CHAPTER II

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THEORETICAL OVERVIEW

Mathematics, as a science emerged from the early experiences of ancient man. These experiences were given by the system noticed in the various phenomena reflected by nature, like the systematic movement of the innumerable heavenly bodies, the beauty of the symmetry found in plants and the presence of various shapes of the objects around. The need for a system to regularize his social life, made man to think of a language for the purpose of interaction and communication.

The country needs today effective and productive citizens who display scientific and constructive thinking and attitudes in all walks of life. This is possible to a great extent, with carefully devised educational curricula, especially on the school mathematics programme.

The education commission (1964-66) has recommended that "Proper foundation in the knowledge of mathematics should be laid at the school".

According to the commission "the advent of automation and cybernetics in this country marks the beginning of the new scientific industrial revolution and makes it all the imperative to devote special attention to the study of mathematics."

According to Kapoor (1993) in his book, 'Some Aspects of School Mathematics', secondary school mathematics is the basic structure on which the whole super structure of mathematics, mathematical sciences, physical sciences, social sciences and technology in the universities and technical institution rests. Weaknesses in the basic structure led in the past and can lead in future to considerable weaknesses in the super structure. To quote Brown, "Mathematics has become the basic fabric of our social order the strength of that fabric, in fact the very survival of our nation – may well depend upon the amount and kind of mathematics taught in our secondary schools. If we take the responsibility lightly, our children will suffer the consequences of our foolish action". Modern development in science and technology require a new habit of original and critical thinking and the foundations of such a habit have to be laid in secondary school mathematics. The teaching of secondary school mathematics should reflect something of its nature. It must bring out its chief characteristics such as abstractness, precision, generality, logical nature etc. and any topics which do not satisfy these standards, have to go. Theorems in mathematics are as true today as they were yesterday but they can become absolute in as much as more general or more interesting or more abstract or more useful theorems may be discovered.

At the university stage, most of the physical and social sciences require the application of mathematics. Ignorance of mathematics will be a great handicap in the study of many other subjects first rate science and technology can be based only on first rate mathematics and whatever is not first rate has to be discarded.

Shukla (1981) in his study "Identification of major skills involved in mathematics teaching at secondary school stage" found that mathematical problem solving can be best achieved when students are.

- Exposed to the problem properly
- Made to recollect known relationships
- Made to find out the necessities required for the solution and
- Are made to analyse the given data in the light of the relationship and necessaries.

A dominant trend of haste to proceed as fast as, or as much as they could have been quite transparent and evident. Such an accelerated style would by implication, hamper sustained effort on the problem - solving stage and processes, with the result, the students would acquire rapid, but, mechanical drill type skills resulting in the total loss of the thrill and the ecstasy of creative problem solving process.

Mathematics is not merely a product but a process. It is not only knowledge it is an activity also. Its static part is important, its dynamic part is even more vital. Not only mathematical facts are to be taught, methods of arriving at these facts are also to be communicated.

Research studies show that when students discover mathematical ideas and invent mathematical procedures, they have a stronger conceptual understanding of connections between mathematical ideas" (Grouws & Cebulla, 2000). Flewelling and Higginson (2005) state that inquiry, investigations and problem solving "give students the opportunity to use their imagination and to get into the habit of doing so. In contrast, traditional text-based tasks provide the student with little or no such opportunity".

"Rich learning opportunities stem from students engagement in challenging tasks… Classroom tasks can be broadly categorized into those that have the potential for learning and those that provide opportunities for students to practice applying their previously learnt knowledge or skills" (Diezmann, 2005).

Investigations are a good way of supplying the first need. Much research points to the need for mathematical inquiry, investigation and problem solving in mathematics lessons. Words such as discovery, investigation activity and problem solving have become very much a part of the language we now use in talking about mathematics teaching. Mathematics is a discovery of relationships and the expression of the relationships in symbolic (or abstract) form. Edith Biggs has written about discovery, investigation and active learning in an almost synonymous way. Thus, “Investigations are central to the reforms advocated internationally to improve mathematical learning and develop children’s mathematical power” (Baroody & Coslick 1998) cited in Diezmann (2005).

The potential value for using problem solving contents is that it may broaden and develop students mathematical thinking and provide them with an impetus for understanding a greater collection of problems of increasing complexity and mathematical abstraction. (Kapur, 2008., Moreno, 2008 and Bell and Sutherland, 2001).

