

Chapter – I



General Introduction



A shock wave (also called shock front or simply “shock”) is a type of propagating disturbance. Like an ordinary wave, it carries energy and can propagate through a medium (solid, liquid, gas or plasma) or in some cases in the absence of a material medium, through a field such as the electromagnetic field. Shock waves are characterized by an abrupt, nearly discontinuous change in the characteristics of the medium. Across a shock there is always an extremely rapid rise in pressure, temperature and density of the flow. In supersonic flows, expansion is achieved through an expansion fan. A shock wave travels through most media at a higher speed than an ordinary wave. Unlike solitons (another kind of nonlinear wave), the energy of a shock wave dissipate relatively quickly with distance. Also, the accompanying expansion wave approaches and eventually merges with the shock wave, partially cancelling it out. Thus the sonic boom associated with the passage of a supersonic aircraft is the sound wave resulting from the degradation and merging of the shock

wave and the expansion wave produced by the aircraft. When a shock wave passes through matter, the total energy is preserved but the energy which can be extracted as work decreases and entropy increases. This, for example, creates additional drag force on aircraft with shocks.

Shock Waves can be –

Normal : at 90° (perpendicular) to the shock medium's flow direction.

Oblique : at an angle to the direction of flow.

Bow : occurs upstream of the front (bow) of a blunt object when the upstream velocity exceeds Mach 1.

Some other terms –

Shock Front : an alternative name for the shock wave itself

Contact Front : in a shock wave caused by a driver gas (for example the “impact” of a high explosive on the surrounding air), the boundary between the driver (explosive products) and the driven (air) gases. The Contact Front trails the Shock Front.

Shock Waves :

Shock waves are the most important distinctive features of supersonic flow of a gas, across which the medium undergoes sudden and often considerable changes in velocity, pressure, density and temperature. The occurrence of shock waves is commonly associated with supersonic flight, explosions and electric discharges. The formation of shock wave can be simply visualized by considering the uniform motion of a piston into a open ended tube filled with virgin gas. Suppose the continuous motion of the piston is approximated by a set of forward-moving pulses, each of short duration. When the piston makes the first short movement forward, a small disturbance is propagated forwards into the gas at the speed of sound. This small amplitude wave (or sound wave) heats the gas slightly and because of the square of the local speed of sound is proportional to the temperature, the second pulse will be propagated as another sound wave at a speed slightly in excess of first one. Similarly the third pulse will be propagated at a speed slightly in excess of second and so on. Thus the discrete pulses cause a train of sound waves of ever increasing velocity to be propagated through the gas. The tendency is for faster moving rearmost waves to catch up with the slower moving foremost ones. In so doing the sound waves

coalesce to form a more powerful shock front moving at a speed which is in excess of the local speed of sound.

Shock waves are the most conspicuous phenomena occurring in non-linear wave propagation. Even without being caused by initial discontinuities, they may appear and be propagated. The underlying mathematical fact is that, unlike linear partial differential equations, non-linear equations often do not admit solutions which can be continuously extended whenever the differential equations themselves remain regular. The problem of shock waves has a bearing on many problems outside of supersonic aeronautics, for example, detonation waves, but also has great importance for several practical aeronautical problems. In fact, shock waves may cause sudden change in the aerodynamic behavior of high speed aircraft affecting not only their balance and stability but also control producing undesirable vibrations.

Rayleigh and Hugoniot first pointed that an adiabatic reversible transition in a shock would violate the principle of conservation of energy. In fact, Hugoniot showed that in the absence of viscosity and heat conduction (outside the shock) conservation of energy implies conservation of entropy in continuous flow and also implies a change of entropy across a shock. Rayleigh pointed out that

the entropy must increase in crossing a shock front and that, for this reason, a rarefaction shock cannot occur in a perfect gas.

It is true, of course, that a shock wave is not a discontinuity in the strict sense. It has a finite thickness across which the physical properties change continuously. If this thickness is small compared with some appropriate macroscopic dimension of the flow field, such as the radius of curvature of a curved shock, the physical relationships may be obtained by an analysis which treats the discontinuity as strict. The assumption that the discontinuity thickness is small compared with a macroscopic dimension is a fundamental one. The term 'structure' as applied to a shock wave, refers to the values of the physical properties of the fluid within the small but finite thickness of the discontinuity. If thermodynamic equilibrium in a substance is disturbed a characteristic time must elapse before equilibrium can be approximately reestablished. This time the velocity of the fluid, define a characteristic distance which is of the order of a molecular mean free path or greater. If the physical and chemical changes, occurring in the discontinuity are sufficiently slow so that the thickness of discontinuity is large compared with this characteristic distance, the concept of their thermodynamic quasi-equilibrium may be considered. In this case, the Navier – Stoke's

equations are applicable. If the discontinuity is thin with physical and chemical changes occurring rapidly, i.e. detonation front, the essential absence of thermodynamic equilibrium must be taken into account (Hayes [15]).

