APPENDIX-6

Theoretical temperature estimation - A sample calculation

Theoretical temperature estimation at the depth \( z \) 359.66 \( \mu \)m and the corresponding longitudinal points

The temperature at various depths \( z \) below the surface and the various corresponding points in the longitudinal directions can be calculated using the equation (3.27) as below

\[
T(r, z, t) = \frac{2Q}{k} \sqrt{\alpha t} \left[ \text{erfc} \left( \frac{z}{2\sqrt{\alpha t}} \right) - \text{erfc} \left( \frac{\sqrt{z^2 + a^2}}{2\sqrt{\alpha t}} \right) \right] e^{-\frac{x^2}{4\sigma^2}} \quad \text{--- (3.27)}
\]

Case depth \( z \) = 359.66 \( \mu \)m = 359.66 X 10\(^{-6}\) m

Thermal diffusivity \( \alpha \) = 1.703 X 10\(^{-5}\) m\(^2\)/sec

Thermal conductivity \( k \) = 52 W/mK

Pulse duration \( t \) = 13.5 msec i.e. Power = 890 Watts

Beam spot size = 1.263 mm

Maximum theoretical case depth = \( \sqrt{\alpha t} \)

\[
= \sqrt{1.703 \times 10^{-5} \times 13.5 \times 10^{-3}} = 4.79 \times 10^{-4} \text{m} = 479 \mu \text{m}
\]

Effective spot size (beam diameter) is considered as 0.86 of the beam spot size.

Therefore, effective spot size = 0.86 X 1.263 =1.086 mm

The effective beam diameter at the maximum theoretical depth is considered as zero.

Therefore, the effective beam diameter at the depth (479 - 359.66) = 0.2713 mm and

The effective beam radius = \( a \) = 0.1356 mm = 0.1356 X 10\(^{-3}\) m
Appendices 157

I- \[ \text{ierfc} \left( \frac{z}{2\sqrt{at}} \right) \]

\[ \text{ierfc} \left( \frac{359.33 \times 10^{-6}}{2 \times 479 \times 10^{-6}} \right) = \text{ierfc}(0.375) \]

\[ = \frac{e^{-0.375^2}}{\sqrt{\pi}} - (0.375)[1 - \text{erf}(0.375)] = 0.9001 - (0.375)(1 - 0.399) \]

\[ = 0.2648 \]

II- \[ \text{ierfc} \left( \frac{\sqrt{z^2 + a^2}}{2\sqrt{at}} \right) \]

\[ \text{ierfc} \left( \frac{\sqrt{(359.33 \times 10^{-6})^2 + (0.1356 \times 10^{-3})^2}}{2 \times 479 \times 10^{-6}} \right) = \text{ierfc}(0.40093) \]

\[ = \frac{e^{-0.40093^2}}{\sqrt{\pi}} - (0.40093)[1 - \text{erf}(0.40093)] = 0.48041 - (0.40093)(1 - 0.42839) \]

\[ = 0.2512 \]

III- \[ \frac{2Q_f}{\sqrt{at}} \]

\[ Q_f = \frac{\text{Power}}{\text{Area}} = \frac{890}{\pi (0.1356 \times 10^{-3})^2} = 154.07 \times 10^6 \text{W/m}^2 \]

\[ \frac{2Q_f}{\sqrt{at}} = \frac{2 \times 154.07 \times 10^6 \times 479 \times 10^{-6}}{52} = 283844.3462 \]

Accounting losses for 70% and substituting for I, II and III we get

\[ T_{(r, z; t)} = 0.3 \times 283844.3462(0.2648 - 0.2512)e^{\left(\frac{r^2}{4at}\right)} = 1158.08453e^{\left(\frac{r^2}{4at}\right)} \]

At \( r=0 \mu m \),

\[ e^{\left(\frac{r^2}{4at}\right)} = 1 \quad \therefore T_{(r, z; t)} = 1158.53K = 885.53^\circ C \]

At \( r=100 \mu m \),

\[ e^{\left(\frac{r^2}{4at}\right)} = 0.989 \quad \therefore T_{(r, z; t)} = 1158.53 \times 0.989 = 1145.82K = 872.82^\circ C \]

Similarly the temperatures at the respective longitudinal distances are calculated by computing \( e^{\left(\frac{r^2}{4at}\right)} \) for that point.