Chapter 9

MHD non-Darcian Mixed Convection Flow with Soret-Dufour Effects, Chemical Reaction and Non-uniform Heat Source/Sink *

9.1 Introduction

An energy flux can be generated not only by temperature gradients but by composition gradients. The energy flux caused by a composition is called Dufour or diffusion-thermo effect. Temperature gradients can also create mass fluxes, and this is the Soret or thermal-diffusion effect. Generally, the thermal-diffusion and the diffusion-thermo effects are of smaller-order magnitude than the effects prescribed by Fouriers or Ficks laws and are often neglected in heat and mass transfer processes. However, there are exceptions. The thermal-diffusion effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight (Hydrogen-Hellium) and of medium molecular weight (Nitrogen-air) the diffusion-thermo effect was found to be of a magnitude such that it cannot be neglected. Kafoussias and Williams (1995) considered the boundary layer-flows in the presence of Soret, and Dufour effects associated with the thermal diffusion and diffusion-thermo for the mixed forced natural convection. Postelnicu (2004) studied simultaneous heat and mass transfer by natural convection from a

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vertical plate embedded in electrically conducting fluid saturated porous medium, using Darcy-Boussinesq model including Soret, and Dufour effects. Seddeek and Salama (2007) studied the effects of temperature dependent viscosity and thermal conductivity on unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Afify (2009) studied the effects of thermal diffusion and diffusion thermo on free convective heat and mass transfer over a stretching surface considering suction or injection. Tai and Char (2010) study the Soret and Dufour effects on free convection flow of non-Newtonian fluids along a vertical plate embedded in a porous medium with thermal radiation. Tak et al. (2010) analyzed the MHD free convection-radiation interaction along a vertical surface embedded in Darcian porous medium in presence of Soret and Dufours effects. Verpati and Gari (2010) studied the Soret and Dufour effects on unsteady MHD flow past an infinite vertical porous plate with thermal radiation. Details literature review in provided in chapter 1.

In view of the above discussions, authors envisage to investigate the steady two-dimensional hydromagnetic mixed convection and mass transfer flow past a semi-infinite vertical permeable surface embedded in a non-Darcy porous medium taking into account the Soret and Dufour effects in the presence of suction or injection, viscous dissipation, thermal radiation, non-uniform heat source/ sink, and first-order chemical reaction. The problem addressed here is a fundamental one that arise in many practical situations such as polymer extrusion process. Non-linear stretching velocity, surface temperature and surface concentration are considered in the present paper. The nonlinearity of the basic equations and additional mathematical difficulties associated with it, have led to the use of numerical method.

9.2 Mathematical Formulations and Governing Equations

We consider the steady mixed convective heat and mass transfer of viscous incompressible and electrically conducting fluid past a vertical non-linear stretching sheet embedded in a fluid saturated non-Darcian porous medium with the plane \( y = 0 \) of a coordinate system as shown in Fig.9.1. A uniform transverse magnetic field \( B(x) \) is imposed along the \( y \)-axis. The temperature and the species concentration vary with the distance from the origin. The fluid properties are assumed to be constant except the density in the buoyancy terms.
of the linear momentum equation which is approximated according to the Boussinesq’s approximation. Under the usual boundary layer approximation, the governing equations describing the conservation of mass, momentum, energy and concentration in the presence of radiation magnetic field and non-uniform heat source/sink can be written as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{9.1}
\]

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{C_b}{\sqrt{k}} u^2 - \frac{\sigma B^2(x)}{\rho} u + g \beta_T (T - T_{\infty}) + g \beta_c (C - C_{\infty}) \tag{9.2}
\]

\[
u \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \frac{1}{\rho c_p} \frac{\partial}{\partial y} \left( \kappa \frac{\partial T}{\partial y} \right) + \frac{\mu}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{1}{\rho c_p} q''' + \frac{D_m k_T}{c_s c_p} \frac{\partial^2 C}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \tag{9.3}
\]

\[
u \frac{\partial C}{\partial x} + \nu \frac{\partial C}{\partial y} = \frac{D_m}{\gamma^2 y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2} - R(C - C_{\infty}) \tag{9.4}
\]