Using mathematical problems has been advocated as a crucial and motivating component of learning and understanding mathematics. Schoenfeld (1992) notes that, when solving mathematical problems, students develop a deeper understanding of mathematics because it helps them to conceptualize the mathematics being learnt. Stanic and Kilpatrick’s (1989) review of problem solving indicates that historically, mathematical problem solving has been instrumental in achieving a variety of goals within the mathematics curriculum. Furthermore, productive problem solving experiences that move children beyond the routine acquisition of isolated techniques are fundamental in developing higher order mathematical thinking and reasoning (Booker & Bond, 2009 & Polya, 1973).

In the book how to solve it (1973) Polya is explicit in characterizing the heuristics of effective problem solving. Essentially he attempts to understand how people think and the strategies they might use when solving problems. Polya (1973) contends that to solve any problem, the characteristics and properties of the problem should be analyzed. Once the problem is understood then a plan is devised and strategies are implemented and finally, opportunities to reflect upon the solution are required.

Though Polya emphasizes the heuristics of problem solving, he also acknowledges the idea of mathematical connectedness and generality. He suggests that by actively engaging with problem, students can develop the ability to understand the generalities associated with problem solving.

Human beings face a multiple dimensional problems in their lives and they try to solve these problems in a particular way in the light of their previously gained knowledge and experiences. In this regard it is essential for the students to be prepared for future or near future challenges by facing real life, or real like problems in their learning environment, and finding appropriate solution of these problems. Each society expects from its education system that it enables the individuals to become an effective problem solver in their real life (Walker & Lofton, 2003; Chin & Chia, 2004). The roots of problem solving learning are found in Dewey’s thoughts, “that learning by experimentation or doing is more lasting” (Dewey, 1938). Actually the problem solving is how to learn independently. It is the most convenient approach to achieve the aims of teaching learning process. In present era problem based learning is extensively used nearly in all areas including mathematics and was first implemented in medical education in 1950s.

Creativity thrives in an environment in which ideas are valued on their own merit, neither on the basis of how they were produced nor who produced them. In Such an environment, irrationally produced ideas are evaluated with the same regard as those resulting from a rational process.

The development of creativity could be accomplished through teaching creatively and teaching for creativity. Schools may offer a flexible learning atmosphere, where children can express themselves freely and positively.

2.1. PROBLEM BASED LEARNING METHOD

In the problem based learning model the students turn from passive listeners of information receivers to active, free self-learner and problem solvers. It also shifts the emphasis of educational programmes from teaching to learning. It enables the students to learn new knowledge by facing the problems to be solved instead of feeling boredom. Problem based learning affect positively certain other attributes such as problem solving, information acquisition, and information sharing with others, group works, and communication. Again problem solving is a deliberate and serious act, involves the use of some novel method, higher thinking and systematic planned steps for the acquisition set goals.

Problem Solving is one of the five Process Standards of NCTM's Principles and Standards for School Mathematics 2000. NCTM then continues to stress that mathematical problem solving, in its broader sense, is nearly synonymous with doing mathematics. Furthermore, the National Council of Supervisors of mathematics (NCSM), in a position paper Essential Mathematics for the 21st century (NCSM Newsletter, June 1988, Vol. XVII, No.4), lists problem solving as its first of twelve components of essential mathematics. They state the following:

Problem Solving- Learning to solve problems is the principal reason for studying mathematics. Problem solving is a process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with non-texted problems. Problem solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams and using trial and error. Student should see alternate solutions to problems; they should experience problems with more than a single solution.

Problem Solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and then be encouraged to reflect on their thinking. By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, becoming a good problem solver can lead to great advantages. Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics programme. Problem solving in mathematics should involve all five content areas: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis & Probability.

What is a Problem?

Until very recently, a major difficulty in discussing problem solving was the lack of any clear-cut agreement as to what constituted a “problem.” This has finally been resolved; most mathematics educators accept the following definition of a problem:

2.1.1. Definition

A problem is a situation, quantitative or otherwise, that confronts an individual or group of individuals, that requires resolution, and for which the individual sees no apparent path to the solution. The key to this definition is the phrase “no apparent path.” As children pursue their mathematical training, what were the problems at an early age becomes exercise and eventually reduce to mere questions.

