Chapter 6

Dominator Equitable Coloring of Graphs
6.1 Introduction

This chapter is aimed to discuss one more variant of proper coloring which has applications to handle the real life situations. We illustrate the usefulness of the concept.

Consider the scheduling problem for a school picnic. To find the minimum number of buses required to accommodate students of different classes along with their teachers in-charge as passengers such that,

1. at least one teacher be in every bus.
2. almost all the students of the same class should be with their teacher in charge in a bus.
3. number of passengers should be almost the same in every bus.

The problem is to find the minimum number of groups with almost equal number of members such that everyone should dominate at least one group.

The above stated task can be handled with the concept of dominator equitable coloring.

The concept of dominator coloring was introduced by S. M. Hedetniemi et al. [28] and Gera [21] in 2006 while the notion of equitable coloring was introduced by W. Meyer [39] in 1973.

Definition 6.1.1. A set $S \subseteq V(G)$ of vertices in a graph $G$ is called a dominating set if every vertex $v \in V(G)$ is either an element of $S$ or is adjacent to an element of $S$.

Definition 6.1.2. The dominating set with minimal cardinality is called the $\gamma$-set and its cardinality is called the domination number which is denoted by $\gamma(G)$.

Definition 6.1.3. A dominator coloring of a graph $G$ is a proper coloring in which every vertex of the graph dominates every vertex of some color class.
Definition 6.1.4. The dominator chromatic number $\chi_d(G)$ is the minimum number of colors required for a dominator coloring of $G$.

Definition 6.1.5. A vertex $v \in V(G)$ is a dominator of a set $D \subseteq V$ if $v$ dominates every vertex in $D$.

Definition 6.1.6. A partition $\Pi = V_1, V_2, \ldots, V_k$ is called a dominator partition if every vertex $v \in V(G)$ is a dominator of at least one $V_i$.

Definition 6.1.7. The dominator partition number $\Pi_d(G)$ equals the minimum $k$ for which $G$ has a dominator partition of order $k$.

Definition 6.1.8. If $\Pi$ is a set of color classes of $G$, i.e., a proper coloring of $G$, then $\Pi$ will be a dominator coloring of $G$.

It is very interesting to note that chromatic number may or may not be equal to the dominator chromatic number.

Illustration 6.1.9. Figure 6.1 shows a dominator coloring of a graph with $\chi(G) \neq \chi_d(G)$.

![Figure 6.1: $\chi_d(G) = 3$ while $\chi(G) = 2$](image)

Proposition 6.1.10. [21] The path $P_n$ of order $n \geq 3$ has

$$\chi_d(P_n) = \begin{cases} 1 + \left\lceil \frac{n}{3} \right\rceil, & n \text{ is } 2, 3, 4, 5, 7 \\ 2 + \left\lceil \frac{n}{3} \right\rceil, & \text{otherwise} \end{cases}$$
Proposition 6.1.11. [22] The complete \( k \)-partite graph \( K_{a_1,a_2,\ldots,a_k} \) has \( \chi_d(K_{a_1,a_2,\ldots,a_k}) = k \).

Proposition 6.1.12. [22] The wheel \( W_n \) has

\[
\chi_d(W_n) = \begin{cases} 
3, & n \text{ is even} \\
4, & n \text{ is odd}
\end{cases}
\]

6.2 Dominator Equitable Coloring- A Variant of Proper Coloring

The dominator equitable coloring is an example of merging two parameters- domination and coloring (precisely equitable coloring). Here the first priority is given to domination and then equitable color partitions of vertex set are formed.

Definition 6.2.1. A proper coloring which is dominator and equitable as well, is called dominator equitable coloring.

Definition 6.2.2. The minimum integer \( k \) for which \( G \) is dominator equitable \( k \) colorable is called dominator equitable chromatic number and is denoted by \( \chi_{de}(G) \).

