6. MODELLING HEALTH CARE WASTE MANAGEMENT

6.1. Modelling Waste Management

Modeling of waste management system is rather less developed, perhaps due to the fact that the process invokes a large number of parameters having unknown behavior. However, need of some predictive tool is clearly visualized by many researchers. With this objective, some mathematical models have been developed but most of them are limited to optimize the waste system to reduce environmental burdens or economic costs and to improve social acceptability. Many of the models are decision support models, using a variety of methods and tools, such as risk assessment, environmental impact assessment, cost benefit analysis, multicriteria decision making or life cycle as part of the decision making process. Keeping these in view, present work is aimed at developing a mathematical model to study the effect of season on waste generation rate from some selected health care facilities.

6.2. Present Case Study – Development of Model

The general technique, by which much of modern engineering modeling proceeds, is by the identification and solution of the set of mathematical equations, mostly differential equations representing rate laws. These equations describe the dynamic behaviour of various key variables, whose values, when the equations are solved, predict the state of the system at any instant of time (Kumar and Upadhyay, 2000). Unfortunately, real environmental systems are horribly complex and involve several interlinked parameters; therefore, it is extremely difficult to measure the various constants of rigorous mathematical models with any accuracy. This is particularly true when the dynamic behavior of such systems is considered. Without the values of all the constants, the set of model equations have no practical importance (Parsons and Dohnal, 1995). As a result, most of the modeling approach (including present one) is empirical in nature.

6.2.1. Relation between Bed Occupancy and Waste Generation Rate

In order to get better understanding of biomedical waste management and treatment in the present case study, firstly, it is assumed that the infectious waste generated from the health
care facility (HCF) should be proportional to the bed occupancy. Therefore, a linear relationship between waste generation rate and bed occupancy is expected (Awad et al., 2004). Even if, bed occupancy is zero, there should be some waste generated for the maintenance of hospital premises. Therefore, a linear relationship with a positive intercept on y-axis, represented by line ‘a’ in Figure 6.1, is expected. However, there may be several other factors that may influence waste generation rate. For example, if the bed occupancy increases to a great extent from its sanctioned strength, slight variation in waste generation rate can easily be envisaged due to changed efficiency of the waste handling system. Thus,

![Figure 6.1. Schematic representation of HCW Generation Rate](image)

... to accommodate effect of other factors in the model, in place of the linear relationship a non-linear curve (curve ‘b’ in Figure 6.1) can be a better choice. It is proposed that the average rate of waste generation, $W_0$ (kg/day), can be represented by the following quadratic equation in a better way.

$$W_0 = k_1 + k_2B + k_3B^2$$

----- (6.1)

where $k_1$, $k_2$, and $k_3$ are constants and $B$ is the average bed occupancy (average number of beds used/day in one month) calculated by using Equation (5.1).
Here it is worth mentioning that Equation (6.1) assumes that \( W_0 \) is the waste generated per bed per day that remains unchanged throughout the year, and there is no any effect of weather / season on its value. Also this rate may be different for number and type of specialties in a particular health care facility. However, the data shows a regular pattern in the variation of waste generation rate during one calendar year (Figures 5.4 to 5.9). Therefore a correction factor \( \psi \) depending on season can be incorporated in Equation (6.1) by using a correction factor that depends on the month of a year. Thus Equation (6.1) can be modified to

\[
W = W_0 \psi \tag{6.2}
\]

where \( \psi \) is a correction factor that depends on the month of a calendar year.

6.2.2. Seasonal Variation in the Rate of Waste generation

The raw data for the monthly waste generation rate (Figures 5.4 to 5.9) indicates that the effect of season is considerable and there are at the most only one maxima or minima in a year. Thus it appears that to account this seasonal variation one needs atleast a polynomial of degree three or a trigonometric function involving sine or cosine function. But the waste generation data shows an uneven trend of positive and negative variation from a mean value, therefore sine or cosine functions are not suitable for the present purpose. Therefore, a polynomial would be more suitable form of equation for the present work.

First of all following polynomial of degree three is considered,

\[
\psi = a + b.x + c.x^2 + d.x^3 \tag{6.3}
\]

where \( \psi \) is the correction factor and \( x \) is the representative number of the month.

For the sake of convenience, months are numbered as 0, 1, 2, ----, 11 for January, February, March, ----, December. Thus for the month of January, \( x \) is zero, for February, \( x \) is 1 and for December, \( x \) is 11. Here it should be noted that \( \psi \) is a correction factor depending on month of year, its numerical value should remain same for a particular month in different years, and that, Equation (6.3) is defined for values of \( x \) raging from 0 to 11 only. However, to make this function continuous round the year, it is required that the
function values and its slopes are equal at \( x = 0 \) and \( x = 12 \) (\( x = 12 \) represent the month of January of the next year).