Problem is a situation that requires analysis and synthesis of previously learned knowledge to resolve. A problem must satisfy the following three criteria:

- Acceptance: The individual accepts the problem. There is personal involvement, which may be due to any of variety of reasons, including internal motivation, external motivation (peer, parent and/or teacher pressure), or simply the desire to experience the enjoyment of solving a problem.
- Blockage: The individual’s initial attempts of solution are fruitless. His or her habitual responses and patterns of attack do not work.

- Exploration: The personal involvement identified in acceptance forces the individual to explore new methods of attack.

In fact, Polya stated, that "solving the routine problem has practically no chance to the mental development of the student." Usually a problem is stated in words, either orally or written. Then, to solve the problem one translates the words into an equivalent problem using mathematical symbols, solves this equivalent problem, and then interprets the answer. This process is summarized in fig.2.1.

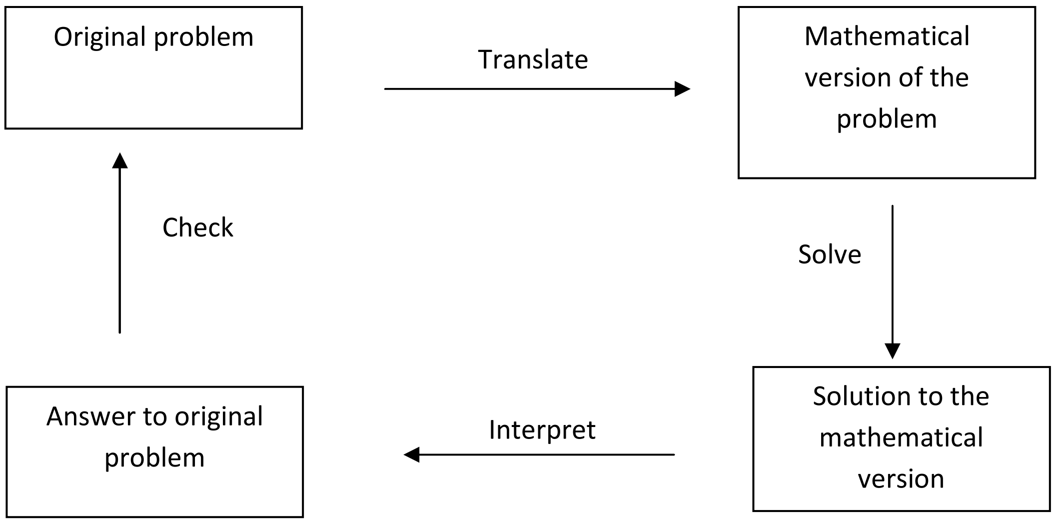

***Fig.2.1:* First steps in becoming a good problem solver by utilizing Polya's four steps.**

2.2 INTRODUCTION TO PROBLEM SOLVING

Polya (1957), the father of modern problem solving died in 1985, leaving mathematics with the important legacy of teaching problem solving. His "ten commandments for teachers" are:

1. Be interested in your subject
2. Know your subject

3. Try to read the faces of your students, try to see the expectations and difficulties and put yourself in their place
4. Realize that the best way to learn anything is to discover it by yourself
5. Give your students not only information, but also know-how mental attitudes & the habit of methodical work
6. Let them learn guessing
7. Let them learn proving
8. Look out for such features of the problem at hand as may be useful in solving the problems to come
9. Let the students find out by themselves as much as is feasible
10. Suggest it, do not force it down their throats

2.3 WHAT IS PROBLEM SOLVING?

Problem solving is a process. It is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation. There are many goals for school mathematics. Two of these are the attainment of information and facts and the ability to use information and facts. The latter ability is an essential part of the problem solving process. In effect, problem solving requires analysis and synthesis. To succeed in problem solving is to learn how to learn.

2.4 WHY A SPECIAL EMPHASIS ON PROBLEM SOLVING?

Little time is devoted to the development of the open-ended thought process that is problem solving. Problem solving is the link between facts and algorithms and real-life problem situations we all face. For most people, mathematics is problem solving! Problem solving shows an interconnection between mathematical ideas. The greater the involvement the better the end product. Thus, a carefully selected sequence of problem solving activities that yield success will stimulate students, leading them to a more positive attitude towards mathematics in general and problem solving in particular. Finally, problem solving is an integral part of the larger area of critical thinking, which is a universally acceptable goal for all education.