Consider a graph \( G \) of order \( n \). Since every vertex is a dominator of itself and the partition of the vertex set into singleton sets as \( V_1 = \{v_1\} \), \( V_2 = \{v_2\} \), \ldots, \( V_n = \{v_n\} \) gives a dominator equitable coloring. Thus, every graph of order \( n \) has a dominator equitable coloring and therefore dominator equitable chromatic number \( \chi_{de}(G) \) is well defined.

The following illustration demonstrates the difference between dominator, equitable and dominator equitable colorings of graph.

Illustration 6.2.3. Figure 6.2 and Figure 6.3 show dominator coloring and equitable coloring of the graph while Figure 6.4 shows a dominator equitable coloring of the same graph. Here \( \chi_d(G) = 7 \) and \( \chi_e(G) = 3 \), but \( \chi_{de}(G) = 10 \).
Figure 6.2: $\chi_d(G) = 7$

Figure 6.3: $\chi_e(G) = 3$
Our first result is quite elementary.

**Theorem 6.2.4.** If a graph $G$ with order $n \geq 2$, then $2 \leq \chi_{de}(G) \leq n$.

**Proof.** Let $G$ be a connected graph of order $n \geq 2$. Then, at least two different colors are needed in a dominator equitable coloring since there are at least two vertices adjacent to each other. If $G$ is a disconnected graph of order $n \geq 2$, then also we need at least two different colors in a dominator equitable coloring since a vertex dominates its own color class. Hence $2 \leq \chi_{de}(G)$. Moreover, if each vertex receives its unique color, then also we have dominator equitable coloring. Hence $\chi_{de}(G) \leq n$. Thus, $2 \leq \chi_{de}(G) \leq n$.

We will have some quick observations.

**Observations:**

1. $\chi_{de}(K_n) = n$.

2. $\chi_{de}(\overline{K_n}) = n$.

3. $\chi_{de}(G) \geq cl(G)$, where $cl(G)$ is the clique number.
Theorem 6.2.5.  
1. \( \chi(G) \leq \chi_{de}(G) \).
2. \( \chi_d(G) \leq \chi_{de}(G) \).
3. \( \chi_e(G) \leq \chi_{de}(G) \).
4. \( \max\{\chi_d(G), \chi_e(G)\} \leq \chi_{de}(G) \).

Proof. The inequality \( \chi(G) \leq \chi_{de}(G) \) is evident as only a proper coloring is needed for a dominator equitable coloring. Since we are considering dominator and equitable coloring, the inequalities (2) and (3) also hold. The inequality (4) holds trivially because for a dominator equitable coloring a vertex has to dominate equitably to all the vertices of a color class.

Theorem 6.2.6. Let \( G \) be a non empty graph. \( \chi_{de}(G) = 2 \) if and only if \( G \) is isomorphic to \( K_{a,b} \) where \( |a - b| \leq 1 \).

Proof. Suppose that \( \chi_{de}(G) = 2 \). Then, \( G \) has dominator equitable 2-coloring. Let \( V_1 \) and \( V_2 \) be the partition of vertices with color 1 and 2 respectively. Since \( G \) has a dominator coloring, each vertex of \( V_1 \) is a dominator of \( V_2 \) and each vertex of \( V_2 \) is a dominator of \( V_1 \). Since \( G \) has an equitable coloring, \( |V_1| - |V_2| \leq 1 \). That is, either \( |V_1| = |V_2| \) or \( |V_1| = |V_2| - 1 \) or \( |V_2| + 1 \). That is, \( G \approx K_{a,b} \) where \( a = b \) or \( a = b - 1 \) or \( a = b + 1 \).

