SYMBOLS

Chapter 1

$X$ The universe of discourse

$\mu_A(x)$ Membership grade of $x$ in fuzzy set $A$

$[0, 1]$ The closed interval of real numbers between 0 and 1

$\{a, b,.....\}$ The set of elements $a, b,....$

$(a, b)$ The ordered pair where $b$ represents fuzzy membership of element $a$

$x \in A$ The element $x$ belongs to set $A$

$\forall$ for phrase “for all”

$A = B$ Set equality

$A \subset B$ $A$ is contained in $B$

$A \cap B$ Intersection of fuzzy sets $A$ and $B$

$A \cup B$ Union of fuzz sets $A$ and $B$

$\overline{A}$ The complement of fuzzy set $A$ with respect to $X$

$\min(a_1,a_2,..,a_n)$ Minimum of $a_1,a_2,..,a_n$

$\max(a_1,a_2,..,a_n)$ Maximum of $a_1,a_2,..,a_n$

$T$ Threshold of nonfuzzy neuron

$w$ Weight vector of fuzzy or nonfuzzy neuron

$w_i(t)$ The $i$th element of $w$ at $t$th time instance

$x$ Input pattern vector of fuzzy or nonfuzzy neuron

$x_i(t)$ The $i$th element of $x$ at $t$th time instance

$net$ Scalar product of weight and input vector

$f^1(\cdot)$ Nonfuzzy neuron's activation operator

$\sum_{i=1}^{n} x_i$ Summation of the $n$ terms $x_1$ through $x_n$
\( g_1 \) The parameter that controls the slope of sigmoidal function

\( w^t \) The transpose of vector \( w \)

\( \mu \) Learning rate of nonfuzzy neuron

\( o(t) \) Actual output of nonfuzzy neuron at \( t \)th time instance

\( o_d(t) \) Desired output of nonfuzzy neuron at \( t \)th time instance

\( e(t) \) Error signal of nonfuzzy neuron at \( t \)th time instance

\( \Delta w \) Change in synaptic weight vector of nonfuzzy neuron

\( \mathbb{R}^n \) \( n \)-dimensional Euclidean space

\( \theta_1 \) Variable activation threshold of the FN

\( s \) State of the FN

\( z \) Net input of the FN

\( g_j(\cdot) \) \( j \)th output function of the FN

\( y_j \) \( j \)th output of the FN

\( f^2(\cdot) \) Activation operator of the FN

\( h(\cdot) \) Aggregation function of the FN

\( t \) Threshold function of the FN

\( c_k \) \( k \)th competitive variable of the FN

**Chapter 2**

\( f(x, y) \) Image function of size \( Q \times Q \)

\( m_{pq} \) The \( (p + q) \)th geometrical moment

\( g(x, y) \) Image function normalized with respect to scale and translation

\( \overline{(x, y)} \) Centroid of the image function \( f(x, y) \)

\( \omega \) Predetermined value for the number of object pixels in the image.
\( I^n \)  
\( \phi \)  
\( S_\phi \)  
\( R_w \)  
\( L \)  
\( \text{int} \ [x] \)  
\( \mu \)  
\( \sigma \)  
\( n_1 \times n_2 \)  
\( x_{ij} \)  
\( y^{[1]}_{ij} \)  
\( w_{[m,n]} \)  
\( s^{[2]}_{pq} \)  
\( \beta_i \)  
\( y^{[2]}_{pqm} \)  
\( \alpha_1 \)  
\( g_{pqm} \)  
\( \Theta_{pqm} \)  
\( m_j \)  
\( T_f^1 \)  
\( w_1 \)  
\( w_2 \)  
\( K \)  
\( y_{ijk} \)

- \( I^n \): \text{n-dimensional unit hypercube}
- \( \phi \): Ring number
- \( S_\phi \): The set of pixels in the \( \phi \) th ring
- \( R_w \): Ring width
- \( L \): Largest ring number
- \( \text{int} \ [x] \): The integer part from a real number \( x \)
- \( \mu \): Sample mean
- \( \sigma \): Standard deviation
- \( n_1 \times n_2 \): Nodes in input and YAGER-MAX FN layer of MFNN
- \( x_{ij} \): The \((i,j)\)th value of an input pattern
- \( y^{[1]}_{ij} \): The output of \((i,j)\)th neuron in the input layer of MFNN
- \( w_{[m,n]} \): The weight function of MFNN
- \( s^{[2]}_{pq} \): The state of the \((p, q)\)th YAGER-MAX FN
- \( \beta_i \): Fuzzification function parameter of MFNN
- \( y^{[2]}_{pqm} \): The \(m\)th output of \((p, q)\)th YAGER-MAX FN
- \( \alpha_1 \): Base length of output function of YAGER-MAX FN
- \( g_{pqm} \): The \(m\)th output function of \((p, q)\)th YAGER-MAX FN
- \( \Theta_{pqm} \): The central point of the \(m\)th output function of \((p, q)\)th YAGER-MAX FN
- \( m_j \): The \(j\)th node of YAGER-MIN FN layer
- \( T_f^1 \): Tolerance factor of the MFNN
- \( w_1 \): The strength of union operator of YAGER-MAX FN
- \( w_2 \): The strength of intersection operator of YAGER-MIN FN
- \( K \): Total number of training patterns
- \( y_{ijk} \): The output of \((i,j)\)th input node for \(k\)th pattern
The output of \( y_{jk}^{[3]} \) YAGER-MIN FN for the \( k \)th pattern