Velocity and heat conduction are neglected but where large gradients of temperature and velocity develop, they become important. Viscosity and heat conduction have the effect of smoothing out the discontinuity. Both viscosity and thermal conductivity present themselves as diffusion phenomena, arising respectively from the molecular transfer of momentum and energy. The former manifests itself in diffusion of vorticity and the later in the diffusion of heat.

Relative to the shock wave, the flow on the upstream side must be supersonic, on the down stream side the flow relative to the shock wave may be either supersonic or subsonic, depending on the inclination to the incident stream of the normal to the wave. If the normal to the wave is parallel to the incident stream, the flow behind is always subsonic relative to the wave. By now, the theory of shock waves in non-conducting and conducting media has been much developed.

Supersonic Flows :

When an object (or disturbance) moves faster than the information about it can be propagated into the surrounding fluid, fluid near the disturbance cannot react or “get out of the way” before the disturbance arrives. In a shock wave the properties of the fluid (density, pressure, temperature, velocity, Mach number) change almost instantaneously. Measurements of the thickness of shock waves have resulted in values approximately one order of magnitude greater than the mean free path of the gas investigated.

Shock waves are formed when the speed of a gas changes by more than the speed of sound. At the region where this occurs sound waves traveling against the flow reach a point where they cannot travel any further upstream and the pressure progressively builds in that region, and a high pressure shock wave rapidly formed. Shock waves are not conventional sound waves; as shock wave takes the form of a very sharp change in the gas properties on the order of a few mean free path (roughly micro-meters at atmospheric conditions) in thickness. Shock waves in air are heard as a loud “crack” or “snap” noise. Over longer distances a shock wave can change from a

nonlinear wave into a linear wave, degenerating into a conventional sound wave as it heats the air and loses energy. The sound wave is heard as the familiar “thud” or “thump” of a sonic boom, commonly created by the supersonic flight of aircraft.

The shock wave is one of the several different ways in which a gas in a supersonic flow can be compressed. Some other methods are isentropic compressions, including Prandtl – Meyer compressions. The method of compression of a gas results in different temperatures and densities for a given pressure ratio, which can be analytically calculated for a non-reacting gas. A shock wave compression results in a loss of total pressure, meaning that it is a less efficient method of compressing gases for some purposes, for instance in the intake of a scramjet. The appearance of pressure-drag on supersonic aircraft is mostly due to the effect of shock compression on the flow.

Due to Nonlinear Steepening :

Shock waves can be formed due to steepening of ordinary waves. The best-known example of this phenomenon is ocean waves that form breakers on the shore. In shallow water, the

speed of surface waves is dependent on the depth of the water. An incoming ocean wave has a slightly higher wave speed near the crest of each wave than near the troughs between waves, because the wave height is not infinitesimal compared to the depth of the water. The crests overtake the troughs until the leading edge of the wave forms a vertical face and spills over to form a turbulent shock (a breaker) that dissipates the wave's energy as sound and heat.

Similar phenomena affect strong sound waves in gas or plasma due to the dependence of the sound speed on temperature and pressure. Strong waves heat the medium near each pressure front, due to adiabatic compression of the air itself, so that high pressure fronts outrun the corresponding pressure troughs. While shock formation by this process does not normally happen to sound wave in Earth's atmosphere, it is thought to be one mechanism by which the solar chromospheres and corona are heated, via waves that propagate up from the solar interior.

Analogies :

A shock wave may be described as the furthest point upstream of a moving object which "knows" about the approach of

the object. In this description, the shock wave position is defined as the boundary between the zone having no information about the shock-driving event, and the zone aware of the shock-driving event, analogous with the light cone described in the theory of special relativity.