The boundary conditions for Eqs. (9.1)-(9.4) are expressed as Zheng et al. (2011)

\[
u(x, 0) = U(x) = C_1 x^m, \quad v(x, 0) = v_w(x) = C_2 x^n, \quad T(x, 0) = T_w(x) = T_{\infty} + C_3 x^r, \tag{9.5}
\]

\[
u(x, 0) = C_w(x) = C_{\infty} + C_4 x^r \tag{9.6}
\]

\[
u(x, \infty) = 0, \quad T(x, \infty) = T_{\infty}, \quad C(x, \infty) = C_{\infty} \tag{9.7}
\]

where \( u \) and \( v \) denote the velocity components in \( x \)- and \( y \)- directions, respectively. \( T \) and \( C \) is the fluid temperature and concentration, respectively. \( \rho \) is the fluid density, \( \mu \) is the dynamic viscosity, \( k \) is the permeability of the porous medium, \( \kappa \) is the thermal conductivity, \( c_p \) is the specific heat at constant pressure, \( g \) is the acceleration due to gravity, \( k_T \) is the thermal diffusion ratio, \( \beta_T \) is the coefficient of thermal expansion, \( \beta_c \) is the coefficient of expansion with concentration, \( C_1, C_2, C_3, C_4 \) are the constants, \( U(x) = C_1 x^m \) is the stretching speed of the plate, \( v_w(x) = C_2 x^n \) is the transverse velocity at the surface, \( B(x) = B_0 x^s \) is the applied magnetic field, where \( s = \frac{m-1}{2}, n = \frac{m-1}{2}, r = 2m - 1 \), and \( C_b \) is the form of drag coefficient which is independent of viscosity and other physical properties of the fluid which depend on the geometry of the medium. The stretching surface has a uniform temperature \( T_w \) and the free stream temperature is \( T_{\infty} \) with \( T_w > T_{\infty} \). Also, it has a uniform concentration \( C_w \) and the free stream concentration is \( C_{\infty} \) with \( C_w > C_{\infty} \). \( D_m \) is the coefficient of mass diffusivity, \( T_m \) is the mean fluid temperature, and \( c_s \) is the concentration susceptibility.

The non-uniform heat source/sink \( q''' \) is modeled as

\[
q''' = \frac{k u_w(x)}{x^p} [A^*(T_w - T_{\infty}) e^{-\eta} + (T - T_{\infty}) B^*], \tag{9.8}
\]
where \(A^*\) and \(B^*\) are the coefficients of space and temperature dependent heat source/sink respectively. Here we make a note that case \(A^* > 0, B^* > 0\) corresponds to internal heat generation and \(A^* < 0, B^* < 0\) corresponds to internal heat absorption. The stream function \(\psi(x, y)\) is defined by \(u = \frac{\partial \psi}{\partial y}\) and \(v = -\frac{\partial \psi}{\partial x}\), such that the continuity Eq. (9.1), is satisfied automatically. We have the following similarity transformations:

\[
\eta = y \sqrt{\frac{m + 1}{2} U(x)} \quad \psi = \sqrt{\frac{2}{m + 1} \nu_x (x) f(\eta)}
\]

\[
\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}
\]

For liquid metals, it has been found that the thermal conductivity varies with temperature in an approximately linear manner in the range from \(0^\circ F\) to \(400^\circ F\). Therefore, the thermal conductivity \(\kappa\) is assumed to vary linearly with temperature which is of the form

\[
\kappa = \kappa_\infty [1 + \epsilon \theta(\eta)]
\]

where \(\epsilon\) is a small parameter. Employing the Rosseland diffusion approximation (Pal (2009)) the radiation heat flux is given by

\[
q_r = -\frac{4 \sigma^* \partial T^4}{3 k^* \partial y}
\]

where \(\sigma^*\) is the Stephan-Boltzmann constant and \(k^*\) is the mean absorption coefficient. Let us introduce the wall temperature excess ratio parameter \(\theta_w = \frac{T_w}{T_\infty}\). Thus \(T^4\) may be expressed as

\[
T^4 = T_\infty^4 \left\{1 + (\theta_w - 1)\theta\right\}^4
\]

Substituting Eqs. (9.9)-(9.10) into the governing Eqs. (9.2)-(9.4) and using Eq. (9.13) we finally obtain a system of non-linear ordinary differential equations with appropriate boundary conditions:

\[
f''' + f f'' - \frac{2}{m + 1} \left[ m f'^2 + M f' + k_1 f' + F^* f'^2 - Gr \theta - Gc \phi \right] = 0
\]

\[
f' \theta' - \frac{2(2m - 1)}{m + 1} \frac{\theta f'}{P_r} + E_c f'^2 + \frac{2}{m + 1} \frac{1 + \epsilon \theta}{P_r} \left( A^* f' + B^* \theta \right) + D f \phi'' + \left(1 + \epsilon \theta\right) \left[ (1 + N_r \{(\theta_w - 1)\theta\}^3) \theta \right]' = 0
\]

