CHAPTER 6

NON-IDEAL ANALYSIS OF THE PROPOSED CIRCUITS

6.1 INTRODUCTION

In previous chapter, the mathematical analysis is considered for the ideal characteristics of the OTRA. However, the ideal behavior of the OTRA is deviated, when it is considered for the practical implementation of the OTRA applications. This chapter describes the non-ideal analysis of the proposed circuits in chapter 4.

6.2 NON-IDEAL MODEL OF THE OTRA

The equivalent circuit models of the ideal and non-ideal OTRA is shown in Figs. 6.1 and 6.2.

![Fig. 6.1 Equivalent circuit model of the ideal OTRA](image1)

![Fig. 6.2 Equivalent circuit model of the non-ideal OTRA](image2)
In OTRA, the output voltage is the difference of two input terminal currents multiplied by the transresistance gain $R_m$, which is ideally infinite. The inverting and non-inverting input terminals of the OTRA are internally grounded as shown in Fig. 6.1 and 6.2. The parasitic capacitances and resistances ($R_p$, $R_n$ and $R_0$) associated with the input and output terminals of the OTRA are very small, grounded and negligible. Thus the most important non-ideality in OTRA is due to the finite transresistance gain $R_m$ [42-44]. The finite transresistance gain $R_m$ along with the frequency limitations associated with the OTRA must be considered in the analysis of the OTRA based circuits. Considering a single pole model for the transresistance gain $R_m$, then $R_m(s)$ can be expressed as

$$R_m = \frac{R_{mo}}{1 + \frac{j\omega}{\omega_o}} \quad (6.1)$$

Where $R_{mo}$ is the DC open loop transresistance gain of the OTRA and $\omega_o$ is the pole angular frequency of the OTRA. For middle and high frequency applications, the transresistance gain $R_m$ in $s$ domain can be expressed as

$$R_m(s) = \frac{R_{mo}}{s}sC_p \; ; \; C_p = \frac{1}{R_0\omega_o} \quad (6.2)$$

Where $C_p$ is called the parasitic capacitance of the OTRA.

**6.3 NON-IDEAL ANALYSIS OF THE GROUNDED RESISTANCE/CAPACITANCE BASED SINUSOIDAL OSCILLATORS**

The generalized configuration proposed in chapter 4 is shown in Fig. 6.3 with the equivalent non-ideal OTRA model. The non-ideal analysis of the generalized configuration shown in Fig. 6.3 gives the following equations. The output voltage at the output terminal of the OTRA shown in Fig. 6.1 can be written as

$$V_o = V_u\left(\frac{Y_4 - Y_3}{Y_2 - (Y_5 + Y_m)}\right) \quad (6.3)$$
Fig. 6.3 Generalized configuration of the proposed oscillators in chapter 4

The following equation is derived by applying KCL at node ‘a’ shown in Fig. 6.3.

\[ V_0 (Y_1 + Y_6) = V_a (Y_1 + Y_3 + Y_4 + Y_6 + Y_7) \]  \hspace{1cm} (6.4)

The non-ideal equation for the generalized configuration shown in Fig. 6.3 is derived by substituting the equation (6.4) in (6.2).

\[ Y_1Y_2 + Y_2Y_3 + Y_2Y_4 + Y_2Y_5 + Y_2Y_7 + Y_1Y_3 + Y_1Y_5 - Y_3Y_5 - Y_4Y_6 - Y_5Y_6 - Y_7Y_8. \]

\[ Y_m(Y_1 + Y_3 + Y_4 + Y_6 + Y_7) = 0 \]  \hspace{1cm} (6.5)

In the above equation, \( Y_i \)'s are the admittances of the passive components and \( Y_m \) is the admittance of the transresistance gain. From the generalized non-ideal characteristic equation (6.5), the non-ideal equations for the proposed circuits in chapter 4 can be derived by substituting the passive components. The non-ideal condition of oscillation and frequency of oscillation for the proposed circuits in chapter 4 is given in Table 6.3. The condition of oscillation and frequency of oscillation for the proposed circuits can be derived independently without using the generalized characteristic equation as given in chapter 4. Likewise, the non-ideal
equations for the condition of oscillation and frequency of oscillation can also derive independently for the proposed circuits in chapter 4.