To get a shock wave something has to be travelling faster than the local speed of sound. In that case some parts of the air around the aircraft are travelling at exactly the speed of sound with the aircraft, so that the sound waves leaving the aircraft pile up on each other, similar to a tailback on a road, and a shock wave forms, the pressure increases, and then spreads out sideways. Because of this amplification effect, a shock wave is very intense, more like an explosion when heard (not coincidentally, since explosions create shock waves). Analogous phenomena are known outside fluid mechanics. For example, particles accelerated beyond the speed of light in a refractive medium (where the speed of light is less than that in a vacuum, such as water) create visible shock effects, a phenomenon known as Cherenkov radiation.

Different Types of Shock Waves :

Moving Shock –

It is a shockwave propagating into a stationary medium. In this case, the gas ahead of the shock is stationary (in the laboratory frame), and the gas behind the shock is supersonic in the laboratory frame. The shock propagates with a wave front which is normal (at right angles) to the direction of flow. The speed of the shock is a function of the original pressure ratio between the two bodies of gas. Moving shocks are usually generated by the interaction of two bodies of gas at different pressure, with a shock wave propagating into the lower pressure gas and an expansion wave propagating into the higher pressure gas.

Detonation Wave –

A detonation wave is essentially a shock supported by a trailing exothermic reaction. It involves a wave traveling through a highly combustible or chemically unstable medium, such as an oxygen-methane mixture or a high explosive. The chemical reaction of the medium occurs following the shock wave, and the chemical energy of the reaction drives the wave forward. A detonation wave follows slightly different rules from an ordinary shock since it is

driven by the chemical reaction occurring behind the shock wave front. On the simplest theory for detonations, an unsupported, self-propagating detonation wave proceeds at the Chapman – Jouguet velocity. A detonation will also cause a shock of type 1, above to propagated into the surrounding air due to the overpressure induced by the explosion. When a shock wave is created by high explosive such as TNT (which has a detonation velocity of 6,900 m/s) it will always travel at high, supersonic velocity from its point of origin.

Detached Shock –

These shocks are curved and form a small disturbance in front of the body. Directly in front of the body, they stand at 90 degrees to the oncoming flow, and then curve around the body. Detached shocks allow the same type of analytic calculations as for the attached shock, for the flow near the shock. They are a topic of continuing interest, because the rules governing the shock's distance ahead of the blunt body are complicated, and are a function of the body's shape. Additionally, the shock standoff distance varies drastically with the temperature for a non-ideal gas, causing large differences in the heat transfer to the thermal protection system of the vehicle. These follow the “strong-shock” solutions of the analytic

equations, meaning that for some oblique shocks very close to the deflection angle limit, the downstream Mach number is subsonic. Such a shock occurs when the maximum deflection angle is exceeded. A detached shock is commonly seen on blunt bodies, but may also be seen on sharp bodies at low Mach numbers.

Attached Shock –

The shock appears as “attached” to the tip of a sharp body moving at supersonic speeds. For examples supersonic wedges and cones with small apex angles. The attached shock wave is a classic structure in aerodynamics because, for a perfect gas and inviscid flow field, an analytic solution is available, such that the pressure ratio, temperature ratio, angle of the wedge angle and the downstream Mach number can all be calculated knowing the upstream Mach number and the shock angle. Smaller shock angles are associated with higher upstream Mach numbers, and the special case where the shock wave is at 90 degrees to the oncoming flow (Normal shock), is associated with a Mach number of 1. These follow the “weak-shock” solutions of the analytic equations.

Recompression Shock –

These shocks appear when the flow over a transonic body is decelerated to subsonic speeds. Transonic wings and turbines

are the examples of the recompression shock. Where the flow over the suction side of a transonic wing is accelerated to a supersonic speed, the resulting recompression can be by either Prandtl – Meyer compression or by the formation of a normal shock. This shock is of particular interest to makers of transonic devices because it can cause separation of the boundary layer at the point where it touches the transonic profile. This can then lead to full separation and stall on the profile, higher drag, or shock-buffet, a condition where the separation and the shock interact in a resonance condition, causing resonating loads on the underlying structure.

Shock in a Pipe Flow –

This shock appears when supersonic flow in a pipe is decelerated. Supersonic ramjet, scramjet and needle valve are the examples of this shock flow. In this case the gas ahead of the shock is supersonic (in the laboratory frame) and the gas behind the shock system is either supersonic (oblique shocks) or subsonic (a normal shocks). The shock is the result of the deceleration of the gas by a converging duct or by the growth of the boundary layer on the wall of a parallel duct.

