Chapter 2

Electrodynamics at the surface of a solid

In photofield emission also like in other electron emissive phenomena, say photoemissions, field emissions, inverse or two photon photoemission, the intensity of photoassisted field emission current (PFEC) depends strongly on the nature of the incidence radiation and its interaction with the solid. Due to this therefore, the exact formulation for the vector potential of the incident radiation is required. This will require a systematic study of the electrodynamics of the surface and bulk of the solid under investigation which will require appropriately defined dielectric response functions for these regions. We have seen in the last chapter that in calculations of PFEC, vector potential is involved in the evaluation of matrix element. This is therefore very important in the study of photofield emission.

In this chapter, we will present the model for dielectric functions and detailed calculations of the vector potential with the help of solutions of Maxwell's equations. There had been several studies on the electrodynamics of the metal surface to define the vector potential which had been used in emission spectroscopies like photoemission, inverse photoemission, two photon photoemissions etc. but we do not find any one of this type for the case of photofield emission studies. However, we find that there is a very strong correlation between photofield emissions and photoemissions especially when we consider the emission of electrons from the surface region only, hence the vector potential used in the process of photoemission should be also applicable in the case of photofield emissions. We will therefore consider in this chapter the details discussion about the electrodynamics of the surface with regard to deriving a formula for the vector potential which will be used in the matrix element for calculating the photofield emission current. The method of calculations will be one as used by Thapa *et. al.* (1994, 1995) in the study of photoemisions. This model had been at first designed by Bagchi and Kar (1978) but for the region of the surface defined by $-\frac{d}{2} \le z \le \frac{d}{2}$.

2. 1. Dielectric model :

For calculation of electromagnetic field the dielectric model is shown in Fig. 2.1. The metal is assumed to occupy the space to the left of $z = 0$ plane. The dielectric constant varies linearly over the surface region $-d < z < 0$, where it is a local function interpolating between the bulk value inside the metal $(z \le -d)$ and the vacuum value (unity) outside $(z \ge 0)$. The dielectric model which is depend on the frequency is given by:

$$\varepsilon_1(z) = \begin{cases} \varepsilon_1 & z \le -d \quad (bulk) \\ 1 + (1-\varepsilon_1)\dfrac{z}{d} & -d \le z \le 0 \quad (surface) \\ 1 & z \ge 0 \quad (vacuum). \end{cases} \tag{2.1}$$

and,

$$\varepsilon_2(z) = \begin{cases} \varepsilon_2 & z \le -d \quad (bulk) \\ -\varepsilon_2\dfrac{z}{d} & -d \le z \le 0 \quad (surface) \\ 0 & z \ge 0 \quad (vacuum). \end{cases} \tag{2.2}$$

Such that, we can write :

$$\varepsilon(\omega,z) = \begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) + i\varepsilon_2(\omega), & z < -d \quad (bulk) \\ 1 - [1-\varepsilon(\omega)]\dfrac{z}{d}, & -d < z < 0 \quad (surface) \\ 1, & z > 0 \quad (vacuum). \end{cases} \tag{2.3}$$

where d is the width of the surface, and $\varepsilon(\omega)$ is a complex dielectric function. The refractive index of the bulk metal $\tilde{n}$ is also a complex one such that $\tilde{n} = \sqrt{\varepsilon_1 + i\varepsilon_2} = n + ik$. The real part $\varepsilon_1(\omega)$ and imaginary part $\varepsilon_2(\omega)$ are defined in the following way to simplify the calculation,

$$\left.\begin{aligned} \alpha_1 &= 1; & \beta_1 &= \frac{(1-\varepsilon_1)}{d} \\ \alpha_2 &= 0; & \beta_2 &= \frac{-\varepsilon_2}{d} \end{aligned}\right\} \tag{2.4}$$

such that

$$\left.\begin{aligned} \varepsilon_1(z) &= \alpha_1 + \beta_1 z \\ \varepsilon_2(z) &= \alpha_2 - \beta_2 z \end{aligned}\right\} \tag{2.5}$$