### Table 6.1. Non-ideal condition of oscillations and frequency of oscillation for the proposed circuits in chapter 4.

<table>
<thead>
<tr>
<th>Proposed designs</th>
<th>Condition of oscillation</th>
<th>Frequency of oscillation ( (\omega_0^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 4.4 (a)</td>
<td>[ R_3(C_2+C_p)(R_3+R_7) = C_4R_3R_7 ]</td>
<td>[ \frac{R_3 + R_7}{R_1R_3R_4(C_2 + C_p)} ]</td>
</tr>
<tr>
<td>Fig. 4.4 (b)</td>
<td>[ R_1(C_1 + C_4 + C_7) + C_pR_5 = C_1R_4 ]</td>
<td>[ R_1R_3(C_4 + C_p(C_1 + C_4 + C_7)) ]</td>
</tr>
<tr>
<td>Fig. 4.4 (c)</td>
<td>[ C_6R_1R_7 = (C_5+C_p) (R_3R_7 + R_1R_7 + R_1R_3) ]</td>
<td>[ \frac{1}{R_1R_3(C_3 + C_p)C_6} ]</td>
</tr>
<tr>
<td>Fig. 4.4 (d)</td>
<td>[ R_1R_5(C_d+C_p) + R_2R_7(C_d+C_p) = R_2R_3R_6R_7(C_4C_6 + C_p(C_4 + C_6)) ]</td>
<td>[ \frac{(R_1 + R_7)(R_2 - R_5)}{R_2R_3R_5R_7(C_4C_6 + C_p(C_4 + C_6))} ]</td>
</tr>
<tr>
<td>Fig. 4.4 (e)</td>
<td>[ R_1R_5(C_d+C_p) + R_2R_7(C_d+C_p) = R_2R_3R_6R_7(C_4C_6 + C_p(C_4 + C_6)) ]</td>
<td>[ \frac{R_4 + R_7}{R_2R_3R_6R_7(C_3C_6 - C_p(C_3 + C_6))} ]</td>
</tr>
<tr>
<td>Fig. 4.4 (f)</td>
<td>Equation (6.6)</td>
<td>Equation (6.7)</td>
</tr>
<tr>
<td>Fig. 4.4 (g)</td>
<td>Equation (6.8)</td>
<td>Equation (6.9)</td>
</tr>
<tr>
<td>Fig. 4.5 (a)</td>
<td>Equation (6.10)</td>
<td>Equation (6.11)</td>
</tr>
</tbody>
</table>

The non-ideal condition of oscillation and frequency of oscillation for the circuit shown in Fig. 4.4 (f) is given in equations (6.6) and (6.7).

\[ R_3C_2 \ (R_3R_6 + R_6R_7 + R_7R_3) = C_4R_7R_3 \ (R_3 + R_6) + C_pR_3R_5(R_3R_7 + R_6R_7 + R_7R_6) \]  \( (6.6) \)

\[ f = \frac{1}{2\pi} \sqrt{\frac{R_1R_6 + R_2(R_3 + R_6 - R_5)}{C_4(C_2 + C_p)R_3R_5R_6R_7}} \]  \( (6.7) \)

The non-ideal condition of oscillation and frequency of oscillation for the circuit shown in Fig. 4.5 (a) is given in below equations.
\[
C_2 \left( \frac{R_2 R_3 + R_1 R_3 + R_2 R_4}{R_1 R_2 R_4} \right) + C_3 + C_p \left( \frac{R_1 + R_3}{R_1 R_3} \right) = \frac{C_2}{R_1}
\] (6.8)

\[
f = \frac{1}{2\pi} \sqrt{\frac{R_1 + R_3}{R_1 R_2 R_3 \left( C_2 C_3 + C_p C_2 \left( R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4 \right) \right)}}
\] (6.9)

Similarly, for the proposed circuit shown in Fig. 4.5 (b), the non-ideal equations for the condition of oscillation and frequency of oscillation are given in equations (6.10) and (6.11).

\[
C_1 \left( \frac{R_2 R_3 + R_1 R_3 + R_2 R_4}{R_2 R_3 R_4} \right) + C_3 + C_p \left( \frac{R_2 + R_3}{R_2 R_3} \right) = \frac{C_1}{R_2}
\] (6.10)

\[
f = \frac{1}{2\pi} \sqrt{\frac{R_2 + R_3}{R_1 R_2 R_3 \left( C_1 C_3 + C_p C_1 \left( R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4 \right) \right)}}
\] (6.11)

The non-ideal equations for the proposed circuits shown in Fig. 4.3, 4.4 and 4.5 can also be derived without using the generalized characteristic equation given in equation (6.5). For example, the non-ideal analysis of the circuit generated from the generalized configuration is shown in Fig. 6.2 is given below.