\[
\frac{1}{S_c} \phi'' - \frac{2(2m - 1)}{m + 1} f' \phi + f \phi' + S_c \theta'' - \frac{2}{m + 1} R_1 \phi = 0
\]
The boundary conditions (9.5), (9.6) and (9.7) become after using (9.9)-(9.10) as

\[
\begin{align*}
  f(0) &= f_w, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad (9.18) \\
  f'(*) &= 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0. \quad (9.19)
\end{align*}
\]

where primes denote differentiation with respect to \( \eta \), \( k_1 = \frac{\nu}{k_c} \) is the porous parameter, \( M = \frac{\sigma B_0^2}{\rho c_1} \) is the magnetic parameter, \( G_c = \frac{g \beta_c (C_w - C_\infty)}{C_1^2 2^{m-1}} \) is the local modified Grashof number, \( Gr = \frac{g \beta (T_w - T_\infty)}{C_1 x^2 m^{m-1}} \) is the local Grashof number, \( F^* = \frac{C_b x \sqrt{k}}{\kappa} \) is the inertia coefficient, \( Nr = \frac{16 \sigma^* T^3 \kappa}{k^*} \) is the thermal radiation parameter, \( Pr = \frac{\mu c_p \kappa}{\nu} \) is the Prandtl number, \( D_f = \frac{D_m k_t (C_w - C_\infty)}{\alpha m (C_w - C_\infty)} \) is the Dufour number, \( S_c = \frac{\nu D_m}{\kappa} \) is the Schmidt number, and \( S_r = \frac{D_m k_t (C_w - C_\infty)}{\alpha m (C_w - C_\infty)} \) is the Soret number, \( f_w = -C_2 \sqrt{\frac{2}{(m+1)\nu C_1}} \) is the suction or injection parameter, \( R_1 = \frac{R e}{\theta} \) is the Chemical reaction parameter.

The quantities of physical interest in this problem are the local skin friction co-efficient, the local Nusselt number, and the local Sherwood numbers, which are defined by

\[
\begin{align*}
  C_f &= \frac{\tau_w}{\left( \frac{\partial u}{\partial y} \right)_{y=0}}, \quad Nu_x = \frac{x q_w}{\kappa (T_w - T_\infty)}, \quad Sh_x = \frac{x m_w}{D_m (C_w - C_\infty)} \quad (9.20)
\end{align*}
\]

where

\[
\begin{align*}
  \tau_w &= \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -\left( \frac{16 \sigma^* T^3}{3 k^*} + \kappa \right) \left( \frac{\partial T}{\partial y} \right)_{y=0}, \quad m_w = -D_m \left( \frac{\partial C}{\partial y} \right)_{y=0}. \quad (9.21)
\end{align*}
\]

Using non-dimensional variables (9.9), (9.10) and relation (9.21) into (9.20), we obtain

\[
\begin{align*}
  C_f Re^{1/2} x = 2 \sqrt{\frac{m+1}{2}} f''(0), \quad Nu_x = -\sqrt{\frac{m+1}{2}} \left( 1 + Nr \theta_w^3 \right) \theta'(0) \quad (9.22)
\end{align*}
\]

\[
\begin{align*}
  Sh_x = -\sqrt{\frac{m+1}{2}} \phi'(0). \quad (9.23)
\end{align*}
\]

The coupled ordinary differential Eqs. (9.14)-(9.17) are of third order in \( f \), and second order in \( \theta \) and \( \phi \) which have been reduced to a system of seven simultaneous equations of first-order for seven unknowns following the method of superposition.

### 9.3 Results and Discussion

Eqs. (9.13)-(9.17) constitute highly non-linear coupled boundary value problem of third and second-order. Thus we have developed most effective numerical shooting technique with fifth-order Runge-Kutta-Fehlberg integration algorithm. The results are presented graphically in Figs. 9.2-9.8 and conclusions are drawn for flow field and other physical
quantities of interest that have significant effects. Comparisons of the present results with previously works are performed and excellent agreement has been obtained. Comparison of our results of $-\theta'(0)$ with those obtained by Abel and Mahesha (2008) (see Table 9.1) show an excellent agreement.

Fig. 9.2 illustrate that the temperature profile for different values of suction/injection parameter $f_w$. It can be seen that the temperature distribution increases with increase in injection parameter $f_w < 0$ and decrease with increase in suction parameter $f_w > 0$. There is no peak formulation for the plots of temperature distribution for any value of the suction/injection parameter $f_w$. Further, it is observed that temperature decreases with increase in the space variable $\eta$ till the boundary condition on temperature satisfies as $\eta \rightarrow \infty$. The skin-friction coefficient decreases as the magnetic parameter $M$ increases, while it increases as a result of increasing the value of Dufour number $D_f$, as shown in Fig.9.3. The effect of the magnetic field on the local Sherwood number, in terms of $-\phi'(0)$, is displayed in Fig. 9.4. It is observed that the local Sherwood number increases with increasing the magnetic field parameter $M$ whereas reverse trend is observed by increasing the value of the Dufour number $D_f$.