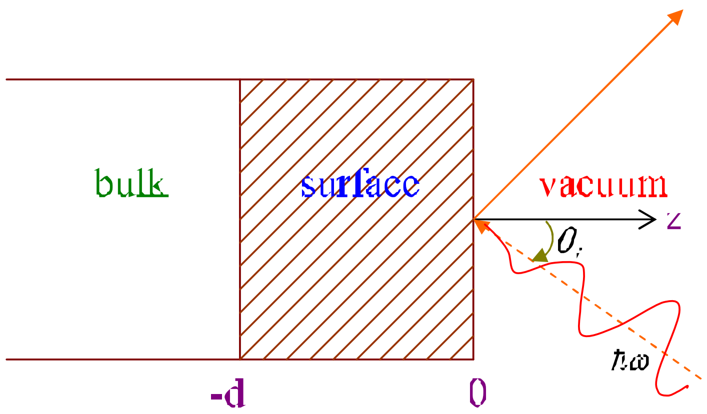

Figure 2. 1 Dielectric model used for the calculation of vector potential. Here d is the width of the surface.

2. 2. Calculation of Electromagnetic Field:

The incident $p-$polarised radiation is considered to be a light of angular frequency ω with θ_i as the angle of incidence on the metal surface defined by the x-y plane as shown in Fig. 2.1. A gauge was chosen in which the scalar potential is set equal to zero i.e. $\phi(\vec{r},t) = 0$, such that the electromagnetic field $\boldsymbol{E}(\boldsymbol{K},\omega,z)$ can be expressed in terms of the vector potential as

$$\boldsymbol{E}(\boldsymbol{K},\omega,z) = -\frac{i\omega}{c}\boldsymbol{A}(\boldsymbol{K},\omega,z), \tag{2.6}$$

where $\boldsymbol{K} = \frac{\omega}{c}\sin\theta_i$ is the parallel component of wave vector.

The magnetic field is related to vector potential by the formula

$$\boldsymbol{B} = \nabla\times\boldsymbol{A}. \tag{2.7}$$

Let $\boldsymbol{A}(\mathbf{r},t) = \boldsymbol{A}(z).e^{i(\boldsymbol{K}.\boldsymbol{\rho}-\omega t)}$, where $\boldsymbol{\rho} = x\hat{x} + y\hat{y}$ and $\boldsymbol{K} = K\hat{x} = \hat{x}\,(\frac{\omega}{c}\sin\theta_i)$.

Therefore, $\frac{\partial}{\partial t}\boldsymbol{A}(\mathbf{r},t) = \boldsymbol{A}(z).e^{i(\boldsymbol{K}.\boldsymbol{\rho}-\omega t)}(-i\omega) = -i\omega\boldsymbol{A}(\mathbf{r},t)$,

$$\text{i.e } \frac{\partial}{\partial t}\langle\rangle = -i\omega\,. \tag{2.8}$$

Also, $\nabla\,\boldsymbol{A}(\mathbf{r},t) = \nabla\,\boldsymbol{A}(z)e^{i(K\hat{x}(x\hat{x}+y\hat{y})-\omega t)}$

$$= \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\left[\boldsymbol{A}(z)\,e^{i(Kx-\omega t)}\right]$$

$$= \left(iK\hat{x} + \hat{z}\frac{\partial}{\partial z}\right)\boldsymbol{A}(\mathbf{r},t)\,.$$

$$\text{Therefore,}\quad \nabla \leftrightarrow \left(iK\hat{x} + \hat{z}\frac{\partial}{\partial z}\right). \tag{2.9}$$

From Maxwell's equations

$$\left.\begin{aligned}
&\nabla . \boldsymbol{D} = 0 \\
&\nabla . \boldsymbol{B} = 0 \\
&\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \\
&\nabla \times \boldsymbol{B} = \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{4\pi}{c}\boldsymbol{J}
\end{aligned}\right\} \tag{2.10}$$

we assume $\mu = 1 \Rightarrow \boldsymbol{B} = \boldsymbol{H}$. Also, $\boldsymbol{J}_{ext} = 0$, i.e. there are no external currents and charges.

