Chapter 3

Numerical study of viscous unsteady flow in a tube with multiple constrictions *

3.1 Introduction

Partial occlusion of blood vessels, referred to as atherosclerosis, a disease in the human cardiovascular system, is one of the most frequently occurring abnormalities. Such occlusion or constriction of the arterial lumen grows inward and restricts the normal supply of blood in different organs of the body. The transport of blood beyond the narrow region is reduced drastically depending on the severity of the constriction. It is well known that localized atherosclerotic constriction in arteries, known as arterial stenosis, is found predominantly in the internal carotid artery. Fluid dynamical factors play an important role in the post-stenotic blood flow. Blockage of more than 70% area reduction of the artery is considered to be clinically significant. The rheology of blood can best be described by Casson’s relationship, and the blood exhibits nonlinear shear stress versus rate of shear characteristics especially at low rates of shear. But at relatively high rates of shear, the viscosity coefficient asymptotically approaches a constant value. Hence, for flow in large blood vessels, where relatively large shear rates can be expected (during systole), a Newtonian description appears to be reasonable. Blood flow in the larger vessels can be modelled quite accurately as a Newtonian fluid [Pedley (1980), Fung (1981)]. Several theoretical and numerical studies pertaining to blood flow through single constriction have been carried out to calculate the velocity and the shear stresses at the walls. Forrester and Young (1970), Lee and Fung (1970), Ahmed and Giddens (1983), Lee (1994) and Ku (1997) examined the effects of flow field on arterial constriction. The disturbances created by the constriction are in the post-stenotic region and
play a key role in the diagnosis of arterial disease. The pressure loss in the post stenotic region can reduce the blood supply through the artery and also impose additional load on the heart. The pressure losses are significant when the internal diameter is reduced beyond 50% of the normal value [Young (1979)]. Wall pressure and shear stress play an important role in case of fluctuations of the flow variables in the blood flow downstream of the stenosis. This can damage and weaken the internal wall (intima) of the artery. The post-stenotic dilatation i.e., widening of the artery in the downstream of the stenosis is due to the increased distensibility of the arterial wall induced by the variation of pressure [Roach (1963), Lighthill (1975)]. Furthermore, the variability in the distal arterial wall shear stress can result in a predilection towards atherosclerosis. Angiograms of patients having coronary symptoms have shown the presence of several stenoses on the same artery. Considering this fact we have studied the flow pattern in tube with multiple constrictions with same height as well as decreasing heights (10% reduction in height of successive constrictions). Talukder et. al. (1977) conducted experimental study of the effects of multiple stenoses on the pressure drop. On the other hand, the steady flow characteristics in a tube with multiple constrictions has been carried out numerically by Damodaran et. al. (1996).

In this paper, we have studied numerically the flow characteristics through a tube with multiple constrictions. A stable two-stage numerical scheme has been developed for this problem in the axi-symmetric approximations. The staggered grid and the finite-difference discretization are employed in the present scheme as done in Chapter 2 of this thesis. Constrictions of 75% area reduction in the flow area are considered as this reduction or blockage
is clinically significant in human circulatory system. Due attention has been paid on wall shear stress, pressure distribution, velocity profiles for the case of four consecutive constrictions. The results obtained in this study are verified by comparing them with experimental and numerical results available for the case of single constriction. Flow separates downstream of each constriction and a large and stronger recirculating region is created after the last constriction. The length of the recirculating region in the downstream of the last constriction depends on the number of constrictions embedded in a tube for a fixed Reynolds number. The average Reynolds number for the human carotid artery and the monkey aorta is usually < 400, whereas for the human and dog aortas the average Reynolds number is < 2000 [Ahmed and Giddens (1983)]. In a typical aorta, one can estimate the Reynolds number, assuming a diameter of 2.5 cm at the root of the aorta and a mean time-averaged flow velocity of 20 ml/s (based on a cardiac output of 61 pm), and compute a magnitude of about 1500. However, the human aorta is a distensible vessel, which has a complex geometry, and the critical Reynolds number determined from experiments in rigid straight cylindrical pipes is not applicable in this situation. Talukder et al. (1977) varied the Reynolds number between 30 and 280 in their experimental in vitro and in vivo studies. These studies indicate that laminar flow studies are relevant for a number of arteries in the human body. A review of the literature indicates that flow remains laminar for Re < 250. For 70% stenoses, the critical upstream Reynolds number for turbulence is ≈ 300. In our paper we provide a detailed analysis on the dynamics of the flow over a range of Reynolds numbers with four consecutive constrictions of 75% area reduction. Our results suggest that the flow be-
comes oscillatory at the Reynolds number 175. At this Reynolds number, the velocity components as well as other flow features like shear stress, pressure, streamlines etc. are fluctuating. The frequency of the oscillations of streamwise velocity component at two positions, e.g. downstream location after the last constriction and the location between two consecutive constrictions are calculated and is found to be 1.1 for both positions.