Conversely, suppose that \( G \approx K_{a,b} \) with bipartition \( V = V_1 \cup V_2 \) where \( a = b \) or \( a = b - 1 \) or \( a = b + 1 \). Assigning color 1 to all the vertices of \( V_1 \) and color 2 to all vertices of \( V_2 \). Each vertex of \( V_1 \) is adjacent to all the vertices of \( V_2 \), the partition is a dominator partition. Clearly, \( V_1 \) and \( V_2 \) are independent sets and \( |V_1| - |V_2| \leq 1 \). Thus, this coloring is a dominator equitable coloring. Hence \( \chi_{de}(G) = 2 \).

Theorem 6.2.7. \( \chi_{de}(K_{m,n}) = 1 + \left\lceil \frac{n}{m+1} \right\rceil \).

Proof. For the graph \( K_{m,n} \), \( \chi_e(K_{m,n}) = 1 + \left\lceil \frac{n}{m+1} \right\rceil \) and \( \chi_d(K_{m,n}) = 2 \) by Proposition 5.2.11 and Proposition 6.1.11 respectively.
Now by Theorem 6.2.5, \( \max\{\chi_d(K_{m,n}), \chi_e(K_{m,n})\} = \chi_e(K_{m,n}) \leq \chi_{de}(K_{m,n}) \). The equitable coloring of \( K_{m,n} \) is also a dominator coloring of \( K_{m,n} \). Hence \( \chi_{de}(K_{m,n}) = \chi_e(K_{m,n}) = 1 + \left\lceil \frac{n}{m+1} \right\rceil \).

We state very obvious results as follows:

**Corollary 6.2.8.**  
1. \( \chi_e(K_{1,n}) = 1 + \left\lceil \frac{n}{2} \right\rceil \).
2. \( \chi_e(K_{n,n}) = 2 \).

**Theorem 6.2.9.** \( \gamma(G) \leq \chi_{de}(G) \leq \gamma(G) + \left\lceil \frac{n - \gamma(G)}{2} \right\rceil \), \( \forall n \geq 3 \).

**Proof.** Since dominating set of a graph \( G \), with \( n \) vertices, provides color classes and each vertex is a dominator of its own color class, dominator equitable coloring needs minimum \( \gamma(G) \) number of colors. Thus, \( \gamma(G) \leq \chi_{de}(G) \).

Now, for the upper bound, let \( G \) be a connected graph with \( n \geq 3 \). Let \( S \) be the minimal dominating set of \( G \) and \( |S| = \gamma(G) \). Each vertex in \( S \) is a dominator of its own color class. Define \( V_1, V_2, \ldots, V_{\gamma(G)} \) to be a coloring of \( G \) in which \( V_i = \{v_i\} \); (\( 1 \leq i \leq \gamma(G) \)) and every vertex in \( V(G) - S \) is adjacent to some vertex \( v_i \).

Since we need equitable coloring also, each of the partition of \( V(G) - S \) should contain at most 2 elements as \( |V_i| = 1 \). Hence \( \chi_{de}(G) \leq |S| + \left\lceil \frac{V(G) - |S|}{2} \right\rceil \). That is, \( \chi_{de}(G) \leq \gamma(G) + \left\lceil \frac{n - \gamma(G)}{2} \right\rceil \).

**Observations:**

We can observe that for a given graph \( G \) and a subgraph \( H \), \( \chi(H) \leq \chi(G) \) and if \( H \) is a spanning subgraph of \( G \) then \( \gamma(H) \geq \gamma(G) \). But the dominator equitable chromatic number problem does not have this kind of property. That is, the dominator equitable chromatic number of the subgraph \( H \) can be smaller or larger than the dominator equitable chromatic number of the graph \( G \).
Let $G_1$ be an empty graph with $n$ vertices, $G_2$ be the star graphs $K_{1,n-1}$ and $G_3$ be the complete graph on $n$ vertices. Clearly, $G_1 \subset G_2 \subseteq G_3$ and $\chi_{de}(G_1) = n$, $\chi_{de}(G_2) = 1 + \left\lceil \frac{n-1}{2} \right\rceil$ and $\chi_{de}(G_3) = n$.