In other words

\[
\psi(0) = \psi(12) \quad \text{----- (6.4)}
\]

and

\[
\left. \frac{d\psi}{dx} \right|_{x=0} = \left. \frac{d\psi}{dx} \right|_{x=12} \quad \text{----- (6.5)}
\]

Thus from Equation (6.5) we get

\[
b + 2c(0) + 3d(0) = b + 2c(12) + 3d(12)^2 \quad \text{----- (6.6)}
\]

or

\[
2c(12) + 3d(12)^2 = 0 \quad \text{----- (6.7)}
\]

or

\[
c = -18d \quad \text{----- (6.8)}
\]

and from Equation (6.4) we have

\[
a + b(0) + c(0) + d(0) = a + b(12) + c(12)^2 + d(12)^3 \quad \text{----- (6.9)}
\]

or

\[
b + 12c + 12^2d = 0 \quad \text{----- (6.10)}
\]

Carrying the value of \( c \) from equation (6.8) to equation (6.10), we get

\[
b + 12(-18d) + 12^2d = 0 \quad \text{----- (6.11)}
\]

or

\[
b = 72d \quad \text{----- (6.12)}
\]

Thus the required correction factor (equation 6.3) reduces to,

\[
\psi = a + 72x - 18dx^2 + dx^3 \quad \text{----- (6.13)}
\]

or

\[
\psi = a + d.(72x - 18x^2 + x^3) \quad \text{----- (6.14)}
\]

In the above equation ‘\( a \)’ is the mean value (approximately equal to unity) about which the correction factor \( \psi \) oscillates and \( d \) is the parameter that sets magnitude of the oscillation.

In this case the function has fixed value of \( x \) (for all values of \( d \)) when \( \psi \) approaches its mean value \( a \) viz. at \( x = 0, 6, \) and \( 12 \). Fig. 6.2 presents \( \psi \) calculated by Equation (6.14) at three different values, \(-0.05, -0.04, \) and \(+0.05\) of \( d \).
However, this is not the actual case. This indicates that we need more number of parameters in the required equation to accommodate the waste generation data i.e., a polynomial of higher degree is required. Therefore, the following polynomial of fourth degree was considered as next possibility

$$\psi = a + b.x + c.x^2 + d.x^3 + e.x^4$$  \hspace{1cm} (6.15)

This equation should also be such that the condition of Equations (6.4) and (6.5) is applied. Therefore for condition at Equation (6.5), we get

$$b + 2c(0) + 3d(0) + 4e(0) = b + 2c(12) + 3d(12)^2 + 4e(12)^3$$  \hspace{1cm} (6.16)

or $$2c(12) + 3d(12)^2 + 4e(12)^3 = 0$$  \hspace{1cm} (6.17)

or $$2c + 3d(12)^2 + 4e(12)^2 = 0$$  \hspace{1cm} (6.18)

or $$c = -(18d + 288e)$$  \hspace{1cm} (6.19)

and for condition at Equation (6.4), we have

$$a + b(0) + c(0) + d(0) + e(0) = a + b(12) + c(12)^2 + d(12)^3 + e(12)^4$$  \hspace{1cm} (6.20)

or $$b(12) + c(12)^2 + d(12)^3 + e(12)^4 = 0$$  \hspace{1cm} (6.21)

or $$b = -(12c + 12^2d + 12^3e)$$  \hspace{1cm} (6.22)

Substituting values of b from equations (6.22) in Equation (6.15), the polynomial reduces to

$$\psi = a - (12c + 12^2d + 12^3e)x + cx^2 + dx^3 + ex^4$$  \hspace{1cm} (6.23)
or \[ \psi = a + c(x^2 - 12x) + d(x^3 - 12^2 x) + e(x^4 - 12^3 x) \] ----- (6.24)

Again, substituting value of \( c \) from Equation (6.19) in Equation (6.24), we get
\[ \psi = a - (18d + 288e)(x^2 - 12x) + d(x^3 - 12^2 x) + e(x^4 - 12^3 x) \] ----- (6.25)

or \[ \psi = a + d(72d + 1728e)x - (18d + 288e)x^2 + dx^3 + ex^4 \] ----- (6.26)

or \[ \psi = a + d(72x - 18x^2 + x^3) + e(1728x - 288x^2 + x^4) \] ----- (6.27)

Now, we have three adjustable parameters \( a, d, \) and \( e \) to accommodate seasonal variation. However, even this equation could not explain the data in a satisfactory manner. Additional degree of freedom in the form of coefficient \( e \), can be used to shift the point of inflection, however, magnitude of oscillation of the function cannot be adjusted in this equation. Therefore, in order to make waste generation data more representative, one more parameter is introduced, i.e., following polynomial of fifth degree is considered
\[ \psi = a + b.x + c.x^2 + d.x^3 + e.x^4 + f.x^5 \] ----- (6.28)