3.2 Equations of motion

We consider an axi-symmetric and laminar unsteady flow in a tube with multiple constrictions at the specified position. Let \((r^*, \theta^*, z^*)\) be the cylindrical polar co-ordinates with \(z^*\)—axis along the axis of symmetry of the tube. The region of interest is \(0 \leq r^* \leq r_0(z^*), \ 0 \leq z^* \leq L^*\) (\(L^*\) being the finite length of the tube). The incompressible two-dimensional Navier-Stokes equations can be taken for the modelling of Newtonian blood flow past multiple constrictions. Let \(u^*\) and \(v^*\) be the axial and radial velocity components respectively, \(p^*\) the fluid pressure, \(\rho\) the constant density and \(\nu\) the kinematic viscosity of the fluid. Let \(U\) be the maximum inflow velocity specified in the inlet section or test section of the tube. We introduce the non-dimensional variables \(t = t^*U/D_0, r = r^*/D_0, z = z^*/D_0, r_0(z) = r_0^*(z^*/D_0)/D_0, u = u^*/U, v = v^*/U, p = p^*/\rho U^2\) where \(D_0\) is the diameter of tube in the unoccluded portion. The governing equations for incompressible fluid flow representing conservation of mass and momentum fluxes may be
expressed in dimensionless variables as

\[ \frac{r}{\partial z} \frac{\partial u}{\partial z} + \frac{\partial vr}{\partial r} = 0, \]  

(3.1)

\[ \frac{\partial u}{\partial t} + \frac{\partial uv}{\partial r} + \frac{\partial u^2}{\partial z} + \frac{uw}{r} = -\frac{\partial p}{\partial z} + \frac{1}{Re} \left[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} \right], \]  

(3.2)

\[ \frac{\partial v}{\partial t} + \frac{\partial v^2}{\partial r} + \frac{\partial uv}{\partial z} + \frac{v^2}{r} = -\frac{\partial p}{\partial r} + \frac{1}{Re} \left[ \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2} \right], \]  

(3.3)

where \( Re = \frac{UD_0}{\nu} \) is the Reynolds number.

### 3.2.1 Boundary conditions

The boundary conditions on the symmetry line of the tube are

\[ \frac{\partial u}{\partial r} = 0, \quad v = 0. \]  

(3.4)

The no-slip boundary conditions are imposed at the solid wall as

\[ u = v = 0 \quad \text{at} \quad r = r_0(z). \]  

(3.5)

The governing equations of our model assume that the flow regime is laminar. This model also assumes the flow to be fully developed far upstream of the multiple constrictions. The flow is considered to be fully developed at the inlet test section of the tube where the inlet section is considered at the position \( z = -L_1 \), \( L_1 \) being the upstream length from the first constriction.

The boundary conditions at the inlet section are

\[ u = 2(1 - r^2), \quad v = 0. \]  

(3.6)
3.2.2 Initial condition

The initial condition is that there is no flow inside the region of the tube except the parabolic velocity profile at the inlet. The flow is gradually developing as time elapses. We have also tested the result taking parabolic velocity profile at all sections of the tube and found that there are no variations of numerical values in the flow variables at the time under consideration.