We have $G_2 \subseteq G_3 \Rightarrow \chi_{de}(G_2) = 1 + \left\lceil \frac{n-1}{2} \right\rceil \leq n = \chi_{de}(G_3)$,
and $G_1 \subset G_2 \Rightarrow \chi_{de}(G_1) = n \geq 1 + \left\lceil \frac{n-1}{2} \right\rceil = \chi_{de}(G_2)$.

That is, $\chi_{de}(G_2) \leq \chi_{de}(G_3)$ and $\chi_{de}(G_1) \geq \chi_{de}(G_2)$.

### 6.3 Dominator Equitable Coloring of Some Standard Graphs

#### Theorem 6.3.1. $\chi_{de}(P_n) = \gamma(P_n) + \left\lceil \frac{n - \gamma(P_n)}{2} \right\rceil$.

**Proof.** Let $S$ be the minimal dominating set of $P_n$ and $|S| = \gamma(P_n)$. Each vertex in $S$ is a dominator of its own color class.

Define $V_1, V_2, \ldots, V_{\gamma(P_n)}$ to be a coloring of $P_n$ in which $V_i = \{v_i\}; (1 \leq i \leq \gamma(P_n))$ and every vertex in $V(P_n) - S$ is adjacent to some vertex $v_i$.

Since we need equitable coloring also, each of the partition of $V(P_n) - S$ should contain at most 2 elements as $|V_i| = 1; (1 \leq i \leq \gamma(P_n))$. Hence $\chi_{de}(P_n) = |S| + \left\lceil \frac{|V(P_n)| - |S|}{2} \right\rceil$.

That is, $\chi_{de}(P_n) = \gamma(P_n) + \left\lceil \frac{n - \gamma(P_n)}{2} \right\rceil$.

#### Theorem 6.3.2. $\chi_{de}(C_n) = \gamma(C_n) + \left\lceil \frac{n - \gamma(C_n)}{2} \right\rceil$, $n \geq 3$.

**Proof.** By Theorem 6.3.1, we have $\chi_{de}(P_n) = \gamma(P_n) + \left\lceil \frac{n - \gamma(P_n)}{2} \right\rceil$. So there is a dominator equitable coloring of the path with vertices $v_1, v_2, \ldots, v_n$ using $\gamma(P_n) + \left\lceil \frac{n - \gamma(P_n)}{2} \right\rceil$ colors. We obtain the cycle $C_n$ from the path $P_n$ by adding the edge $v_1v_n$. 

and so each vertex of \( C_n \) will be dominated. In \( C_n \), \( v_1 \) and \( v_n \) are dominated by themselves or by \( v_2 \) and \( v_{n-1} \) respectively. Now, \( \gamma(C_n) = \gamma(P_n) \) as the dominating set of \( C_n \) is same as that of the path \( P_n \). Thus, we assign the dominator equitable coloring of \( C_n \) using the same number of colors as in the dominator equitable coloring of \( P_n \). Hence, \( \chi_{de}(C_n) = \gamma(C_n) + \left\lceil \frac{n - \gamma(C_n)}{2} \right\rceil \), \( n \geq 3 \). ■

**Theorem 6.3.3.** If \( G \approx H_n, CH_n \), then \( \chi_{de}(G) = \gamma(G) + \left\lceil \frac{n - \gamma(G)}{2} \right\rceil \).

**Proof.** Let \( S \) be the minimal dominating set of \( G \) and \(|S| = \gamma(G)\). Each vertex in \( S \) is a dominator of its own color class.

Define \( V_1, V_2, \ldots, V_{\gamma(G)} \) to be a coloring of \( G \) in which \( V_i = \{v_i\}; (1 \leq i \leq \gamma(G)) \) and every vertex in \( V(G) - S \) is adjacent to some vertex \( v_i \).