Magnetogasdynamics :

The field of magnetogasdynamics is basically a study of the interaction between electromagnetic field and moving electrically conducting gases. The mathematical model for studying the interaction between gasdynamic phenomena and the magnetic field includes equations from gasdynamics and electromagnetic theory.

In most practical problems, we are interested more in the resultant effect due to the motion of a large number of particles, rather than in the motion of an individual particles in the fluid. Therefore, we use a macroscopic analysis of the fluid rather than view it in terms of microscopic quantities. Hence the fundamental equations used in analyzing the dynamics of conducting fluid are based on the conservation of mass, momentum and energy together with Maxwell's equations and the electromagnetic forces as well as the ordinary gasdynamical forces.

In most of the electromagnetic problems involving conductors the Maxwell's displacement currents are ignored (Anile and Greco [1], Pai [39, 40] and Spitzer [61]). The magnetic permeability of the medium considered in magnetogasdynamics differ

only slightly from unity and therefore, is taken here as unity in the application.

The field equations are then

$$\nabla \times \bar{\mathbf{E}} = -\frac{1}{c} \bar{\mathbf{H}}_t, \quad (1.1)$$

$$\nabla \times \bar{\mathbf{H}} = \frac{4\pi}{c} \bar{\mathbf{J}} = \frac{4\pi}{c} \sigma \left(\bar{\mathbf{E}} + \frac{\bar{\mathbf{u}} \times \bar{\mathbf{H}}}{c} \right), \quad (1.2)$$

$$\nabla \cdot \bar{\mathbf{H}} = 0, \quad (1.3)$$

where c is the velocity of light, $\bar{\mathbf{u}}$ is the velocity of fluid, $\bar{\mathbf{H}}$ is the magnetic field, σ is the electrical conductivity, $\bar{\mathbf{E}}$ is the electric intensity and $\bar{\mathbf{J}}$ is the current density.

We consider σ being uniform in the medium.

Substituting the equation (1.3) in equation (1.2) we get

$$\bar{\mathbf{H}} - \nabla \times (\bar{\mathbf{u}} \times \bar{\mathbf{H}}) = \frac{c^2 \nabla^2 \bar{\mathbf{H}}}{4\pi\sigma}. \quad (1.4)$$

In case of infinite electrical conductivity, the equation (1.4) becomes

$$\bar{\mathbf{H}} + (\bar{\mathbf{u}} \cdot \nabla) \bar{\mathbf{H}} + \bar{\mathbf{H}} (\nabla \cdot \bar{\mathbf{u}}) - (\bar{\mathbf{H}} \cdot \nabla) \bar{\mathbf{u}} = 0. \quad (1.5)$$

Using equation (1.3) in equation (1.5) we get

$$\bar{\mathbf{H}} + (\bar{\mathbf{u}} \cdot \nabla) \bar{\mathbf{H}} + \bar{\mathbf{H}} (\nabla \cdot \bar{\mathbf{u}}) = 0. \quad (1.6)$$

The equation (1.6) is used in conjunction with gas dynamic flow equation to incorporate the effect of magnetic field interaction.

Limited by the ability to analyze the system of non-linear differential equations, pioneering efforts in this field were concentrated on the propagation of gas dynamic shocks and electromagnetic waves. In order to investigate the hydrodynamic shocks, the electrical conductivity of the medium is assumed to have infinite value. This assumption implies that the self-induction will prevent changes to magnetic field of the medium at rest (Hoffman and Teller [19] and Kulikovskii and Liubimov [26]). Also the governing equations degenerate into the non-convex hyperbolic system, for which the characteristic surface may have unexpected singularities, making the wave structure much more complex than aerodynamic shocks (Christov *et al.* [12], Courant and Hilbert [13], Hirschler and Steiner [18] and Jeffrey and Taniuti [20]). Ideal magneto-gasdynamics offers impressive potential applications, but also generates many unanswered questions and uncertainties (Kantrowitz and Perschek [22]).

Shock Wave in Magnetogasdynamics :

If a conducting fluid moves in a magnetic field, electric fields are induced and electric currents flow. The magnetic field exerts forces on these currents which considerably modify the flow. In many problems the energy in the electric field is much smaller than that in magnetic field. In these cases, we may express all the electromagnetic quantities in terms of magnetic field. As a result, we consider only the interaction between the magnetic field and the gas dynamic field. This analysis forms the subject matter of the well known 'magnetogasdynamics' and this interaction is of prime importance in most of the astrophysical and geophysical problems and in the behavior of interstellar gaseous masses.