3.2.3 Transformation of basic equations

We consider a co-ordinate stretching in the radial direction, which transforms the tube of multiple constrictions into a straight circular tube, defined by

\[ R = \frac{r}{r_0(z)}. \]  \hspace{1cm} (3.7)

The function \( r_0(z) \) is defined as

\[ r_0(z) = L_0 - \sum_{i=1}^{n} A_i \exp\left(-\frac{(x - d_i)^2}{2(0.25S)^2}\right), \]  \hspace{1cm} (3.8)

where \( r_0(z) \) denotes the radius of the tube in the constricted region. Here \( S \) is the spread between two consecutive constrictions, \( A_i \), the height of the constriction, \( d_i \), the distance of the constrictions from the inlet section of the tube and \( L_0 \) being the unconstricted radius of the tube. A schematic diagram of the multiple constricted tube geometry considered in this analysis is given in Fig.3.1 along with all relevant quantities. The tube under consideration is taken to be of finite length 40 for low Reynolds number flow. But suitable length is taken for the case of high Reynolds numbers so that the reattachment length is independent of this downstream distance. In the
present study, we have taken \( n = 4 \) and \( S = 3 \) so that the four consecutive constrictions with a distance 3 apart are developed. The transformed equations according to the equation (3.7) for continuity and momentum are then

\[
Rr_0(z) \frac{\partial u}{\partial z} - R^2 \frac{\partial r_0(z)}{\partial z} \frac{\partial u}{\partial R} + \frac{\partial v R}{\partial R} = 0, \tag{3.9}
\]

\[
\begin{align*}
\frac{\partial u}{\partial t} + \frac{1}{r_0(z)} \frac{\partial uv^2}{\partial z} - R \frac{\partial u}{\partial R} \frac{\partial r_0(z)}{\partial z} + \frac{u v}{r_0(z)} &= -\frac{\partial p}{\partial z} + \frac{1}{r_0(z)} \frac{\partial R}{\partial R} \frac{\partial r_0(z)}{\partial z} \\
+ \frac{1}{Re} \frac{\partial u}{\partial z} + \frac{1}{r_0(z)} \frac{\partial u}{\partial R} \frac{\partial r_0(z)}{\partial z} + 3R \frac{\partial r_0(z)}{\partial z} &+ \frac{R}{r_0(z)} \left( \frac{\partial r_0(z)}{\partial z} \right)^2 \frac{\partial u}{\partial R} - \frac{R}{r_0(z)} \frac{\partial u}{\partial z} \frac{\partial r_0(z)}{\partial z} \\
- R \frac{\partial r_0(z)}{\partial z} \frac{\partial^2 u}{\partial R \partial z} - \frac{R}{r_0(z)} \frac{\partial r_0(z)}{\partial z} \frac{\partial^2 u}{\partial R \partial z} + R^2 \left( \frac{\partial r_0(z)}{\partial z} \right)^2 \frac{\partial^2 u}{\partial R \partial z} \tag{3.10}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial v}{\partial t} + \frac{1}{r_0(z)} \frac{\partial v^2}{\partial z} - R \frac{\partial uv}{\partial R} \frac{\partial r_0(z)}{\partial z} + \frac{v^2}{r_0(z)} &= -\frac{1}{r_0(z)} \frac{\partial p}{\partial R} + \frac{1}{Re} \frac{1}{r_0(z)} \frac{\partial^2 v}{\partial R^2} \\
+ \frac{1}{r_0(z)} \frac{\partial v}{\partial z} - \frac{v}{r_0(z)} \frac{\partial^2 v}{\partial R^2} + 3R \frac{\partial r_0(z)}{\partial z} &+ \left( \frac{\partial r_0(z)}{\partial z} \right)^2 \frac{\partial v}{\partial R} - \frac{R}{r_0(z)} \frac{\partial r_0(z)}{\partial z} \frac{\partial^2 v}{\partial R \partial z} \\
- \frac{R}{r_0(z)} \frac{\partial r_0(z)}{\partial z} \frac{\partial^2 v}{\partial R \partial z} - \frac{R}{r_0(z)} \frac{\partial^2 r_0(z)}{\partial R^2} &+ R^2 \left( \frac{\partial r_0(z)}{\partial z} \right)^2 \frac{\partial^2 v}{\partial R \partial z} \tag{3.11}
\end{align*}
\]

