APPENDIX - A

The literature [106], [202] of fixed-step-size (FSS-LMS) algorithm reflects a trade-off between the misadjustment and convergence-rate, which depicts that a small step-size (SS) produces small misadjustment, but at the cost of longer convergence time. Under time-varying environment, the optimum value of step-size in FSS-LMS algorithm strikes a balance between the amount of lag-noise and gradient-noise [98]. However, the optimum value of step-size can not be determined *a priori* due the unknown channel parameters. Therefore in KVSS-LMS algorithm [91], the variable step size (VSS) is attuned using

$$\mu'(n) = \bar{\alpha}\mu'(n-1) + \bar{\gamma}\underline{\underline{e^{2}(n-1)}} \tag{A.1}$$

In KVSS-LMS algorithm, high prediction-error causes the SS to increase in order to achieve fast tracking, while low prediction-error leads to reduction in SS to yield small misadjustment. The SS elevates or alleviates as the MSE rises or falls, which allow adaptive-filtering configuration to chase changes in TV system, as well as to reduce steady-state error. It also reduces sensitivity of misadjustment to the level of nonstationarity. This technique is heuristically sound and has resulted in various ad hoc methods, where the choice of convergence parameters rests upon the magnitude of estimation-error, polarity of the successive samples of estimation-error, measurement of the cross-correlation of estimation-error with input-samples. However, VSS-LMS algorithms are found to be sensitive to noise [95], [63] in the low signal-to-noise ratio environment because the SS update of these algorithms are directly calculated using instantaneous-error, which is couupted by noise. Further in AVSS-LMS algorithm [96], the VSS is controlled using

$$\mu'(n) = \bar{\alpha}\mu'(n-1) + \bar{\gamma}\theta^{2}(n-1) \tag{A.2}$$

$$\theta(n-1) = \bar{\alpha}_{A}\theta(n-2) + (1-\bar{\alpha}_{A})\underline{\underline{\{e(n-1)e(n-2)\}}} \tag{A.3}$$

Here, the error auto-correlation is usually a fine gauge of proximity to the optimal-value, which eliminates the effects of uncorrelated noise-sequences on SS updating. In beginning stages of adaptive tuning, the error auto-correlation estimate is relatively high, which in turn results in a high value of SS. However, the small error auto-correlation leads to a small step-size under the optimum conditions. It results in effective adjustment of SS, while sustaining the immunity against independent noise, for the flexible control of misadjustment. The AVSS-LMS algorithm [96] shows substantial convergence-rate improvement over KVSS-LMS algorithm [91] and FSS-LMS algorithm [202] under the stationary environment for the low SNR as well as the high SNR values. However, the performance of AVSS-LMS algorithm is comparable to the FSS-LMS and KVSS-LMS adaptive algorithms under the nonstationary conditions. But in SVSS-LMS algorithm [104], the VSS is adjusted using the following recursive relation by adjusting the control-parameters $\bar{\rho}_w$ *and* $\bar{\alpha}_w$.

$$\mu'(n) = \mu'(n-1) + \bar{\rho}_w \vec{\psi}^T(n)\vec{x}(n)e(n) \tag{A.4}$$

$$\vec{\psi}(n) = \bar{\alpha}_w \vec{\psi}(n-1) + e(n-1)\vec{x}(n-1) \tag{A.5}$$

The above equation can be rewritten in expanded form as

$$\begin{aligned}\vec{\psi}^T\{n\} = {} & \bar{\alpha}_w^{\bar{Q}}\vec{\psi}^T\{n-\bar{Q}\} + \bar{\alpha}_w^{\bar{Q}-1}e(n-\bar{Q})\vec{x}^T(n-\bar{Q}) + \\ & \ldots\ldots \quad + \bar{\alpha}_w e(n-2)\vec{x}^T(n-2) + e(n-1)\vec{x}^T(n-1)\end{aligned} \tag{A.6}$$

For $0 \le \bar{\alpha}_w < 1$ and $\bar{Q} \to high\ value$, the Eq. (A.6) can be approximated as

$$\begin{aligned}\vec{\psi}^T\{n\} \approx {} & \underline{\underline{\bar{\alpha}_w^{\bar{Q}-1}e(n-\bar{Q})\vec{x}^T(n-\bar{Q}) +}} \quad \ldots\ldots \\ & \underline{\underline{+\ \bar{\alpha}_w^2 e(n-3)\vec{x}^T(n-3) + \bar{\alpha}_w e(n-2)\vec{x}^T(n-2) + e(n-1)\vec{x}^T(n-1)}}\end{aligned} \tag{A.7}$$

This algorithm [104] outperforms the Mathews' algorithm [92], when both are set to track the random-walk channel under the similar conditions.