PREFACE

The present thesis is an outcome of my research work carried out during the period 2005-2008 in the Department of Postgraduate Studies and Research in Mathematics at M. L. K. (P.G.) College, Balrampur-271201 (U.P.) under the supervision of Dr. S. B. Misra, Reader and Head of the department.

The whole thesis has been divided into seven chapters and each chapter has been divided into sections. In numbering the equations the decimal notation has been employed. References to the equations are of the form (CSE) where C, S and E stand for the corresponding chapter, section and equation respectively. If C coincides with the chapter in hand, it is omitted, the numbers in the square brackets refer to the references given at the end of the chapter. The notation \( \frac{k}{h} \) means the subtraction from the former term obtained by the interchange of the indices \( k \) and \( h \). For instance \( \Omega_{kh} - \frac{k}{h} = \Omega_{kh} - \Omega_{hk} \), the symbols \( \partial_{\dot{x}}, \dot{\partial}_{\dot{x}}, (\ ) \) and \( \mathcal{E} \) denote the partial differentiation with respect to \( x^{\dot{k}} \), the partial differentiation with respect to \( \dot{x}^{\dot{k}} \), the Berwald’s covariant differentiation and the Lie-differentiation with respect to a given infinitesimal transformation respectively.

The first chapter of the thesis is introductory, in which an account of the nature and content of the preliminary details has been given. Some results and definitions useful for the later work have been mentioned therein. The Finsler space \( F^n_z \) equipped with non-symmetric connection parameter, \( \Gamma'_{jk}(x,\dot{x}) \neq \Gamma'_{ij}(x,\dot{x}) \)
that is based on non-symmetric fundamental metric tensor 
\( g_{\alpha}(x,\dot{x}) \neq g_{\beta}(x,\dot{x}) \), has been defined in this chapter. We have 
introduced the concepts of \( \alpha \) - covariant differentiation and \( \beta \) 
covariant differentiation of a tensor quantity with respect to \( x^k \) 
and have obtained the curvature tensors \( R_{ijk}^h(x,\dot{x}) \) and \( \tilde{R}_{ijk}^h(x,\dot{x}) \) 
due to the duality in the nature of the covariant derivatives.

The second chapter of the thesis deals with a recurrent 
Finsler space equipped with non-symmetric connection. The 
chapter has been divided into five sections, the first section is 
introductory in nature, the second section deals with \( R^+ \) - 
recurrent \( F_n^* \) of second order. In this section we have derived 
results in the form of theorems telling about the relationships 
which hold in between the recurrence tensor field and the 
curvature tensor \( R_{ijk}^{+h} \), we have also obtained Bianchi identities in 
such a Finsler space. The third section of the chapter deals with a 
special \( R^+ \) - birecurrent \( F_n^* \), we have stated that an \( F_n^* \) is said to be 
special \( R^+ \) - birecurrent of first kind or of second kind according 
as \( R^{+h}_{ijk} |_{\ell m} = \lambda_{\ell} R^{+h}_{ijk} |_{\ell} \) or. \( R^{+h}_{ijk} |_{\ell m} = \lambda_{\ell} R^{+h}_{ijk} |_{\ell} \) hold respectively. 
In this section we have established that if a \( R^+ \) - birecurrent \( F_n^* \) of 
first order be a special \( R^+ \) - birecurrent of second kind then such 
an \( F_n^* \) is either flat or in such an \( F_n^* \) the recurrence vector is a 
covariant constant. We have also derived the characterizing 
conditions for a \( R^+ \) - birecurrent space of first kind and a special 
birecurrent \( F_n^* \) of second kinds. In this section, we have also 
derived the conditions which are satisfied by the Ricci tensor \( R_{jk} \).
and the contracted tensor $R$ of a special $R^+$ - birecurrent $F_n^+$ of first and second kinds. The fourth section of the chapter deals with generalised birecurrent $R^+$ - $F_n^+$ of first and second kinds. In this section we have established the conditions which are satisfied by the curvature tensor $R_{ijk}^h$ and the scalar $R$ along with the identity which holds for the curvature tensor $R_{ijk}^h$ under the assumption that $\Gamma_{ijk}^h$ is first order recurrent in such a space. The fifth and the last section of the chapter deals with Bianchi and Veblen identities in a Finsler space $F_n^+$ equipped with non-symmetric connection.