### 3.3 Finite-Difference formulation

Finite-difference discretization of the above equations (3.9)-(3.11) with appropriate boundary conditions are solved numerically by control-volume based finite difference method. The details of numerical techniques and algorithm are given in Chapter 2. Discretization of the continuity equation at \((i,j)\) cell
delivers
\[ R_j r_0(z_i) \frac{u_{i,j}^n - u_{i-1,j}^n}{\delta z} - R_j^2 \frac{\partial r_0(z_i)}{\partial z} utc - ubc + \frac{R_j v_{i,j}^n - R_{lj-1} v_{i,j-1}^n}{\delta R} = 0 \quad (3.12) \]

where \( utc, ubc \) are defined in Chapter 2. Considering the source, convective and diffusive terms at the \( n \)-th time level, the momentum equation in \( z \)-direction in finite difference form may be put as
\[ \frac{v_{i,j}^{(n+1)} - v_{i,j}^n}{\delta t} = \frac{p_{i,j}^n - p_{i+1,j}^n}{\delta z} + \frac{R_j}{r_0(z_i)} \frac{\partial r_0(z_i)}{\partial z} \frac{p_t - p_b}{\delta R} + U.cd_{i,j} \quad (3.13) \]

where the terms \( p_t, p_b \) and \( U.cd_{i,j} \) all are defined in Chapter 2. The diffusive terms are discretized centrally as given in Chapter 2.

The finite difference equation approximating the momentum equation in the \( R \)-direction is
\[ \frac{v_{i,j}^{(n+1)} - v_{i,j}^n}{\delta t} = \frac{1}{r_0(z)} \frac{p_{i,j}^n - p_{i,j+1}^n}{\delta R} + V.cd_{i,j} \quad (3.14) \]

where \( V.cd_{i,j} = \frac{1}{Re} Diff v_{i,j}^n - Conv v_{i,j}^n \).
Here \( V.cd_{i,j}^n \) is the discretization of convective and diffusive terms of \( v \)-momentum equation at the \( n \)-th time level at cell \((i,j)\). The diffusive and the convective terms in the \( v \)-momentum equation are differenced similar to that in \( u \)-momentum for the convective flux as given in Chapter 2. The final form of the Poisson equation for pressure is
\[ (A + B + C + D)p_{i,j}^n - Ap_{i+1,j}^n - Bp_{i-1,j}^n + A_1 p_{i+1,j+1}^n - A_1 p_{i+1,j-1}^n \\
- A_2 p_{i-1,j+1}^n + A_2 p_{i-1,j-1}^n - (C - A_1 + A_2)p_{i,j+1}^n - (D + A_1 - A_2)p_{i,j-1}^n \\
= - \left[ \frac{\text{Div} v_{i,j}^n}{\delta t} + R_j r_0(z_i) \frac{U.cd_{i,j}^n - U.cd_{i-1,j}^n}{\delta z} + \frac{R_{lj} V.cd_{i,j}^n - R_{lj-1} V.cd_{i,j-1}^n}{\delta R} \right] \quad (3.15) \]
where \( A, B, C, D, A_1, A_2 \) are given in Chapter 2. \( \text{Div}^n_{ij} \) is the finite-difference representation of the divergence of the velocity field at cell \((i,j)\). The equation (3.15) is solved iteratively, by the SOR (successive over relaxation) method by exploiting appropriate boundary conditions. After performing a few iteration steps with the pressure equation, the pressure-velocity corrections are invoked. The method is continued until it achieves a satisfactory level of divergence value (in this case, we fixed the divergence value at 0.00001). The pressure-velocity correction scheme is given in Chapter 2. The updated velocities obtained after solving pressure equation and then solving the updated momentum equations. The time-step \( \delta t \) is calculated by the two criteria viz. CFL condition (Courant, Friedrichs and Lewy condition) and grid Fourier condition. So the time step must satisfy the following criteria

\[
\delta t \leq \min \left[ \frac{\delta z}{|u|}, \frac{\delta R}{|v|} \right]_{ij},
\]

where minimum is taken in the global sense and

\[
\delta t \leq \min \left[ \frac{\text{Re}}{2} \frac{\delta z^2 \delta R^2}{(\delta z^2 + \delta R^2)} \right]_{ij}.
\]