2.5 WHAT ARE HEURISTICS?

Problem solving consists of a series of tasks and thought processes that are loosely linked together to form what is called a set of heuristics or a heuristic pattern. There are a set of suggestions and questions that a person must go through in order to resolve a dilemma. Heuristics are general and are applicable to all classes of problems. They provide the direction needed by all people to approach, understand, and obtain answers to problems that confront them. There is no single set of heuristics for problem solving. What is important is that students learn some set of carefully developed heuristics and they develop the habit of applying these heuristics in all problem solving situations. We must do more than merely hand the heuristics to the students, rather instructions must focus on the thinking that the problem solver goes through as he or she considers a problem. It is the process-not the answer-that is problem solving. Applying heuristics is a difficult skill in itself. We must spend time showing students how and when to use each of the heuristics-a prescriptive approach rather than a merely descriptive one.

The term "Heuristic" is derived from a Greek word "Heuisco" which means I find. Here the child is put in place of a discoverer. The method involves finding out by the student instead of merely telling of everything by the teacher. Discovery learning is not like traditional classroom learning. It consists of three main attributes (Bicknell, Holmes and Hoffman, 2000).

- Through exploration and problem solving students create, integrate and generalize knowledge.
- Student driven, interest based activities which the student determines the sequence and frequency.
- Activities to encourage integration of new knowledge into the learner's existing knowledge base.
- Discovery learning conducted through Polya's heuristic approach helped the learner to be active rather than passive.
- Learning is process based rather than fact based.
- Feedback was obtained and hence it motivated the child.
- Understanding is deeper.
- The child is able to create new problems when confronted with an open ended question.

2.6 PROBLEM SOLVING STRATEGIES FROM GEORGE POLYA

Polya (1887 – 1985) was one of the most famous mathematics educators of the 20th century. Polya believed that the skill of problem solving could and should be taught – it is not something that you are born with. He identifies four principles that form the basis for any serious attempt at problem solving:

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back (reflect)