The third chapter of the thesis deals “Projective Entities” in a Finsler space $F_n^+$ equipped with non-symmetric connection. In this chapter we have obtained the necessary and sufficient condition in order that the skew symmetric part of the connection coefficient $\Gamma_{jk}^i$ and that of the projective connection coefficient be identical along with the identities which are satisfied by the projective entities $Q_{jkl}^i$ analogous to those of Bianchi along with the Bianchi identity which such entities satisfy. Here it is notable that the projective entities $Q_{jkl}^i$ is not a tensor quantity but however it is homogeneous of degree zero in its directional arguments. The fourth and the last section of the chapter deals with recurrence and decomposition of the projective entities $Q_{jkl}^i$ and have obtained the identities which are satisfied by the projective entities $Q_{jkl}^i$ under the decomposition rule $Q_{ijk}^h = \gamma_i^h \psi_{jk}$ where $\gamma_i^h$ and $\psi_{jk}$ are non-null tensor fields such that $\gamma_i^h \lambda_i = \beta_h$.
where $\beta_h$ and $\lambda_h$ are respectively the non-null decompose and recurrence vector fields.

The fourth chapter of the thesis is concerned with the study of generalised birecurrent spaces, in this chapter we have generalised an $R^h$-birecurrent space in which Cartan third curvature tensor satisfies the generalised birecurrent condition with respect to Cartan's connection. The chapter has been divided into five sections, like the previous chapters, the first section of this chapter is introductory in nature, the second section of the chapter is concerned with $R^h$-generalised birecurrent spaces, in this section we have stated that if $R^i_{jkhm\ell} = \lambda_\ell R^i_{jkh\ell} + a_{\ell m} R^i_{jkh}$ or $R^i_{jkh\ell} = \lambda_m R^i_{jkh\ell} + a_{\ell m} R^i_{jkh}$ (where $\lambda_\ell$ and $a_{\ell m}$ are non-zero covariant vector and covariant tensor field of order 2) then the space satisfying either of two conditions is respectively, termed as $R^h$-generalised birecurrent of first and second kind, whereas if the space satisfies $R^i_{jkhm\ell} = \lambda_\ell R^i_{jkhm}$ or $R^i_{jkhm\ell} = \lambda_m R^i_{jkh\ell}$ (where $\lambda_\ell$ is a non-zero covariant vector field) then the space under consideration will be called as special $R^h$-generalised birecurrent space of the first and second kind respectively. In this section we have established that $R^h$-generalised birecurrent and $R^h$-special generalised birecurrent spaces of the two kinds are also Ricci $R^h$-generalised and Ricci $R^h$-special generalised recurrent but not conversely. In this continuation we have also established that the tensors $H^i_h$, $H'_h$, the vector $H_k$ and the scalar $H$ of $R^h$-generalised and $R^h$-special generalised birecurrent Finsler space of the two kinds are respectively h-generalised and h-special.
generised birecurrent of the two kinds. In this section we have also obtained the necessary and sufficient conditions in order that the Berwald’s curvature tensor $H^i_{jkh}$ and Cartan curvature tensor $K^i_{jkh}$ of an $R^h$-generalised birecurrent Finsler space of first kind/of second kind/ $R^h$-special generalised of first kind and that of second kind be respectively h-generalised birecurrent of first kind/ h-generalised birecurrent of second kind/h-special generalised birecurrent of first kind and h-special generalised birecurrent of second kind. The third section of the chapter deals with normal projective curvature tensor while the fourth section deals with generalised normal projective recurrent Finsler space.