Denoting the right hand side of (3.16) and (3.17) by \( \delta t_1 \) and \( \delta t_2 \), respectively we find that both these inequalities are satisfied if the time step \( \delta t \) satisfies

\[
\delta t \leq \min \left[ \delta t_1, \delta t_2 \right].
\]

Hence, in our computations we take

\[
\delta t = c \min \left[ \delta t_1, \delta t_2 \right],
\]

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where $c$ is a constant lying between 0.2 to 0.4. A typical value of $\delta t$ is 0.005 for $\delta x = 0.05$ and $\delta R = 0.05$ for the Reynolds number smaller than 200.

### 3.4 Results and discussions

The present study first considers steady flow in a tube with single constriction to establish the grid density and the sensitivity of the inlet as well as outlet conditions on the predicted flow quantities like separation and reattachment points, shear stress based on comprehensive computations to available experimental data. An uniform grid distribution of size 0.05 is taken in both directions of the tube. The downstream length of the multiple constrictions was taken sufficiently long to permit the flow to redevelop into a fully developed flow i.e., the condition $\frac{du}{dz} = 0$ is properly satisfied. With increasing Reynolds number and also the area reduction, the length of flow development downstream of the four constrictions increased. The tube geometry chosen in this investigation is simple and may not correspond to the arterial stenosis. But the flow in this configuration exhibits many features that characterize the post-stenotic flow in a realistic artery. Using a Laser Doppler Anemometer, Ahmed and Giddens (1983) carried out their experimental study in a tube with single constriction taking the internal diameter of the tube as 2 and the spread of the constriction as twice the internal diameter of the tube. The measurements were taken in tube with 25% and 75% area reduction. A comparison of axial velocity is made with the present results obtained using two-stage algorithm for the case of single constriction at the axial distance $z=0$ for the Reynolds number 500 (Fig.3.3(a)) and the position $z=2.5$ shown.
in Fig. 3.3(b) where \( z \) is the distance from the centre of the constriction. Our results agree well with both of the numerical results of Damodaran et al. (1996) and the experimental results of Ahmed and Giddens (1983) for 25% area reduction in case of single constriction at \( Re=500 \) but differ slightly for 75% area reduction. The flow field showed an oscillation in the shear layer for 75% area reduction. This seems to be the cause of discrepancy in the results for the severe area reduction. The axial velocity close to the wall is less than zero beyond the stenosis because of the appearance of recirculating flow as seen clearly in the Fig 3(b). With this verification of our numerical code we use this for calculating flow variables in different Reynolds numbers.

<table>
<thead>
<tr>
<th>grid</th>
<th>property</th>
<th>( R \rightarrow 0.0988 )</th>
<th>0.2965</th>
<th>0.4942</th>
<th>0.6919</th>
<th>0.8896</th>
</tr>
</thead>
<tbody>
<tr>
<td>1200x40</td>
<td>( \psi )</td>
<td>0.0099</td>
<td>0.0853</td>
<td>0.2172</td>
<td>0.3680</td>
<td>0.4804</td>
</tr>
<tr>
<td>600x20</td>
<td>( \psi )</td>
<td>0.0099</td>
<td>0.0854</td>
<td>0.2173</td>
<td>0.3681</td>
<td>0.4806</td>
</tr>
<tr>
<td>1200x40</td>
<td>( \omega )</td>
<td>0.4256</td>
<td>1.2044</td>
<td>2.0045</td>
<td>2.8449</td>
<td>3.7330</td>
</tr>
<tr>
<td>600x20</td>
<td>( \omega )</td>
<td>0.4465</td>
<td>1.2169</td>
<td>2.0104</td>
<td>2.8465</td>
<td>3.7328</td>
</tr>
</tbody>
</table>