![Non-ideal OTRA based oscillator circuit](image)

Fig. 6.4 Non-ideal OTRA based oscillator circuit

The currents at the inverting and non-inverting terminal of the OTRA shown in Fig. 6.4 can be written as

\[
P = \frac{R_m(I_P + I_n)}{R_o}
\]
\[ I_+ = \frac{V_a}{R_3} \]  
(6.12)

\[ I_- = \frac{V_a}{R_3} + V_a sC_4 \]  
(6.13)

From the terminal relation of the OTRA, the following equation can be written as

\[ V_o sC_p = V_a R_3 \left( \frac{1 - sC_1 R_3}{R_3} \right) \]  
(6.14)

By applying Kirchhoff’s Current Law (KCL) at the node \( V_a \) in Fig. 6.4

\[ V_o sC_1 R_3 = V_a (1 + sR_3 (C_4 + C_7 + C_1)) \]  
(6.15)

From the equations (6.14) and (6.15), the non-ideal characteristic for the circuit shown in Fig. 6.2 can be written as

\[ s^2 \left( R_3 R_5 \left( C_1 C_4 + C_p (C_1 + C_4 + C_7) \right) \right) + s \left( R_3 (C_1 + C_4 + C_7) + C_p R_5 - C_1 R_5 \right) + 1 = 0 \]  
(6.16)

The non-ideal condition of oscillation and frequency of oscillation can be derived from the above equation.

\[ R_3 (C_1 + C_4 + C_7) + C_p R_5 = C_1 R_5 \]  
(6.17)

\[ f = \frac{1}{2\pi} \sqrt{\frac{1}{R_3 R_5 \left( C_1 C_4 + C_p (C_1 + C_4 + C_7) \right)}} \]  
(6.18)

6.4 NON-IDEAL ANALYSIS OF THE PROPOSED QUADRATURE SINUSOIDAL OSCILLATORS

The proposed quadrature sinusoidal oscillator in chapter 4 is shown in Fig. 6.5. From the Fig. 6.5 and from the non-ideal characteristics of the OTRA stated in equations (6.1) and (6.2), the output terminal voltage \( V_{01} \) can be written as
Similarly, the output voltage \( V_{o2} \) at the output terminal of the OTRA 2 is given in equation (6.20).

\[
V_{01} = V_{02} \left( \frac{R_1 R_{m1}}{R_5 (R_{m1} - R_1 - sC_2 R_1 R_{m1})} \right) \tag{6.19}
\]

Fig. 6.5 Non-ideal model of the quadrature sinusoidal oscillator circuit proposed in chapter 4, Fig. 4.6

\[
V_{01} = \frac{(sC_4 R_4 + 1 + sC_4 R_{m2}) R_2}{(sC_4 R_4 + 1) R_{m2}} V_{02} \tag{6.20}
\]

Fig. 6.6 Non-ideal model of the quadrature sinusoidal oscillator circuit proposed in chapter 4, Fig. 4.7
The non-ideal condition of oscillation and frequency of oscillation of the proposed circuit in Fig. 6.5 is given in equations (6.21) and (6.22).

\[
\text{C.O: } R_1 R_4 C_4 = R_3 R_5 (C_{p2} + C_4) \quad (6.21)
\]

\[
\text{F.O: } f = \frac{R_i}{\sqrt{C_{p2} C_4 R_1 R_4 R_5 - R_2 R_3 R_5 (C_{p1} C_{p2} - C_{p2} C_2 - C_{p1} C_4 - C_2 C_4)}} \quad (6.22)
\]

\(C_{p1}\) and \(C_{p2}\) are the parasitic capacitances associated with the OTRA 1 and OTRA 2 respectively.

Similarly, the condition of oscillation and frequency of oscillation for the proposed quadrature sinusoidal oscillator circuit shown in Fig. 6.6 is given in equation (6.23) and (6.24).

\[
\text{C.O: } R_1 R_4 C_1 = R_3 R_2 (C_{p2} + C_1) \quad (6.23)
\]

\[
\text{F.O: } f = \frac{R_i}{\sqrt{R_2 R_3 R_5 (C_1 C_4 + C_1 C_{p1} + C_4 C_{p2} + C_{p1} C_{p2}) - C_1 C_{p1} R_1}} \quad (6.24)
\]

\(C_{p1}\) and \(C_{p2}\) are the parasitic capacitances of the OTRA 1 and OTRA 2. It can be easily verified that, the condition of oscillation and frequency of oscillation equations (5.23), (5.24), (5.28) and (5.29) are given in chapter 5 from the ideal characteristics of OTRA can be derived by substituting \(C_{p1} = C_{p2} = 0\) in the equations (6.21) - (6.24).