\((R_k, d_k)\) The \( k \)th training pair

\(\{V_{nm}(\cdot)\}\) The set of orthogonal complex polynomials

\(\rho\) The length of vector from origin to \((x, y)\) pixel

\(\psi\) An angle between vector \(\rho\) and \(x\)-axis in counterclockwise direction

\(R_{nm}(\rho)\) The radial polynomial

\(A_{nm}\) Zernike moments of order \(n\) with repetition \(m\)

\(V_{nm}^*(\cdot)\) Complex conjugate of \(V_{nm}(\cdot)\)

\(|A_{nm}|\) The magnitude of Zernike moment

\(\vartheta\) An angle of rotation

**Chapter 3**

\(F_R, F_E, F_D\) and \(F_C\) The first, second, third and fourth layer of FHLSNN

\(V\) and \(W\) The matrices in which end points of hyperline segments are stored

\(\theta_2\) The maximum length of hyperline segment

\((R_h, d_h)\) The \(h\)th input training pair

\(e_j\) The \(j\)th hyperline segment

\(V_j, W_j\) Two end points of the \(j\)th hyperline segment

\(e_j(\cdot)\) The fuzzy membership function of \(e_j\)

\(l_1\) Euclidean distance between \(R_h\) and \(W_j\)

\(l_2\) Euclidean distance between \(R_h\) and \(V_j\)

\(l\) Euclidean distance between \(W_j\) and \(V_j\)

\(x\) The sum of \(l_1, l_2\)

\(f^3(\cdot)\) Three-parameter ramp threshold function
\( \gamma_1 \) The sensitivity parameter of fuzzy hyperline segment membership function in the FHLSNN

\( d_k \) The output of \( k \)th node of \( F_D \) which also represents a class \( d_k \)

\( U \) The matrix used to store binary weights between \( F_E \) and \( F_D \) layers

\( p \) The number of neurons in \( F_D \) and \( F_C \)

\( m \) The number of neurons created in \( F_E \) layer

\( c_k \) The nonfuzzy output of \( k \)th \( F_C \) node

\( T_1 \) The maximum of \([d_1, d_2, \ldots, d_k]\)

\( r_1, r_2 \) The constants used in the equations of hyperlines

\( a_i, b_i \) The variables used in equations of hyperlines

\( p_t \) The point of intersection of two hyperlines

\( \gamma \) The sensitivity parameter of fuzzy hyperbox membership function in FMN

\( \lambda \) The maximum size of fuzzy hyperbox in FMN

**Chapter 4**

\( F_R, F_E \) and \( F_C \) The first, second and third layer of FHLSCNN

\( V \) and \( W \) The matrices in which end points of hyperline segments are stored

\( R_h \) The \( h \)th input pattern

\( e_j \) The \( j \)th hyperline segment

\( V_j, W_j \) Two end points of the \( j \)th hyperline segment

\( e_j() \) The fuzzy membership function of \( e_j \)

\( l_1 \) Euclidean distance between \( R_h \) and \( W_j \)

\( l_2 \) Euclidean distance between \( R_h \) and \( V_j \)
\( l \)  Euclidean distance between \( W_j \) and \( V_j \)

\( x \)  The sum of \( l_1, l_2 \)

\( f^3(\cdot) \)  Three-parameter ramp threshold function

\( \gamma_1 \)  The sensitivity parameter of \( e_j(\cdot) \)

\( U \)  The matrix to store binary weights between \( F_E \) and \( F_C \)

\( p \)  The number of clusters created in \( F_C \)

\( m \)  The number of hyperline segments created in \( F_E \)

\( c_k \)  The fuzzy output of \( k \)th \( F_C \) node

\( \gamma \)  The sensitivity parameter of fuzzy hyperbox membership function in FMCN

\( \theta \)  The maximum size of fuzzy hyperbox in FMCN

\( \xi_1 \)  The parameter that puts bound on maximum length of hyperline segment \( 0 \leq \xi_1 \leq 1 \)

\( \alpha_2 \)  The centering parameter in FHLSCNN algorithm

\( \beta_2 \)  The bunching factor in FHLSCNN algorithm

\( R_p \)  The set of patterns used in current pass

\( R_c \)  The set of patterns clustered in current pass

\( R_n \)  The set of patterns that will be used in next pass

**Chapter 5**

\( F_R, F_M, F_N \) and \( F_O \)  The first, second, third and fourth layer of FHSNN

\( m_j \)  The \( j \)th hypersphere

\( m_j(\cdot) \)  The membership function of \( m_j \)

\( C_j \)  The center point of \( m_j \)

\( C \)  The matrix in which center points of hyperspheres are stored
Threshold input of $m_j$

The radius of $m_j$

The maximum limit on radius of $m_j$

The training set

The $h$th training pattern

The class of the $h$th training pattern

Three-parameter ramp threshold function of FHSNN

The sensitivity parameter of $m_j(\cdot)$

The small constant chosen, $\delta = 0.0001$

Euclidean distance between $R_h$ and $C_j$

The maximum size of the hyperbox in GFMN