Since $\boldsymbol{J} = \boldsymbol{J}_{ind} = \sigma \boldsymbol{E}$, the last of Maxwell's equations gives

$$\nabla \times \boldsymbol{B} = \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{4\pi\sigma}{c}\boldsymbol{E}$$

$$= \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{4\pi\sigma}{c}\frac{1}{(-i\omega)}\frac{\partial \boldsymbol{E}}{\partial t}, \qquad \left(\because \ \frac{\partial \boldsymbol{E}}{\partial t} = -i\omega \boldsymbol{E}\right)$$

$$= \frac{1}{c}\left(1 + \frac{4\pi\sigma i}{\omega}\right)\frac{\partial \boldsymbol{E}}{\partial t} = \frac{\varepsilon}{c}\frac{\partial \boldsymbol{E}}{\partial t}, \tag{2.11}$$

where $\varepsilon = 1 + \dfrac{4\pi i \sigma}{\omega}$.

In terms of the vector potential, Eq. (2.7) can be written as

$$\nabla \times \nabla \times \boldsymbol{A} = \nabla(\nabla . \boldsymbol{A}) - \nabla^2 \boldsymbol{A} = -\frac{\varepsilon}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2}. \tag{2.12}$$

Substituting Eqs. (2.8) and (2.9) in (2.12) we have,

$$\left(-K^2 + \frac{\partial^2}{\partial z^2}\right)\boldsymbol{A}(z) + \frac{\varepsilon\omega^2}{c^2}\boldsymbol{A}(z) - \left(i\boldsymbol{K} + \hat{z}\frac{\partial}{\partial z}\right)\left[\left(i\boldsymbol{K} + \hat{z}\frac{\partial}{\partial z}\right).\boldsymbol{A}(z)\right] = 0 \tag{2.13}$$

There arise two cases of polarisation. The *s*-polarisation and the *p*-polarisation. We will consider the case of *p*-polarisation in which, $\boldsymbol{A}$ is in the *xz*-plane and $\boldsymbol{B} \parallel \mathbf{y}$. Hence, the *x*- and *z*- components of $\boldsymbol{A}$ can be deduced from Eq. (2.13).

Firstly taking the x-component of $\boldsymbol{A}$:

$$\left(-\boldsymbol{K}^2+\frac{\partial^2}{\partial z^2}\right)A^x(z)\hat{x}+\frac{\varepsilon\omega^2}{c^2}A^x(z)\hat{x}-\left(iK\hat{x}+\hat{z}\frac{\partial}{\partial z}\right)\left[\left(iK\hat{x}+\hat{z}\frac{\partial}{\partial z}\right).A^x(z)\hat{x}\right]=0$$

or, $$\frac{d^2A^x(z)\hat{x}}{dz^2}+\frac{\varepsilon\omega^2}{c^2}A^x(z)\hat{x}=iK\frac{dA^z(z)\hat{z}}{dz}$$

i.e. $$\frac{d^2A_x}{dz^2}+\frac{\varepsilon\omega^2}{c^2}A_x=iK\frac{dA_z}{dz}, \qquad (2.14)$$

where $A_x=A^x(z)\hat{x}$, $\quad A_z=A^z(z)\hat{z}$.

Now the z-component of A will give,

$$\left(-\boldsymbol{K}^2+\frac{\partial^2}{\partial z^2}\right)A^z(z)\hat{z}+\frac{\varepsilon\omega^2}{c^2}A^z(z)\hat{z}-\left(iK\hat{x}+\hat{z}\frac{\partial}{\partial z}\right)\left[\left(iK\hat{x}+\hat{z}\frac{\partial}{\partial z}\right).A^z(z)\hat{z}\right]=0$$

or, $$-\boldsymbol{K}^2A^z(z)\hat{z}+\frac{d^2A^z(z)\hat{z}}{dz^2}+\frac{\varepsilon\omega^2}{c^2}A^z(z)\hat{z}-iK\frac{dA^x(z)\hat{x}}{dz}-\frac{d^2A^z(z)\hat{z}}{dz^2}=0$$

i.e. $$\left(\frac{\varepsilon\omega^2}{c^2}-\boldsymbol{K}^2\right)A_z=iK\frac{dA_x}{dz}. \qquad (2.15)$$

We also note from Eq. (2.7) that

$$B=\nabla\times A=\left(iK\hat{x}-\hat{z}\frac{\partial}{\partial z}\right)\times\left(A^x\hat{x}+A^z\hat{z}\right)=\frac{\partial A_x}{\partial z}-iKA_z,$$

where $A_x=A^x\hat{x}$ and $A_z=A^z\hat{z}$.

