_s\bar{h}}{6}d\bar{z} - \int\int_{\Omega^e}\left[\frac{\bar{h}}{2}\left(\frac{1-\lambda\ c_r\bar{K}_s\bar{h}}{6}\right)\right]\frac{\partial N_i}{\partial\theta}\ d\theta\ d\bar{z}$$

$$+\dot{\eta}\left[\int_{\Gamma^e} N_i\sin\theta d\bar{z} - \int\int_{\Omega^e}\left[\sin\theta.\frac{\partial N_i}{\partial\theta}d\theta\right]d\bar{z}\right]$$

$$+\varepsilon\dot{\xi}\left[\int_{\Gamma^e} N_i\cos\theta d\bar{z} - \int\int_{\Omega^e}\left[\cos\theta.\frac{\partial N_i}{\partial\theta}d\theta\right]d\bar{z}\right] \qquad (2.29)$$

Single integral terms can be removed since integration is being carried out over a closed loop.

Simplifying Eq. (2.29) we have

$$\int\int_{\Omega^e}\frac{\bar{h}^3}{12\bar{\mu}}\left[1-\lambda c_r\left(1+\frac{\bar{K}_s\bar{h}}{2}\right)\right]\left[\frac{\partial N_i}{\partial\theta}\frac{\partial N_j}{\partial\theta}+\frac{\partial N_i}{\partial\bar{z}}\frac{\partial N_j}{\partial\bar{z}}\right]\bar{p}_j d\theta d\bar{z}$$

$$= \int\int_{\Omega^e}\frac{\bar{h}}{2}\frac{\partial N_i}{\partial\theta}\left(1-\frac{\lambda c_r\bar{K}_s\bar{h}}{6}\right)d\theta d\bar{z} - \dot{\eta}\int\int_{\Omega^e}\frac{\partial N_i}{\partial\theta}\sin\theta d\theta d\bar{z} - \varepsilon\dot{\xi}\int\int\frac{\partial Ni}{\partial\theta}\cos\theta\, d\theta\, d\bar{z} \quad (2.30)$$

This can be written in matrix form as:

$$[K_f]^e\ \{\bar{p}\}^e = \{R_1\}^e + \{R_2\ \}^e + \varepsilon\{R_3\ \}^e \qquad (2.31)$$

where $R_{1i}^e = \int\int_{\Omega^e}\frac{\partial N_i}{\partial\theta}.\frac{\bar{h}}{2}\left(1-\frac{\lambda c_r\bar{K}_s\bar{h}}{6}\right)d\theta d\bar{z}$

$$R_{2i}^e = -\dot{\eta}\int\int_{\Omega^e}\frac{\partial N_i}{\partial\theta}.\sin\theta d\theta d\bar{z} \qquad R_{3i}^e = -\dot{\xi}\int\int_{\Omega^e}\frac{\partial N_i}{\partial\theta}.\cos\theta d\theta d\bar{z}$$

and

$$[K_{ij}]^e = \int\int_{\Omega^e}\left[\frac{\bar{h}^3}{12\bar{\mu}}1-\lambda c_r\left(1+\frac{\bar{K}_s\bar{h}}{2}\right)\right]\left[\frac{\partial N_i}{\partial\theta}\frac{\partial N_j}{\partial\theta}+\frac{\partial N_i}{\partial\bar{z}}\frac{\partial N_j}{\partial\bar{z}}\right]\bar{p}_j d\theta\, d\bar{z}$$

$[K_{ij}]^e$ is the element stiffness matrix. Ω^e refers to domain of the eth element and Γ^e refers to the boundary of the e^{th} element.

Numerical integration is carried out by using Gauss quadrature formula.

In the case of multilobe bearing, each of the lobes can be treated as a partial bearing with an active film from the leading edge tot the trailing edge.

where $\frac{\partial p}{\partial \theta} = 0$

Then Eq. (2.28) becomes

$$\iint_{\Omega^e} \left\{ \frac{\partial}{\partial \theta} \left[\frac{h^{-3}}{12\bar{\mu}} \left\{ 1 - \lambda c_r \left(1 + \frac{\bar{K}_s \bar{h}}{2} \right) \frac{\partial \bar{p}}{\partial \theta} \right\} + \frac{\partial}{\partial \bar{z}} \frac{h^{-3}}{12\bar{\mu}} \left\{ 1 - \lambda c_r \left(1 + \frac{\bar{K}_s \bar{h}}{2} \right) \frac{\partial \bar{p}}{\partial \bar{z}} \right\} \right] \right.$$

$$\left. - \frac{\partial}{\partial \theta} \left[\frac{\bar{h}}{2} \left[1 - \frac{\lambda c_r \bar{K}_s \bar{h}}{6} \right] \right] - \dot{\eta} \cos\,(\theta + \beta) + \varepsilon \dot{\xi} \sin\,(\theta + \beta) \right\} N_i d\theta d\bar{z} = 0 \qquad (2.32)$$

where β locates the leading edge of each lobe.