Since we need equitable coloring also, each of the partition of \( V(G) - S \) should contain at most 2 elements as \(|V_i| = 1; (1 \leq i \leq \gamma(G))\). Hence \( \chi_{de}(G) = |S| + \left\lceil \frac{V(G) - |S|}{2} \right\rceil \). That is, \( \chi_{de}(G) = \gamma(G) + \left\lceil \frac{n - \gamma(G)}{2} \right\rceil \). ■

**Theorem 6.3.4.** \( \chi_{de}(W_n) = 1 + \left\lceil \frac{n}{2} \right\rceil \).

**Proof.** By Theorem 6.2.5, \( \max\{\chi_e(W_n), \chi_c(W_n)\} = \chi_e(W_n) \leq \chi_{de}(W_n) \). We have, \( \chi_e(W_n) = 1 + \left\lceil \frac{n}{2} \right\rceil \) by Theorem 5.5.1. Hence \( 1 + \left\lceil \frac{n}{2} \right\rceil \leq \chi_{de}(W_n) \). Now, the equitable coloring of \( W_n \) is also a dominator coloring of \( W_n \). Hence, \( \chi_{de}(W_n) = 1 + \left\lceil \frac{n}{2} \right\rceil \). ■

The lower bound of statement (4) of Theorem 6.2.5 achieved by \( K_{1,n}, K_{m,n} \) while the upper bound of Theorem 6.2.9 is sharp for \( K_n, P_n, C_n, W_n, H_n \) and \( CH_n \) etc.

**Illustration 6.3.5.** The Petersen graph \( P(5,2) \) and its dominator equitable coloring is shown in Figure 6.5.

**Illustration 6.3.6.** The complement of Petersen graph, \( \overline{P(5,2)} \) and its dominator equitable coloring is shown in Figure 6.6.
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Figure 6.5: $\chi_{de}(P(5, 2)) = 6$

Figure 6.6: $\chi_{de}(P(5, 2)) = 5$

Theorem 6.3.7. For a Prism $Y_n \approx P(n, 1)$,

$$
\chi_{de}(Y_n) = \begin{cases} 
  n, & \text{n is even} \\
  n + 1, & \text{n is odd}
\end{cases}
$$

Proof. Let $V(P(n, 1)) = \{u_i, v_i; 1 \leq i \leq n\}$ and $E(P(n, 1)) = \{u_i u_{i+1}, u_i v_i, v_i v_{i+1}; 1 \leq i \leq n\}$. Each $P(n, 1)$ contains $\frac{n}{2}$ cycles of order 4 and each $C_4$’s can have a dominator equitable coloring using two different colors. Thus, all such $C_4$’s can have a dominator equitable coloring using $\frac{n}{2} \cdot 2 = n$ colors. Hence, $\chi_{de}(Y_n) = n$; $n$ is even.

But when $n$ is odd, each $P(n, 1)$ contains $\left\lceil \frac{n}{2} \right\rceil$ cycles of order 4. To assign the dominator equitable coloring to these cycles we need $n - 1$ colors, except for the vertices
$u_n$ and $v_n$. But these vertices dominate each other and we can assign two new colors to these vertices. Thus, for the dominator equitable coloring of $P(n, 1)$, we need $n - 1 + 2$ colors. Hence, $\chi_{de}(Y_n) = n + 1$; $n$ is odd.

**Illustration 6.3.8.** Dominator equitable coloring of Prisms $P(8,1)$ and $P(11,1)$ are shown in Figure 6.7 and Figure 6.8

![Figure 6.7: $\chi_{de}(P(8,1)) = 8$](image)

![Figure 6.8: $\chi_{de}(P(11,1)) = 12$](image)