Dividing Eq. (2. 14) by ε and differentiating with respect to z, we have

$$\frac{1}{\varepsilon}\frac{d^3A_x}{dz^3}-\frac{iK}{\varepsilon}\frac{d^2A_z}{dz^2}+\frac{\omega^2}{c^2}\frac{dA_x}{dz}=0. \qquad (2.16)$$

Also Eq. (2. 15) gives $$iK\left(\frac{\varepsilon\omega^2}{c^2}-\boldsymbol{K}^2\right)A_z-K^2\frac{dA_x}{dz}=0. \qquad (2.17)$$

Equating Eqs. (2.16) and (2.17) results in Landau and Lifshitz equation, viz.

$$\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon}\frac{\partial \boldsymbol{B}}{\partial z}\right)+\left(\frac{\omega^2}{c^2}-\frac{\boldsymbol{K}^2}{\varepsilon}\right)\boldsymbol{B}=0\,. \tag{2.18}$$

We must solve this equation under the boundary condition that both B and $\frac{\partial B}{\partial z}$ are continuous. Once B is known, the electric field components can be found by using Maxwell's equation, i.e.

$$-i\omega \boldsymbol{E}=\frac{c}{\varepsilon}(\nabla\times \boldsymbol{B})$$

$$=\frac{c}{\varepsilon}\left\{\left(iK\hat{x}+\hat{z}\frac{\partial}{\partial z}\right)\times\left(B\hat{y}\right)\right\}.$$

In terms of Cartesian components,

$$\left.\begin{aligned} E_x &= \frac{1}{ik\varepsilon}\frac{\partial B}{\partial z}\\ E_z &= -\frac{K}{k\varepsilon}B=-\frac{\sin\theta_i}{\varepsilon}B \end{aligned}\right\}. \tag{2.19}$$

To solve Eq. (2.18), we follow the prescription of Landau and Lifshitz and use the substitution $B(z)=u(z)\sqrt{\varepsilon(z)}$ and by substituting $\frac{\omega^2}{c^2}=k^2$ and $K=k\sin\theta_i$, we obtain

$$\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon}\right)\left(\frac{\partial B}{\partial z}\right)-\frac{\partial^2 B}{\partial z^2}\frac{1}{\varepsilon}+\frac{k^2}{\varepsilon}\left(\varepsilon-\sin^2\theta_i\right)B=0$$

or $-\frac{1}{\varepsilon}\frac{\partial\varepsilon}{\partial z}\frac{\partial}{\partial z}\left(u\sqrt{\varepsilon}\right)-\frac{\partial}{\partial z}\left(\frac{\partial u\sqrt{\varepsilon}}{\partial z}\right)+k^2\left(\varepsilon-\sin^2\theta_i\right)u\sqrt{\varepsilon}=0\,.$

The final result gives

$$\frac{d^2u}{dz^2}-k^2\left(\varepsilon-\sin^2\theta_i\right)u-\left[\frac{1}{2\varepsilon}\frac{d^2\varepsilon}{dz^2}-\frac{3}{4}\frac{1}{\varepsilon^2}\left(\frac{d\varepsilon}{dz}\right)^2\right]u=0\,. \tag{2.20}$$

In Eq. (2.20), clearly $\frac{d^2\varepsilon}{dz^2} = 0$ everywhere except for $z = -d$ and 0, where it blows up. Now substituting the values of dielectric constants for each region in Eq. (2.20), we can obtain the expression for the magnetic field for the three regions of the metal.

In the first region, $z \geq 0$ (vacuum), where $\varepsilon = 1$, we have

$$\frac{d^2u}{dz^2} - k^2\cos^2\theta_i u = 0, \tag{2.21}$$

whose solution is $B = B_0 e^{-ik_z z} + B_0'' e^{ik_z z}$. (2.22)

In the second region, $z \leq -d$ and $\varepsilon = \varepsilon_1 + i\varepsilon_2 = \hat{n}$

$$\frac{d^2u}{dz^2} - k^2\hat{n}^2\cos^2\gamma u = 0. \tag{2.23}$$

The solution is (absorbing $\hat{n}$ into the constant coefficient)

$$B = B_o' e^{-ik(\hat{n}\cos\gamma)z}. \tag{2.24}$$

For the third region, $-d \leq z \leq 0$ (surface),

$$\frac{d^2u}{dz^2} + k^2\left(\varepsilon - \sin^2\theta_i\right)u - \frac{3}{4}\frac{1}{\varepsilon^2}\left(\frac{d\varepsilon}{dz}\right)^2 u = 0. \tag{2.25}$$