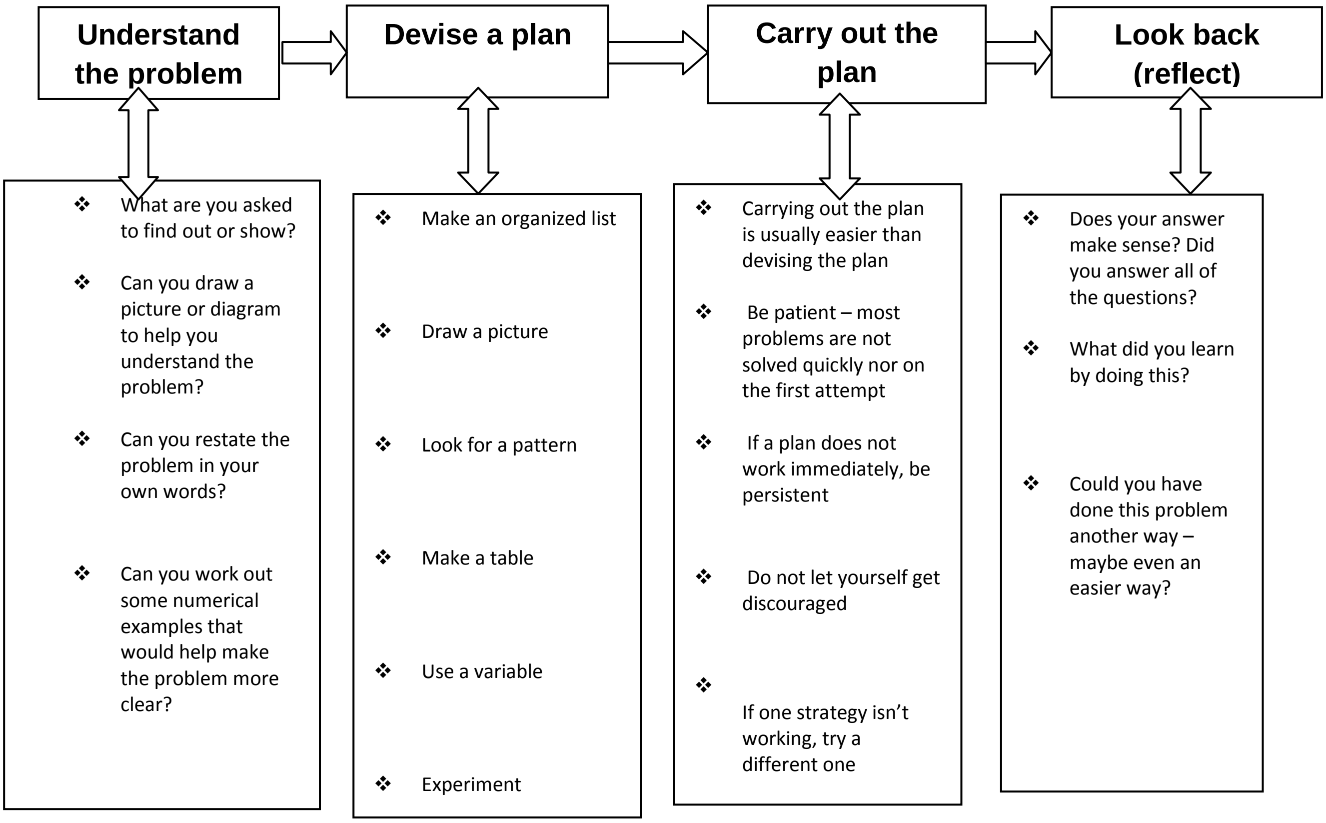

***Fig.2.2*: Steps of Polya's approach**

Yager (2000) has stated that we live in a dynamic society in which social, political and technological conditions are changing continuously, so educators should analyze and evaluate the trends in order to decide an appropriate curricula and method of instruction which will make students ready for real life situation. Today, it is recognized that every person must be empowered to suggest possible explanations, to propose ways to test personal or class, to collect and interpret data obtained, to communicate the process and results to others. In this era of unprecedented breakthroughs in technology and constant change in many aspects of life, educators are challenged more than ever before with the need to develop students who will be adaptable in fast-changing environments. This calls for equipping students with better thinking skills and learning abilities. Concomitant with the quest for the development of skills pertaining to creativity and enterprise is the call for a paradigm shift in education.

2.7 DEFINITION OF MATHEMATICAL CREATIVITY

Runco (1993) describes creativity as a multifaceted construct involving both "divergent and convergent thinking, problem finding and problem solving, self-expression, intrinsic motivation, a questioning attitude, and self-confidence". (Haylock, 1987) summarized many of the attempts to define mathematical creativity. One view "includes the ability to see new relationships between techniques and areas of application and to make associations between possibly unrelated ideas" (Tammadge, as cited in Haylock, 1987). The Russian psychologist Krutetskii characterized mathematical creativity in the context of problem formation (problem finding), invention, independence, and originality (Haylock & Krutetskii, 1976). Others have applied the concepts of fluency, flexibility, and originality to the concept of creativity in mathematics (Haylock, 1997; Jensen, 1973; Kim et al., 2003 & Tuli, 1980).

2.8 DEVELOPMENT OF MATHEMATICAL CREATIVITY

Mathematical creativity is difficult to develop if one is limited to rule-based applications without recognizing the essence of the problem to be solved. The visionary classrooms described by leaders in the National Council of Teachers of Mathematics (NCTM) (2000) enable students to confidently engage in complex mathematical tasks…draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. (NCTM, 2000).

Runco & Chand (1995) explain that finding a problem is the starting point and the key to producing creative products. In current years, finding a problem has sometimes been considered as a creative process in itself

2.9 MATHEMATICAL PROBLEM CREATING ABILITY

Sastry, DSN (2008) expressed his observations on problem posing ability and problem creating ability as follows. Problem posing ability when students began posing their own original mathematical questions and see these questions became the focus of discussion, their perception of the subject was profoundly alerted. When they got to spend time working on these questions, their ownership of the experience produces excitement and motivation. The discussions and activities that follow will help students expand their problem posing repertoire and promote the habit of creating new problems.

Problem posing requires more than the mere tweaking of pre existing questions. He suggested seven basic ways to change a problem to create new problems i.e., change the numbers, change the geometry, change the operation, change the objects under study remove a condition or add new conditions, remove or add content and repeat a process.

Kapur (1978) defined mathematical creativity in terms of ten criteria. Recognize patterns in numbers and space, tend to generalize particular case, see that all intuitive generalization may not be traced, draw a large number of essentially different conclusion for a given hypothesis, see that some parts can be deduced from others on the dependence of facts on one another, insist on precise definition and formulations of problem, make mathematical models, tend to symbolize, ask mathematical questions, recognize the possibility of a large number of answers to a question.