The film variation up to trailing edge of each lobe in non-circular bearing is given by.

$\bar{h} = 1 + \varepsilon_1 \cos(\theta + \beta_1)$ for first lobe

$\bar{h} = 1 + \varepsilon_2 \cos(\theta + \beta_2)$ for second lobe

$\bar{h} = 1 + \varepsilon_3 \cos(\theta + \beta_3)$ for third lobe

2.2.3 Global System Equations

The element fluidity matrices are assembled to get global matrices. The final global equation for the entire flow field can be written as

$$[K_f]\,[\bar{p}] = \{R_1\} + \{R_2\} + \{R_3\} \qquad (2.33)$$

2.2.4 Boundary Conditions

The boundary conditions for each lobe may be given by

$$\bar{p}(\theta, \bar{z}) = 0 \quad \text{at } \theta = 0, \theta_2^i$$

$$\bar{p}(\theta, \bar{z}) = 0 \quad \text{at } \bar{z} = \pm 1 \qquad (2.34)$$

$$\frac{\partial p}{\partial \theta}(\theta, \bar{z}) = 0 \text{ at } \theta = \theta_2^i$$

where θ_2^i is the unknown extent of positive pressure fluid for the i[th] lobe.

Solution of Eq. (2.33) after applying boundary conditions (2.34) gives nodal pressures and nodal flows

2.3 ELASTO HYDRODYNAMIC ANALYSIS

2.3.1 General

The bearing liner is a finite length cylinder subjected to hydrodynamic loading due to the fluid film pressure on its internal surfaces. The distribution of the fluid film pressure is such that it causes the bush to deform in all directions, i.e., radial, axial and circumferential. It is however seen that the bush in a journal bearing is usually enclosed in a housing which is comparatively rigid.

2.3.2 Finite Element Formulation

The bearing liner is discretized by 8 nodded hexagonal isoparametric elements. In the case of circular bearing, the bearing liner is discretized into 64 elements (16 elements in the circumferential direction and 4 elements in the axial direction) (Fig. 2.6), in the case of two lobe journal bearing into112 elements (28 elements in the circumferential direction and four elements in the axial direction) (Fig 2.7) and in the case of three lobe journal bearing into144 elements (36 in the circumferential direction and four elements in the axial direction) (Fig 2.8). The displacement components are considered to vary linearly in the elements. Using the elasticity theory the expression for the potential energy of an element when bearing is subjected only to traction forces, T_{tr} is given by

$$\pi^e = 1/2 \iiint [\delta]^{e^T} [J]^{e^T} [D]^e [J]^e [\delta]^e \; r d\theta \, dr \, dz$$

$$- \iint [T_{tr}]^e [\delta]^e \; r d\theta \, dz \qquad (2.35)$$

The displacement δ for the bearing liner is

$$(\delta) = \begin{bmatrix} V_\theta(r,\theta,z) \\ V_r(r,\theta,z) \\ V_z(r,\theta,z) \end{bmatrix}$$

The element equation can be obtained by minimizing the potential energy over each element.