Using Eqs. (2.4) and (2.5), Eq. (2.25) becomes

$$\frac{d^2u}{dz^2} + k^2(b' + cz)u - \frac{3}{4}\frac{1}{(b' + cz)^2}c^2u = 0, \tag{2.26}$$

where $b' = \alpha_1 + i\alpha_2 = b - \sin^2\theta_i$ is a short hand notation introduced purely for convenience. Eq. (2.26) has the solution $u(z) = Az'^{3/2} + Bz'^{-1/2}$.

But $\varepsilon(z) = b' + cz = cz'$. Therefore, $B(z) = Cz'^2 - D = C\left[z + \frac{(\alpha_1 + i\alpha_2)}{\beta_1 + i\beta_2}\right]^2 - D$

i.e. $$B(z) = C\left|z - \frac{b'}{\varepsilon}\right|^2 + D, \qquad -d \le z \le 0. \tag{2.27}$$

The constants C and D are determined by matching the boundary conditions at $z = -d$ and $z = 0$.

At $z = 0$, from Eqs. (2.22) and (2.27) we have,

$$\left.\begin{aligned} B_0 + B_0^r &= C\left(\frac{b'}{\varepsilon}\right)^2 + D \\ ik\cos\theta_i\left[-B_0 - B_0^r\right] &= 2C\left(\frac{b'}{\varepsilon}\right) \end{aligned}\right\}. \tag{2.28}$$

At $z = -d$, from Eqs. (2, 24) and (2. 27) we have,

$$\left.\begin{aligned} C\left(-d - \frac{b'}{\varepsilon}\right)^2 + D &= B_0' e^{ik(\alpha-i\beta)d} \\ 2C\left(-d - \frac{b'}{\varepsilon}\right) &= -ik(\alpha + i\beta)B_0' e^{-ik(\alpha+i\beta)(-d)} \end{aligned}\right\}. \tag{2.29}$$

Calculations from Eqs. (2.28) and (2.29) shows the results as:

$$\frac{B_0^r}{B_0} = \frac{-1 - \cos\theta_i\left\{\dfrac{\varepsilon}{\alpha + i\beta} - \dfrac{ikd}{2}(1-\varepsilon)\right\}}{1 + \cos\theta_i\left\{\dfrac{\varepsilon}{\alpha + i\beta} - \dfrac{ikd}{2}(1+\varepsilon)\right\}}, \tag{2.30}$$

$$\frac{C}{B_0} = \frac{ik\cos\theta_i}{d}(1-\varepsilon)\frac{1}{1 - \cos\theta_i\left\{\dfrac{\varepsilon}{\alpha + i\beta} - \dfrac{ikd}{2}(1-\varepsilon)\right\}}, \tag{2.31}$$

$$\frac{D}{B_0} = \varepsilon\cos\theta_i\frac{\left|\dfrac{2}{(\alpha - i\beta)} - ikd\left(\dfrac{\varepsilon}{1-\varepsilon}\right)\right|}{1 - \cos\theta_i\left[\dfrac{\varepsilon}{\alpha + i\beta} - \dfrac{ikd}{2}()(1-\varepsilon)\right]}, \tag{2.32}$$

and, $$\frac{B_0'}{B_0} = \frac{2\varepsilon\cos\theta_i}{\alpha+i\beta}\frac{e^{-ikd(\alpha+i\beta)/2}}{1+\cos\theta_i\left\{\frac{\varepsilon}{\alpha-i\beta}-\frac{ikd}{2}(1+\varepsilon)\right\}}. \tag{2.33}$$

Therefore, the expression for magnetic field can be obtained as

$$\frac{B(z)}{B_0} = \begin{cases} e^{-ikz\cos\theta_i} + \frac{B_0''}{B_0}e^{ikz\cos\theta_i} & z > 0 \\ \left(\frac{C}{B_0}\right)\left[z - \frac{d}{1-\varepsilon}\right]^2 & -d \le z \le 0 \\ \frac{B_0'}{B_0}e^{ik(\alpha+i\beta)z} & z \le -d \end{cases} \tag{2.34}$$

where B_0 = amplitude of the magnetic field.