**Theorem 6.3.9.** $\chi_{de}(P_n \circ K_1) = 2n - \left\lfloor \frac{n}{2} \right\rfloor$. 
Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of $P_n$ and $v'_1, v'_2, \ldots, v'_n$ be the vertices corresponding to $K_1$ which are adjacent to each $v_i; i = 1, 2, \ldots, n$. Now the graph $P_n \circ K_1$ contains $\left\lfloor \frac{n}{2} \right\rfloor$ number of edge disjoint $P_3$’s and dominator equitable coloring of these $P_3$’s can be done using $2 \left\lfloor \frac{n}{2} \right\rfloor$ colors. The remaining $2n - 3 \left\lfloor \frac{n}{2} \right\rfloor$ vertices are the dominator of themselves. So we need $2n - 3 \left\lfloor \frac{n}{2} \right\rfloor + 2 \left\lfloor \frac{n}{2} \right\rfloor$ colors in order to assign the dominator equitable coloring of the graph $P_n \circ K_1$. Hence, $\chi_{de}(P_n \circ K_1) = 2n - 3 \left\lfloor \frac{n}{2} \right\rfloor + 2 \left\lfloor \frac{n}{2} \right\rfloor = 2n - \left\lfloor \frac{n}{2} \right\rfloor$. ■

Illustration 6.3.10. Dominator equitable coloring of $P_9 \circ K_1$ is shown in Figure 6.9.

![Figure 6.9: $\chi_{de}(P_9 \circ K_1) = 18 - \left\lfloor \frac{9}{2} \right\rfloor = 14$](image)

Theorem 6.3.11.

$$\chi_{de}(C_n \circ K_1) = 2n - \left\lfloor \frac{n}{2} \right\rfloor.$$ 

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of $C_n$ and $v'_1, v'_2, \ldots, v'_n$ be the vertices corresponding to $K_1$ which are adjacent to each $v_i; i = 1, 2, \ldots, n$. Now the graph $C_n \circ K_1$ contains $\left\lfloor \frac{n}{2} \right\rfloor$ number of edge disjoint $P_3$’s and dominator equitable coloring of these $P_3$’s can be done using $2 \left\lfloor \frac{n}{2} \right\rfloor$ colors. The remaining $2n - 3 \left\lfloor \frac{n}{2} \right\rfloor$ vertices are the dominator of themselves. So we need $2n - 3 \left\lfloor \frac{n}{2} \right\rfloor + 2 \left\lfloor \frac{n}{2} \right\rfloor$ colors in order to assign the dominator equitable coloring of the graph $C_n \circ K_1$. Hence, $\chi_{de}(C_n \circ K_1) = 2n - 3 \left\lfloor \frac{n}{2} \right\rfloor + 2 \left\lfloor \frac{n}{2} \right\rfloor = 2n - \left\lfloor \frac{n}{2} \right\rfloor$. ■

Illustration 6.3.12. Dominator equitable coloring of $C_{10} \circ K_1$ and $C_{11} \circ K_1$ are shown in Figure 6.10 and Figure 6.11.
Theorem 6.3.13.

\[\chi_{de}(P_2 \Box P_n) = \begin{cases} 
2 \left\lfloor \frac{n}{2} \right\rfloor + 2, & n \text{ is odd} \\
\frac{n}{2}, & n \text{ is even}.
\end{cases}\]

Proof. Let \(V(P_2) = \{u_1, u_2\}\) and \(V(P_n) = \{v_1, v_2, \ldots, v_n\}\). In \(P_2 \Box P_n\), let us denote \((u_1, v_i)\) as \(x_i\) and \((u_2, v_j)\); \(1 \leq i, j \leq n\) as \(y_j\) and the vertices \(x_i, x_{i+1}, y_{i+1}, y_i\) form a cycle \(C_4\). We know \(\chi_{de}(C_4) = 2\).
If \( n \) is odd, there are \( \left\lfloor \frac{n}{2} \right\rfloor \) such cycles. Thus, we need \( 2 \left\lfloor \frac{n}{2} \right\rfloor \) colors to assign dominator equitable coloring of such cycles. Now the vertices \( x_n \) and \( y_n \) dominates itself. Hence \( \chi_{de}(P_2 \square P_n) = 2 \left\lfloor \frac{n}{2} \right\rfloor + 2. \)