In this section we have obtained conditions telling as to when generalised normal projective recurrent Finsler space is normal projective recurrent/normal projective birecurrent/normal projective birecurrent of first and second kinds, in the sequel we have also established that in a generalised normal projective and in a normal projective recurrent Finsler space the tensors $H^r_{jkh}, H^i_{kh}, N^i_{jk}, H^i_n, H_k$ and the scalar $H$ are recurrent. In this section we have also established the necessary and sufficient condition in order that (1). a generalised normal projective recurrent Finsler space is generalised recurrent, (2). a normal projective recurrent Finsler space is recurrent, (3). a normal projective birecurrent Finsler space is birecurrent and (4). a normal projective generalized birecurrent Finsler space of the two kinds is generalized birecurrent of the two kinds. The fifth and the last section of the chapter deals with identities in a generalized and special generalized birecurrent Finsler space.
The fifth chapter of the thesis has been devoted to the study of APST Riemannian manifold with second order generalized structure. We have firstly defined a generalized almost contact metric structure manifold $M^n$ and subsequently have derived a special form of Nijenhuis tensor in such a structure and have also derived certain identities and relationships which hold in such a structure. We have also established certain other relationships which hold in a differentiable manifold and also in a generalized almost contact metric manifold in the form of theorems and corollaries. We have established the necessary condition in order that a generalized almost contact metric manifold be completely integrable and have obtained relationship which hold in a completely integrable generalized almost contact metric structure manifold $M^n$.

The sixth chapter of the thesis deals with horizontal and complete lifts from a manifold to its cotangent bundle. Yano has studied and defined manifolds with $f^{(4,2)}$ structure, Yano and Ishihara further studied the geometry of tangent and cotangent bundles in a differentiable manifold, Mishra has defined Hsu-structure. In this chapter we have studied complete and horizontal lifts from a manifold with Hsu-$(6,4)$ structure to its cotangent bundle. If $M$ be an $n$-dimensional differentiable manifold of class $C^\infty$ and if there be on $M$ a tensor field $f(\neq 0)$ of type $(1,1)$ satisfying $f^6 - \lambda^r f^4 = 0$ where $\lambda$ is a non-zero complex number and $r$ is some finite integer, in such a manifold if we write $\ell = f^2 / \lambda^r$ and $m = 1 - f^2 / \lambda^r$ where $I$ denotes the unit tensor field, then it can easily be seen that $\ell^2 = \ell, m^2 = m, \ell + m = I$ and
\[ \ell m = ml = 0. \] This means that the operators \( \ell \) and \( m \) when applied to the tangent space of \( M \) are complementry projection operators. Therefore, there will exist complementry distributions \( L^* \) and \( M^* \) corresponding to the projection operators \( \ell \) and \( m \) respectively. If the rank of \( f' \) is constant everywhere and equals to \( r \) then the dimensions of \( L^* \) and \( M^* \) are \( r \) and \( (n-r) \) respectively. We call such a structure as Hsu-(6,4) structure of rank \( r \). We have established the necessary and sufficient condition in order that the complete lift \( f^c \) of a (1) tensor field \( f \) admitting Hsu-(4,2) structure in \( M \) may have the similar structure in the cotangent bundle \( C_{TM} \). We have also established that the Nijenhuis tensor of the complete lift of \( f^c \) vanishes if the Lie-derivative of the tensor field \( f^4 \) with respect to \( X \) and \( Y \) are both zero and \( f \) acts as an Hsu-structure on \( M \). In the last we have established that if \( f \) be a tensor field of type (1,1) satisfying Hsu-(6,4) structure on the manifold \( M \) then horizontal lift \( f^H \) of \( f \) also admits the same structure in the cotangent bundle \( C_{TM} \).

The seventh and the last chapter of the thesis has been devoted to the aspects of invariant submanifold of a \( f_\lambda \)-Hsu manifold with complemented frames. Upadhya and Gupta have defined and studied the \( f_\lambda \)-manifold and Yano has obtained certain results on invariant submainfold of an \( f_\lambda \)-manifold with complemented frames. In the present chapter we have studied the invariant submanifolds of an \( f_\lambda \)-Hsu manifold with complemented frames the chapter has been divided into six sections, the first section is introductory. In section-2 we have
defined and studied the normality of an $f_\lambda$–Hsu structure with complemented frames and have established the conditions under which an $f_\lambda$–Hsu structure with complemented frames becomes normal. In section-3 of the chapter we have derived the necessary and sufficient condition in order that a manifold $M$ may admit an $f_\lambda$–Hsu structure with complemented frames. In section-4 of the chapter we have obtained the relationship in between the integrability of Hsu-structure and the normality of $f_\lambda$–Hsu structure with complemented frames. In sections 5 and 6 results on in variant submanifold of a normal $f_\lambda$–Hsu manifold with complemented frames have been established.

A selected bibliography consisting of the references of a number of books and papers on the subject has been given in the end.