We now compute the electric field from Eqs. (2.34), using Eq. (2.19). Recall that for the incident electromagnetic wave in vacuum, *the amplitude of the electric field is the same as the amplitude of the magnetic field, ie* $E_0 = B_0$. We also recall our model of linear dielectric variation,

$$\varepsilon(z) = \begin{cases} 1 & z > 0 \\ 1 + \frac{(1-\varepsilon)}{d}z & -d < z < 0 \\ \varepsilon = \varepsilon_1 - i\varepsilon_2 & z < -d \end{cases} \tag{2.35}$$

Therefore, the x-component of electric field are given as

$$\frac{E_x(z)}{E_0} = \left[-e^{-ik\cos\theta_i(z)} + \left(\frac{B_0''}{B_0}\right)e^{ik\cos\theta_i(z)}\right]\cos\theta_i; \qquad z > 0$$

$$\text{or,}\quad \frac{E_x(z)}{E_0}=\begin{cases}\dfrac{1}{ik\left[1+\dfrac{(1-\varepsilon)}{d}z\right]}\left(\dfrac{C}{B_0}\right)2\left[z+\dfrac{d}{1-\varepsilon}\right] & z>0\\[2ex] \dfrac{-2\cos\theta_i}{1+\cos\theta_i\left\{\dfrac{\varepsilon}{\alpha+i\beta}-\dfrac{dki}{2}(1-\varepsilon)\right\}} & -d<z<0\\[2ex] -ik(\alpha+i\beta)\left(\dfrac{B'}{B_0}\right)e^{-ik(\alpha+i\beta)z} & z<-d\end{cases}\tag{2.36}$$

Notice that the x - component of the electric field in the interface region ($-d\le z\le 0$) is a constant quantity and is independent of z. On the other hand, for the z-component of the electric field, we obtain electric field for different regions as follows:

For vacuum region, i.e. $z\ge 0$,

$$\frac{E_z(z)}{E_0}=-\sin\theta_i\left[e^{-ik\cos\theta_i(z)}+\frac{B_0}{B_0}e^{ik\cos\theta_i(z)}\right]$$

$$\frac{E_z(z)}{E_0}=-\sin\theta_i\left[e^{-ik\cos\theta_i(z)}+e^{ik\cos\theta_i(z)}\frac{-1+\cos\theta_i\left\{\dfrac{\varepsilon}{\alpha+i\beta}-\dfrac{dki(1+\varepsilon)}{2}\right\}}{1+\cos\theta_i\left\{\dfrac{\varepsilon}{\alpha+i\beta}-\dfrac{dki(1+\varepsilon)}{2}\right\}}\right]$$

In the long wavelength limit, kd<<1, and $z\ge 0$,

$$\frac{E_z(z)}{E_0}=-\sin\theta_i\left[1+\frac{-1+\dfrac{\varepsilon\cos\theta_i}{\alpha+i\beta}}{1+\dfrac{\varepsilon\cos\theta_i}{\alpha+i\beta}}\right]$$

$$\text{or,}\quad \frac{E_z(z)}{E_0}=-\frac{\varepsilon\sin 2\theta_i}{\left[\varepsilon-\sin^2\theta_i\right]^{\frac{1}{2}}+\varepsilon\cos\theta_i}\qquad \textit{for } z>0,\tag{2.37}$$

For the surface region, i.e. $-d\le z\le 0$,

$$\frac{E_z(z)}{E_0} = \frac{-\sin\theta_i}{1+(1-\varepsilon)\frac{z}{d}}\left[\frac{C}{B_0}\left(z-\frac{d}{1-\varepsilon}\right)^{-2}-\frac{D}{B_0}\right]$$

$$= \frac{-\sin\theta_i d}{1-(1-\varepsilon)\frac{z}{d}}\left[-\frac{ik\cos\theta_i}{d}\cdot\frac{(1-\varepsilon)\left[z+\frac{d}{2}\left(\frac{1+\varepsilon}{1-\varepsilon}\right)\right]^{-2}}{1+\cos\theta_i\left\{\frac{\varepsilon}{\alpha+i\beta}-\frac{ikd}{2}(1+\varepsilon)\right\}} + \varepsilon\cos\theta_i\times\frac{\left[\frac{2}{\alpha-i\beta}+ikd\left(\frac{\varepsilon}{1-\varepsilon}\right)\right]}{1+\cos\theta_i\left\{\frac{\varepsilon}{\alpha+i\beta}-\frac{ikd}{2}(1+\varepsilon)\right\}}\right]$$