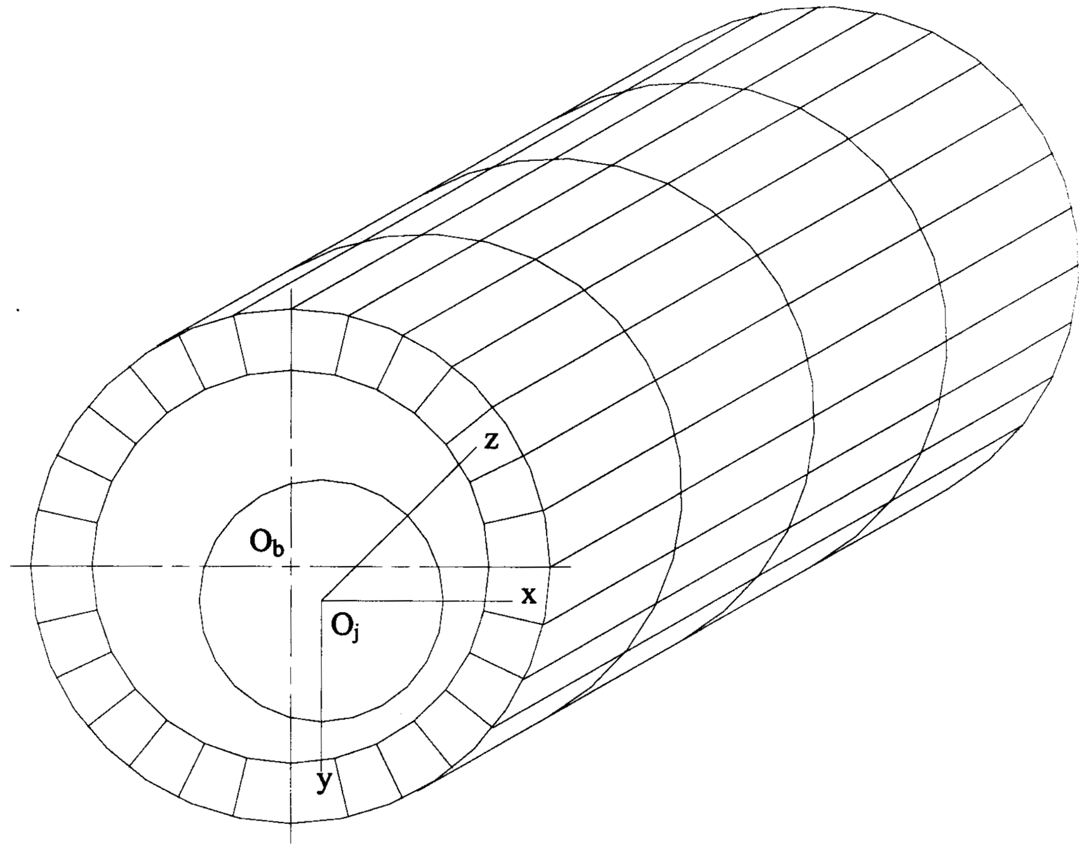

Fig. 2.6 Discretization of Circular Bearing

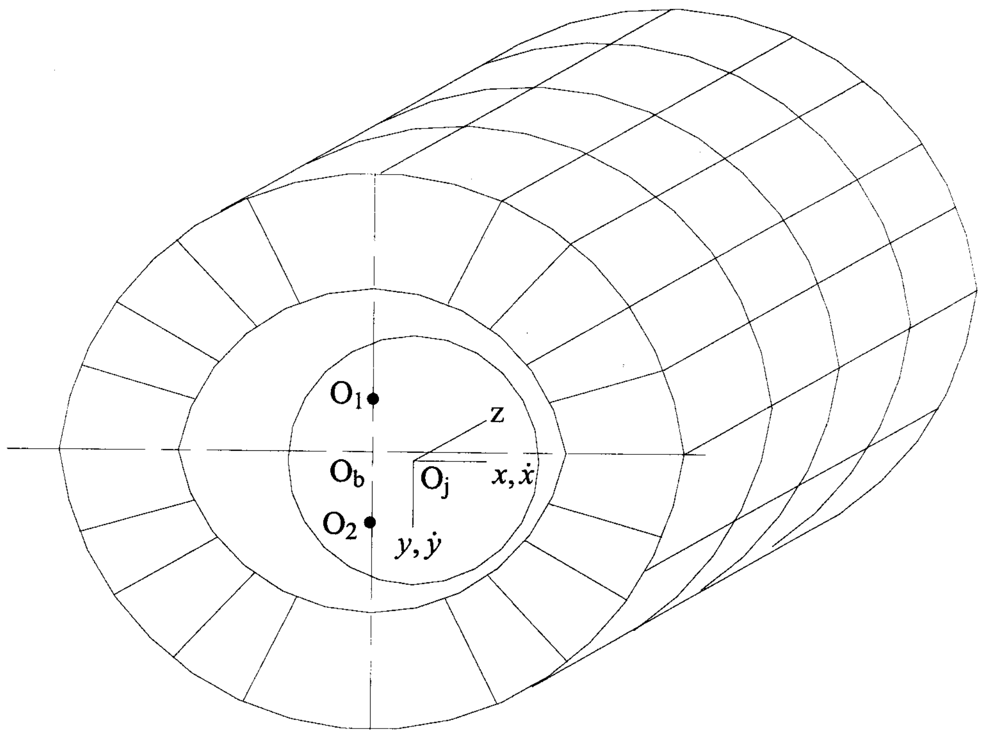

Fig. 2.7 Discretization of Two Lobe Bearing

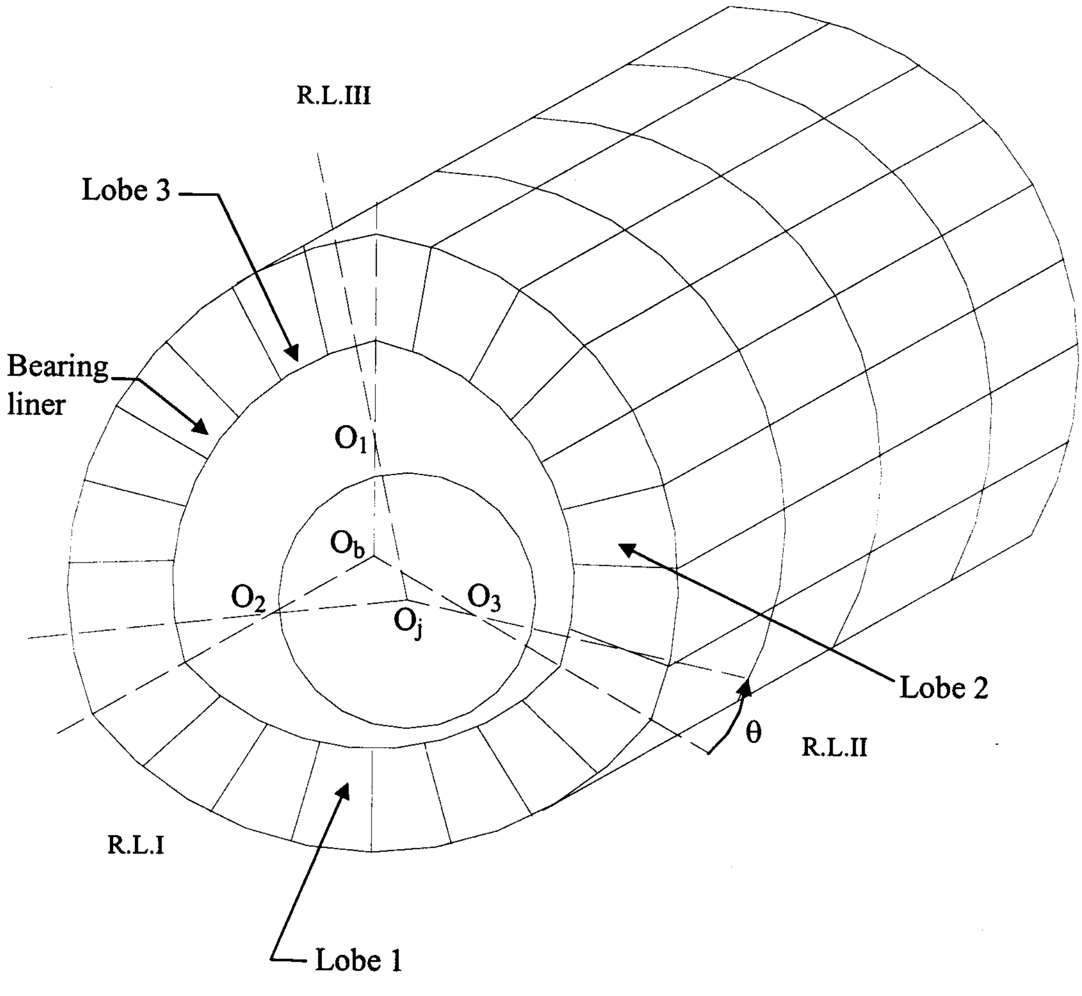

Fig. 2.8 Discretization of Three Lobe Bearing

The displacement field $[\delta]^e$ in e^{th} element can be written as

$$[\delta]^e = \begin{bmatrix} V_\theta \\ V_r \\ V_z \end{bmatrix} = [N]^e [\bar{d}]^e \tag{2.36}$$

where $[N]^e$ represent the element shape function matrix.

$$V_\theta = \sum_{i=1}^{m} V_{\theta i} \bar{N}_i, \quad V_r = \sum_{i=1}^{m} V_{ri} \bar{N}_i, \quad V_z = \sum_{i=1}^{m} V_{zi} N_i$$

and

$$[d]^e = [V_{\theta 1}, V_{r1}, V_{z1}, \ldots\ldots\ldots V_{\theta m}, V_{rm}, V_{zm}]^T \tag{2.37}$$

where m is the number of nodes per element.