Again this equation should be such that condition of Equation (6.4) and (6.5) is satisfied. Therefore for equal slope of the function at \( x = 0 \) and \( x = 12 \) (Equation 6.5), we get
\[ b + 2c(0) + 3d(0) + 4e(0) + 5f(0) = b + 2c(12) + 3d(12)^2 + 4e(12)^3 + 5f(12)^4 \] ----- (6.29)

or \[ 2c(12) + 3d(12)^2 + 4e(12)^3 + 5f(12)^4 = 0 \] ----- (6.30)

or \[ 2c + 3d(12) + 4e(12)^2 + 5f(12)^3 = 0 \] ----- (6.31)

or \[ c + 18d + 288e + 4320f = 0 \] ----- (6.32)

or \[ c = -(18d + 288e + 4320f) \] ----- (6.33)

and for condition expressed in Equation (6.4), we have
\[ a + b(0) + c(0) + d(0) + e(0) + f(0) = a + b(12) + c(12)^2 + d(12)^3 + e(12)^4 + f(12)^5 \] ----- (6.34)

or \[ b(12) + c(12)^2 + d(12)^3 + e(12)^4 + f(12)^5 = 0 \] ----- (6.35)

or \[ b = -(12c + 12^2 d + 12^3 e + 12^4 f) \] ----- (6.36)
Combining equations (6.28), (6.33), and (6.36), the required polynomial to express the correction factor \( \psi \) becomes

\[
\psi = a - (12c + 12e + 12f)x + cx^2 + dx^3 + ex^4 + fx^5
\]

or

\[
\psi = a + c(x^2 - 12x) + d(x^3 - 12^2 x) + e(x^4 - 12^3 x) + f(x^5 - 12^4 x)
\]

or

\[
\psi = a - (18d + 288e + 4320f)(x^2 - 12x) + d(x^3 - 12^2 x) + e(x^4 - 12^3 x) + f(x^5 - 12^4 x)
\]

or

\[
\psi = a + (72d + 1728e + 31104f)x - (18d + 288e + 4320f)x^2 + dx^3 + ex^4 + fx^5
\]

or

\[
\psi = a + d(72x - 18x^2 + x^3) + e(1728x - 288x^2 + x^4) + f(31104x - 4320x^2 + x^5)
\]

Thus we have an equation with four parameters \((a, d, e, f)\) that can be adjusted to obtain the correction factor by regression analysis using available data.

Next, the values of constants \(a, d, e,\) and \(f\) of Equation (6.41), and \(k_1, k_2, k_3,\) of Equation (6.1), were determined for individual health care facilities by using NPSOL. NPSOL is a collection of FORTRAN 77 subroutines designed to solve nonlinear programming problems (Gill et al., 1986). The subroutines of the package can be used as a tool for minimizing a multivariate non-linear function (with more than 900 variables). The multivariate non-linear optimization technique is a well known technique generally used for finding optimum values of parameters of a system of nonlinear equations. A detailed analysis of this method can be seen in any text or reference book on numerical techniques such as Golub and Ortega (1992). This technique relies upon formation of an objective function that represents all the model equations. In the present case, we have twelve predicted values of waste generation rates, \(W_0, \psi\), in terms of seven coefficients \((k_1, k_2, k_3, a, d, e,\) and \(f)\), corresponding to twelve data point of one particular hospital in a calendar year. Now, our objective is to find values of seven coefficients in such a way that the absolute value of difference between actual and predicted values of waste generation rate (the discrepancy) is minimal. To achieve this, the following objective function was minimized to determine the optimal values of the coefficients.
\[ O(k_1, k_2, k_3, a, d, e, f) = \sum_{i=0}^{11} \left[ W_{actual} - W_0(k_1, k_2, k_3, B)\psi(a, d, e, f, x) \right]^2 \] ----- (6.42)

In the above equation \( \psi \) is calculated by Equation (6.41) and \( W_0 \) by Equation (6.1). Here it should be noted that the objective function is the sum of squares of all the discrepancies in one calendar year.

To begin with, an initial guess values of the coefficients \( k_1, k_2, k_3, a, d, e, \) and \( f \) were assumed. The software calculates the value of \( W_0\psi \) at this assumed value. Squares of the difference between \( W_{actual} \) and \( W_0\psi \) are the discrepancy between the data collected from hospitals and that calculated by the model equations. Now, the objective of the computer program is to minimize sum of all the discrepancies, calculated by Equation (6.41), by adjusting values of coefficients \( (k_1, k_2, k_3, a, d, e, \) and \( f) \).

In other words it can be said that the present technique is equivalent to the least square multivariate-nonlinear-regression analysis. The programming to obtain numerical values of coefficients of Equations (6.1) and (6.41) is given in Appendix ‘E’.