The equations of motion for one dimensional magnetogasdynamic flow in a perfectly conducting fluid are as under :

(i) The continuity equation is

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{i \rho u}{r} = 0, \quad (1.7)$$

where $i = 0, 1, 2, \dots$ for planar, cylindrically and spherically symmetric flows, respectively; u and ρ are fluid velocity and density at time t and at distance r from the plane, axis or centre of symmetry.

(ii) The momentum equation in planar symmetry with magnetic field h perpendicular to the flow, is

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\mu h}{\rho} \frac{\partial h}{\partial r} = 0, \quad (1.8)$$

where, μ is the magnetic permeability of the medium and p is the pressure.

The momentum equation in cylindrical or spherical symmetry with azimuthal magnetic field h is

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\mu h}{\rho r} \frac{\partial (hr)}{\partial r} = 0. \quad (1.9)$$

The momentum equation in cylindrical symmetry with axial magnetic field h is

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\mu h}{\rho} \frac{\partial h}{\partial r} = 0. \quad (1.10)$$

If the fluid is not perfectly conducting, but weakly conducting, the momentum equation in cylindrical symmetry takes the form

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \frac{\partial \mathbf{u}}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\sigma \mu^2 h^2 \mathbf{u}}{\rho} = 0, \quad (1.11)$$

where σ is the electrical conductivity and h is axial or azimuthal magnetic field.

(iii) The magnetic field equation is

$$\frac{\partial h}{\partial t} + \mathbf{u} \frac{\partial h}{\partial r} + h \frac{\partial \mathbf{u}}{\partial r} + \frac{j h \mathbf{u}}{r} = 0, \quad (1.12)$$

where for planar symmetry, $j = 0$;

for cylindrical symmetry with azimuthal magnetic field (h), $j = 0$;

for cylindrical symmetry with axial magnetic field (h), $j = 1$;

and for spherical symmetry with azimuthal magnetic field (h), $j = 1$.

(iv) The energy equation is

$$\frac{\partial}{\partial t} (p \rho^{-\gamma}) + \mathbf{u} \frac{\partial}{\partial r} (p \rho^{-\gamma}) = 0, \quad (1.13)$$

where γ is the ratio of specific heats of the fluid.

The study of magnetogasdynamic shock wave was systematically begun in the year 1950 with the paper of Hoffmann and Teller [19]. The basic properties of magnetogasdynamic shock waves, as determined by the conservation laws (the Rankine – Hugoniot relations), have been development further by Pai [39], Ray [48] and Vishwakarma and Yadav [71] and many others; although the more complex question of their existence in nature has yet to be exhaustively treated. The relations connecting the flow variables on the two sides of the shock surface (the generalized Rankine – Hugoniot relations) in the coordinate system in which the velocity in front of the shock wave is zero, are as follows (Pai [39]) :

$$h_2 (U - u_2) = h_1 U , \quad (1.14)$$

$$\rho_2 (U - u_2) = \rho_1 U , \quad (1.15)$$

$$p_2 + \frac{1}{2} \mu h_2^2 + \rho_2 (U - u_2)^2 = p_1 + \frac{1}{2} \mu h_1^2 + \rho_1 U^2 , \quad (1.16)$$

$$\begin{aligned} \frac{1}{2} (U - u_2)^2 + \frac{\gamma p_2}{(\gamma - 1) \rho_2} + \frac{\mu h_2^2}{\rho_2} \\ = \frac{1}{2} U^2 + \frac{\gamma p_1}{(\gamma - 1) \rho_1} + \frac{\mu h_1^2}{\rho_1} , \end{aligned} \quad (1.17)$$

where the subscripts '1' and '2' correspond to the values of the quantities just ahead and just behind the shock surface, respectively, and U is the shock velocity.

If the fluid is weakly conducting, the magnetic field is continuous across the shock (Sakurai [53]).