It is observed from Fig. 9.5 that the local skin-friction coefficient is reduced due to increase in the magnetic field strength, as expected, since the applied magnetic field tends to impede the flow motion and thus reduce the surface friction force, whereas reverse effect is observed by increasing the value of the Soret number $Sr$. Fig. 9.6 shows clearly that the presence of magnetic field gives enhanced local Nusselt number, in terms of $-\theta'(0)$, in a manner that the local Nusselt number increases as $M$ increases, whereas reverse trend is observed as the value of the Soret number is increased. It is found that increasing the value of $k_1$ is to decrease the local skin-friction coefficient whereas reverse trend is observed by increasing the value of $A^*$ as shown in Fig. 9.7. The influences of $B^*$ and $k_1$ on Sherwood number are shown in Fig. 9.8. It is observed from this figure that the effects of increasing $k_1$ is to increase the Sherwood number, whereas no significant effect is seen by increasing the value of the temperature-dependent heat source/sink parameter $B^*$.

### 9.4 Conclusions

The problem of two-dimensional MHD non-Darcy mixed convection flow due to a non-linear stretching sheet in the presence of variable thermal conductivity, Soret and Dufour
effects was investigated. The numerical results obtained are compared with previously reported results available in the open literature and the present results are found to be in excellent agreement. From the present investigation, the following conclusion could be drawn:

(i) Increase in the magnetic field parameter $M$ is to decrease the skin-friction coefficient $f''(0)$, whereas Soret and Dufour effects is to increase the value of $f''(0)$

(ii) The presence of space-dependent heat source/sink parameter $A^*$ is to increase skin friction coefficient, whereas no significant effect is seen by increasing the value of temperature-dependent heat source/sink parameter $B^*$.

(iii) Inclusion of Soret effect on the Sherwood number decrease $-\theta'(0)$ while the increase of porous parameter $k_1$ is to decrease skin-friction coefficient.

Table 9.1: Comparison of wall temperature gradient $-\theta'(0)$ for various values of $Pr$ with $M = k_1 = E_c = F^* = A^* = B^* = \epsilon = D_f = S_r = F^* = Gr = Gc = R_1 = S_c = Nr = 0.0$ and $m = 1.0$

<table>
<thead>
<tr>
<th>Pr</th>
<th>Abel and Mahesha (2008)</th>
<th>Present Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>3.0</td>
<td>1.92368</td>
<td>1.923683</td>
</tr>
<tr>
<td>10.0</td>
<td>3.72067</td>
<td>3.720674</td>
</tr>
</tbody>
</table>
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Figure 9.1: Schematic diagram of the physical model and coordinate system.

\[ u = u_w(x) = c_1 x^n, \quad T_w(x) = T_\infty + c_3 x^f, \quad c_w(x) = c_4 x^e \] 
(surface conditions)

Figure 9.2: Effect of suction/injection parameter \( f_w \) on temperature profile.
Figure 9.3: Influence of magnetic parameter $M$ on the dimensionless skin-friction coefficient $f''(0)$ with $D_f$.

Figure 9.4: Influence of magnetic parameter $M$ on the local Sherwood number $-\phi'(0)$ with $D_f$. 
Figure 9.5: Influence of magnetic parameter $M$ on the dimensionless skin-friction coefficient $f''(0)$ with $S_r$.

Figure 9.6: Influence of magnetic parameter $M$ on the local Nusselt number $-\theta'(0)$ with $S_r$. 
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Figure 9.7: Influence of porous parameter $k_1$ on the dimensionless skin-friction coefficient $f''(0)$ with $A^*$. 

Figure 9.8: Influence of porous parameter $k_1$ on the local Sherwood number $-\phi'(0)$ with $B^*$. 

Parameters:

- For Figure 9.7: $Gr=2.0$, $Gc=6.0$, $m=2.0$, $f^*=0.1$, $D_1=0.04$, $S_f=0.4$
- For Figure 9.8: $Pr=0.71$, $S_f=0.22$, $c=0.5$, $E_f=0.1$, $N_r=0.2$, $\theta_1=1.8$

Conditions:

- For Figure 9.7: $A^*=-0.4$, $R_1=2.0$, $f_1=0.5$, $M=0.4$
- For Figure 9.8: $B^*=-0.4$, $r=2.0$, $f_1=0.5$, $M=0.4$