Let us consider when the wavelength of light is much larger than d, then $kd \ll 1$,

$$\frac{E_z(z)}{E_0}\underset{kd\to 0}{\simeq}\frac{-d\,2\sin\theta_i\cos\theta_i}{1+(1-\varepsilon)\frac{z}{d}}\cdot\frac{\frac{\varepsilon}{\alpha+i\beta}}{1+\cos\theta_i\frac{\varepsilon}{\alpha+i\beta}}$$

$$= -\frac{\sin 2\theta_i}{(\alpha+i\beta)+\varepsilon\cos\theta_i}\cdot\frac{\varepsilon d}{d+(1-\varepsilon)z}.$$

Noting that $\alpha-i\beta=\tilde{n}\cos\gamma=\sqrt{\varepsilon-\sin^2\theta_i}$, we obtain

$$\frac{E_z(z)}{E_0}\underset{kd\to 0}{\simeq} -\frac{\sin 2\theta_i}{\left[\varepsilon-\sin^2\theta_i\right]^{\frac{1}{2}}+\varepsilon\cos\theta_i}\cdot\frac{\varepsilon d}{d-(1-\varepsilon)z}, \qquad \textit{for } -d\le z\le 0 .$$

(2.38)

Again in the bulk region, i.e. $z \le -d$,

$$\frac{E_z(z)}{E_0}=\frac{-\sin\theta_i}{\varepsilon}\left[\left(\frac{B_0'}{B_0}\right)e^{-ik(\alpha+i\beta)z}\right.$$

$$= \frac{\sin\theta_i}{\varepsilon}\left[\frac{2\varepsilon\cos\theta_i}{(\alpha - i\beta)}\,\frac{e^{-ik(\alpha+i\beta)d}\;e^{-ik(\alpha-i\beta)z}}{1-\cos\theta_i\left\{\dfrac{\varepsilon}{\alpha - i\beta} - \dfrac{dki(1+\varepsilon)}{2}\right\}}\right].$$

In the long wavelength limit $(k\,d << 1)$ and for $z \leq -d$, therefore

$$\frac{E_z(z)}{E_0} = \frac{-\sin 2\theta_i}{(\alpha - i\beta)}\,\frac{1}{1+\dfrac{\varepsilon\cos\theta_i}{(\alpha - i\beta)}}$$

$$= -\frac{\sin 2\theta_i}{\left[\varepsilon - \sin^2\theta_i\right]^{\frac{1}{2}} + \varepsilon\cos\theta_i}, \qquad \text{for } z \leq -d. \tag{2.39}$$

Thus the normal component of the electric field or vector potential in the long wavelength limit $\left(\frac{\omega d}{c} \to 0\right)$ are given by

$$\tilde{A}_n(z) = \frac{E_z(z)}{E_0} = \begin{cases} -\dfrac{\varepsilon\sin 2\theta_i}{\left[\varepsilon - \sin^2\theta_i\right]^{\frac{1}{2}} + \varepsilon.\cos\theta_i}, & z \geq 0 \\ -\dfrac{\sin 2\theta_i}{\left[\varepsilon - \sin^2\theta_i\right]^{\frac{1}{2}} + \varepsilon\cos\theta_i}\,\dfrac{\varepsilon.d}{d + (1-\varepsilon)z}, & -d \leq z \leq 0 \\ -\dfrac{\sin 2\theta_i}{\left[\varepsilon - \sin^2\theta_i\right]^{\frac{1}{2}} - \varepsilon\cos\theta_i}, & z \leq -d. \end{cases} \tag{2.40}$$

Eq. (2.40) will be used in the calculation of PFEC by using the Eq. (1. 1) given in chapter 1 by evaluating at first the matrix element for low photon energies less than work function of solid. The reason for this being that the vector potential occurs in the matrix element for calculating photoassisted field emission current.