If \( n \) is even there are exactly \( \frac{n}{2} \) disjoint cycles of length 4. Thus, we need \( 2 \cdot \frac{n}{2} \) colors to assign dominator equitable coloring of such cycles. Hence \( \chi_{de}(P_2 \square P_n) = n; \) for even \( n. \)

**Illustration 6.3.14.** Dominator equitable coloring of \( P_2 \square P_7 \) is shown in Figure 6.12.

![Figure 6.12: \( \chi_{de}(P_2 \square P_7) = 2 \left\lfloor \frac{7}{2} \right\rfloor + 2 = 8 \)](image)

**Theorem 6.3.15.** \( \chi_{de}(P_2 \times P_n) = 2 \cdot \chi_{de}(P_n). \)

**Proof.** Let \( V(P_2) = \{u_1, u_2\} \) and \( V(P_n) = \{v_1, v_2, \ldots, v_n\}. \) In \( P_2 \times P_n, \) let us denote \((u_1, v_i)\) as \( x_i \) and \((u_2, v_j)\) as \( y_j; 1 \leq i, j \leq n \) and the vertices \( x_1, y_2, x_3, y_4, x_5, y_6, \ldots, y_n \) and \( y_1, x_2, y_3, x_4, \ldots, x_n \) forms two disjoint paths, each of length \( n. \) We know \( \chi_{de}(P_n) = \gamma(P_n) + \left\lceil \frac{n - \gamma(P_n)}{2} \right\rceil. \) Since these two paths are disconnected, the colors we assigned in one of the paths cannot be assigned to the other. Hence we need \( 2 \cdot \chi_{de}(P_n) \) colors for the dominator equitable coloring of \( P_2 \times P_n. \) Hence, \( \chi_{de}(P_2 \times P_n) = 2 \cdot \chi_{de}(P_n). \)

**Illustration 6.3.16.** Dominator equitable coloring of \( P_2 \times P_7 \) is shown in Figure 6.13.

**Theorem 6.3.17.** \( \chi_{de}(P_2[P_n]) = 4. \)

**Proof.** Let \( V(P_2) = \{u_1, u_2\} \) and \( V(P_n) = \{v_1, v_2, \ldots, v_n\}. \) Let us denote \((u_1, v_i)\) as \( x_i \) and \((u_2, v_j)\) as \( y_j \) and \( x_i \) is adjacent to each \( y_j \) and \( x_{i+1}; 1 \leq i, j \leq n. \) Let us assign the
coloring as \( c(x_{2k+1}) = 1, c(x_{2k}) = 2, c(y_{2k+1}) = 3 \) and \( c(y_{2k}) = 4; \ k \in \mathbb{N} \). The vertices \( x_{2k+1} \) dominate the vertices \( y_{2k} \) and the vertices \( x_{2k} \) dominate the vertices \( y_{2k+1} \). Hence the coloring is both dominator and equitable. Hence, \( \chi_{de}(P_2 \times P_7) = 10 \).

Illustration 6.3.18. Dominator equitable coloring of \( P_2[P_5] \) is shown in Figure 6.14.

6.4 Concluding Remarks

The dominator equitable coloring is a concept which possesses the blend of dominator coloring and equitable coloring. We have investigated the dominator equitable chromatic number for some standard graphs and also obtained some bounds on this parameter as well as the graphs achieving these bounds. In addition to this we have also proved some results for the larger graphs obtained from standard graphs by means of some graph operations.
6.5 Future Scope of Research

- To obtain new families of graph which admit dominator equitable coloring.

- To identify the relation between dominator equitable coloring and other graph theoretic parameters like clique number, independent domination number, redundancy number, global domination number etc.

- To prove some characterizations for dominator equitable coloring of graphs.

- To introduce edge analogue of dominator equitable coloring.