Since in the equilibrium position, potential energy of the system is minimum, we have

i.e.,
$$\sum_{e=1}^{n_e} \begin{Bmatrix} \dfrac{\partial \pi^e}{\partial V_{\theta i}} \\ \dfrac{\partial \pi^e}{\partial V_{ri}} \\ \dfrac{\partial \pi^e}{\partial V_{zi}} \end{Bmatrix} = 0 \tag{2.38}$$

where e = 1, 2, …….. n_e and i =1, ……, m

n_e = number of elements

Using the above condition, system equations are reduced to

$$\sum_{e=1}^{n_e} \left[\iiint [J]^{eT} [D]^e [J]^e r d\theta \; dr \; dz - \iint [N][T_{tr}]^e r d\theta \; dz \right] = 0 \tag{2.39}$$

where

$$
[J]^e = \begin{bmatrix} \frac{1}{r}\left[\frac{\partial}{\partial\theta}\right] & \frac{1}{r} & 0 \\ 0 & \frac{\partial}{\partial r} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial r} & \frac{1}{r}\left[\frac{\partial}{\partial\theta}\right] & 0 \\ \frac{\partial}{\partial r} & 0 & \frac{1}{r}\left[\frac{\partial}{\partial\theta}\right] \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial r} \end{bmatrix}
$$

$$
[D] = \begin{bmatrix} D_2 & D_2 & D_1 & 0 & 0 & 0 \\ D_1 & D_2 & D_2 & 0 & 0 & 0 \\ D_2 & D_1 & D_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & D_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & D_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & D_3 \end{bmatrix}
$$

and

$$D_1 = E(1-\gamma)(1+\gamma)(1-2\gamma)$$

$$D_2 = E\gamma/(1+\gamma)(1-2\gamma)$$

$$D_3 = E/2\ (1+\gamma)$$

the above element equation may be written in the matrix form as

$$[K]^e\ [d]^e = [F]^e \qquad (2.40)$$

where $[K]^e$ is the element stiffness matrix and is given by

$$[K]^e = \iint [J]^{eT} [D]^e [J]^e \, r d\theta \, dr \, dz \qquad (2.41)$$

$$[F]^e = \iint [N]^{eT} [T_{tr}]^e \ r d\theta \, dz \qquad (2.42)$$

$$[T_{tr}]^e = [0\ [T_{tr}]^e\ 0] \qquad (2.43)$$

By non dimensionalising, using

$r = \bar{r}t_h, z = t_h\bar{z}, [D] = E[\bar{D}], d = C[\bar{d}]$ and $p = \mu\varpi_j(R_j / C)^2 \bar{p}$ and by using the general assembly procedure, the following global system equations are obtained

$$[K][\bar{d}] = \bar{\psi}[\bar{F}] \tag{2.44}$$

where

$$\bar{\psi} = \frac{\mu_0 \varpi_j (R_j / C)^3}{E} \frac{t_h}{R_j}$$

[K] Stiffness Matrix

[d] Displacement Matrix

[F] Force matrix

2.3.3 Boundary Conditions for The Deformations

Here it is assumed that the bearing liner is contained in a comparatively rigid housing. So the outer surface of the housing does not deform, implying that the nodes in contact with the rigid surface are restrained from moving.

Hence
$$\begin{Bmatrix} \bar{V}_{\theta i} \\ \bar{V}_{ri} \\ \bar{V}_{zi} \end{Bmatrix} = \{0\} \tag{2.45}$$

where 'i' is the number of nodes on the bearing liner of rigid housing interface.

2.3.4 Modification of Film Thickness

For the flexible journal bearing system, the non-dimensional fluid film thickness is expressed as

$$\bar{h} = 1 + \varepsilon \cos\theta + \bar{V}_r \tag{2.46}$$

where $\bar{V}_r$ is the non-dimensional deformation of the bearing bush in the radial direction.