6.5 NON-IDEAL ANALYSIS OF THE PROPOSED SQUARE WAVEFORM GENERATORS

To consider the non-ideal and parasitic effects of the proposed circuits shown in Figs. 4.8 (a) and (b), an equivalent circuit model of the OTRA built with two current feedback amplifiers (CFAs) is shown in Fig. 6.7. A practical CFA (AD 844 AN IC) can be modeled as a positive second-generation current conveyor (CCII+) cascading a voltage buffer with finite parasitic resistances and non-zero current tracking errors.

Figure 6.7 reveals a more detailed circuit model of the OTRA, where \(R_x\) and \(R_z\) are the parasitic resistances and \(\alpha\) represents the current tracking error factor from the terminal \(T_z\) with respect to the inverting input terminal. The standard values of \(R_x, R_z\)
and \( \alpha \) are given in AD844AN datasheet are \( \alpha = 0.98 \), \( R_x = 50 \, \Omega \), and \( R_z = 3 \, \text{M}\Omega \). The resulted expressions of the related currents are indicated in Fig. 6.7. The voltage tracking error effect between the CCII+ input terminals are skipped (eliminated) in the circuit model, because of the non-inverting terminal for each CCII+ are connected to ground. The non-ideal model of the proposed square waveform generator in chapter 4, Fig. 4.8 (a) is shown in Fig. 6.8. For the proposed circuit shown in Fig. 4.8 (a), the non-ideal analysis gives the following equations.

\[
\begin{align*}
V_{th} &= \left( \frac{\alpha R_1}{\alpha R_1 + R_z \left( \frac{R_z}{R_X + R_z} + \alpha \right)} \right) V_{sat}^+ \\
V_{tl} &= \left( \frac{\alpha R_1}{\alpha R_1 + R_z \left( \frac{R_z}{R_X + R_z} + \alpha \right)} \right) V_{sat}^-
\end{align*}
\]

(6.25)

(6.26)

The equations for the on-duty cycle and off-duty cycle can be expressed as
Substituting these higher and lower threshold voltage equations \((6.25)\) & \((6.26)\) in equation \((6.27)\) and \((6.28)\), the non-ideal time period for the proposed circuit in chapter 4, Fig. 4.8 (a) can be expressed as

\[
T = 2R_2C \ln \left(1 + \frac{2\alpha R_1}{R_2 \left(\frac{R_z}{R_x + R_z} + \alpha\right)} \right)
\]  
\[(6.29)\]
The non-ideal equation for the second proposed circuit shown in Fig. 4.8 (b), chapter 4 is given in equation (6.30).

\[
T = R_2 C \ln \left[ 1 + \frac{2\alpha R_{11}}{R_2 \left( \frac{R_Z}{R_X + R_Z} + \alpha \right)} \right] \times \left[ 1 + \frac{2\alpha R_{12}}{R_2 \left( \frac{R_Z}{R_X + R_Z} + \alpha \right)} \right]
\]  

(6.30)

It can be easily verified that, the equation (6.29) and (6.30) are reduced to equation (5.40) and (5.47) by substituting the ideal OTRA characteristics, when \( \alpha \approx 1 \) and \( R_X \approx 0 \).

6.6 SUMMARY

In this chapter, the non-ideal analysis of all the proposed circuits is given. All the proposed circuits are re-examined by considering the non-ideal model of the OTRA. The non-ideal condition of oscillation and frequency of oscillation for the sinusoidal oscillator circuits realized from the generalized configuration is presented in a Table. A generalized non-ideal characteristic equation is derived for the proposed sinusoidal oscillator circuits. From this non-ideal characteristic equation, the non-ideal condition of oscillation and non-ideal frequency of oscillation is derived. Similarly, the same procedure is followed to derive the non-ideal condition of oscillation and frequency of oscillation for the quadrature sinusoidal oscillators.

At last, the non-ideal time period equations are derived for the square waveform generators by considering the parasitic resistances and current tracking errors of practical model of OTRA.