Review of Literature :

One of the interesting properties of the shock waves is the problem of determining the differential effects of shock fronts on the rear flow field. To this problem Thomas [65] developed a tensorial approach which was further extended by Kanwal [23, 24] for three dimensional shocks in stationary, pseudo-stationary and unsteady flows of non-conducting gases. The problem of vorticity, magnetic field, shock waves in reactive hydrodynamics, jump condition generated by a shock have also been solved by several authors like Arora and Sharma [5], Arora [6], Arora *et al.* [7], Baty and Tucker [8], Bhardwaj [9], Blythe [10], Hayes [15], Jordan [21], Kanwal [25], Lin and Szeri [31], Muralidharan and Sujith [35], Ram [46], Sekhar and Sharma [55], Singh *et al.* [57, 58 59], Somogyi and Roberts [60], Sujith [63] and Truesdell [66].

However, in many cases, one may be interested to know the position and the strength of the shock at any instant of time. Anile [2] developed the method of generalized wave front expansion (GWE) which accurately predicts the evolution of weak shock fronts (treated as discontinuities moving in space) without explicitly calculating the flow behind it. The method is based on an asymptotic expansion in a neighbourhood of the wave front and provides an efficient tool for solving approximately the problem of propagating weak shock wave. Russo [51] discussed the general features of GWE method and made a comparison between the results obtained by GWE and the exact solution obtained by shock fitting technique. He found that under appropriate regularity assumption GWE method provides an approximation of exact solution to the desired order for short times. Later, Anile and Russo [3, 4] extended this method to higher order correction and derived an infinite hierarchy of couple transport equations along the wave front (rays) for the shock amplitude and the jumps of the field gradients. Madhumita and Sharma [32, 33] employed a different approach to describe the kinematics of a shock wave of arbitrary strength by considering an infinite sequence of transport equations for the variation of jump in the field variable and

their space derivative across the shock and used a truncation procedure similar to that proposed by Maslov [34].

A considerable amount of work has also been done on the shock structure. A lot of work on the shock structure was carried out by Goldman and Sirovich [14] and Kuznetsov *et al.* [27]. Wave fronts which are concave in the direction of propagation exhibit different kinds of behavior depending on the strength of the wavefront. Generally, wave front propagates normal to itself and therefore has a tendency to converge. The shocks of weak strength are called weak shocks. Focusing of weak shock is an important problem. This problem of focusing of weak shocks was studied by Wanner *et al.* [74]. Observes of atomic explosions are also known to have seen shock waves of strong strength, called blast wave. The similarity solution for the strong shock wave in various gasdynamic regimes was present by Helliwell [16] and Singh *et al.* [56]. Ram [47] provided a closed form self similar solution to a MHD flow disturbed by propagating blast waves. Several numerical and analytical methods have been developed to determine the self similar solution of the wave propagation problem e.g. Pandey and Sharma [41, 42], Pandey *et al.* [43], Poslavskii [45], Rogers [50], Sedov [54], Stanyukovich

[62] and Zel'dovich and Raizer [77]. Taylor and Cargill [64] studied the problem of semi-similar expansion waves in magnetogasdynamics flows. Lock and Mestel [29] discussed the possibility of self-similar, imploding, finite annular z-pinch solutions to the equations of ideal magnetogasdynamics. Murata [36] provided analytical solution for spherical blast wave problem, when the density of the gas ahead of the shock front varies as a power of the distance from the origin.

The collapse of a cylindrical shock wave, which is an example of a self-similar solution of the second kind, was studied by many authors. Among them e.g. Lazarus and Richtmyer [28], Lazarus [30], NiCastro [38], Payne [44], Sakurai [52], Sedov [54], Truesdell and Rajagopal [67], Tyagi and Sujith [68, 69], Van Dyke and Guttman [70], Xue [75] and Yousaf [76] have described alternative approaches for determination of the similarity exponent and presented high-accuracy results for the implosion problems. Chisnell [11] provided an analytical description of converging shock waves by replacing the previous approach of numerical solutions of the ordinary differential equations by a theoretical study of the singular points of the differential equations. The effect of thermal radiation upon classical self similar solution of the collapse of cylindrical

converging shock waves was discussed by Hirschler and Gretler [17] and Zic *et al.* [78]. Nath *et al.* [37] have studied numerically the problem of shock wave propagation in a self-gravitating radiative magneto-hydrodynamic medium and obtained the effect of magnetic field and radiation on the flow field. A detailed survey about various topics related to shock waves can be seen in an excellent review article by Razani [49]. Vrsnak and Lulic [72, 73] have studied the formation of coronal MHD shock waves in the field of basic mechanism and pressure pulse mechanism.

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