DECLARATION

I declare that the thesis entitled SCHRÖDINGER KITTEN STATES OF A QUBIT-OSCILLATOR SYSTEM: GENERATION AND QUANTUM PROPERTIES IN THE PHASE SPACE submitted by me for the degree of Doctor of Philosophy (Ph.D.) is the record of work carried out by me during the period from November 2011 to June 2018 under the guidance of Dr. Ranabir Chakrabarti, Professor (retd.), Department of Theoretical Physics, University of Madras and has not formed the basis for the award of any Degree, Diploma, Associateship, Fellowship, Titles in this University or any other University or other similar Institution of Higher Learning.

Place: 
Date: 
Signature of the Candidate: B. Virgin Jenisha
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