The Table 3.1 shows results of different grid distributions in a multiply constricted tube at \( Re=10 \) with 25% area reduction. This shows that the results for the grid size 600 x 20 are reasonably good. The actual computations for the constricted case have been carried out on a grid 1000 x 40, while for higher Reynolds number a grid size of 1200 x 40 has been chosen.
Wall pressure distribution is important because the post-stenotic dilatation due to arterial damage caused by the variation of pressure associated with the complex flow structure developed in this region. With increasing occlusion and reduced cross sectional area, the blood flow dynamics past the stenosis becomes complex. Due to Bernoulli effect, the throat of the stenosis is the site for flow with increased velocity and reduced pressure. With increasing degree of stenosis, the reduction in pressure at the throat can decrease significantly and hence vessel may collapse.

The non-dimensional pressure distribution along the wall of the tube in the axial direction for multiple constrictions (four constrictions) with same heights ($A_i = 0.5, i = 1, 2, 3, 4$) for 75% area reduction at $Re=50$ is plotted in Fig.3.4(a). The curve shows a rapid fall in pressure as the constriction is approached and recovers slightly after the last constriction. The fluctuations of wall pressure are key ingredients in acoustical techniques for detecting arterial stenosis [Ask et.al. (1995)]. As mentioned earlier, the separated flow becomes periodic at the Reynolds number 175 and the corresponding pressure also exhibits oscillation at this Reynolds number (shown in Fig.3.4(b)). Fig.3.5(a) exhibits the variation of centreline velocity in axial direction at $Re=50$ for 75% area reduction in a tube with four symmetric constrictions of same height. Due to formation of recirculation zone near the wall, the maximum centreline velocity occurs slightly in the downstream of the constriction. From this figure it is seen that a larger distance is required for the centreline velocity to recover its initial value. The centreline velocity fluctuates with axial distance ($z$) at $Re=175$ (see Fig.3.5(b)). This indicates that the flow field becomes oscillatory.
Experimental studies suggest that wall shear stress is higher on the outer wall of curvature, whereas it is low in the inner wall of curvature. In the aorta, atherosclerotic lesions occur along the inner wall of curvature where there is low shear stress. Cholesterol is actually synthesized in the arterial wall and diffuses into the lumen where it is washed away by the blood stream. In the region of high wall shear stresses (and hence velocity gradients), more cholesterol is deposited on the surface of the lumen initiating atheroma development [Caro et al. (1971)]. With further growth in stenosis, flow separation, pressure fluctuations and turbulent stresses may induce rupture at the distal site resulting in thrombus growth and occlusion of the vessel or embolism in the distal flow field creating cardiac arrest or neurological deficit. With unsteady flow in arterial curvature and bifurcation sites, there are regions in which the flow is reversed during part of the cardiac cycle. Hence the wall shear stress varies from a large magnitude in one direction to negative values during part of the cycle. Very low oscillating shear stress indicates atherosclerotic lesions [Ku (1997)]. The stress on the wall (denoted by $\tau_w$) of the artery plays an important role in arterial disease. In Fig. 3.6(a), the distribution of wall shear stresses ($\tau_w$) for the Reynolds numbers 50 and 100 in the tube are shown for 75% area reduction. The largest magnitude of the wall stress or skin friction is found near the lip of the first constriction which is consistent with the high streamwise velocity at this location. The maximum magnitude of the wall shear stresses for the second, third and fourth constrictions are found near the lip of each constriction but their values are practically equal. The negative values of the wall shear stresses indicate the separating region. In this study, we also calculate the shear stress for the constrictions with decreas-
ing area reduction ($A_1 = 0.3, A_2 = 0.27, A_3 = 0.24, A_4 = 0.21$) for Reynolds number $Re=100$. The effects of decreasing heights (10% reduction in height of each constriction) on the wall shear stress distribution at $Re=100$ for 50% area reduction in the first constriction is very clear from Fig.3.6(b). It is evident from Fig.3.6(b) that the peak value of $\tau_w$ decreases with the decreasing heights. On the other hand, the oscillations of the wall shear are exhibited at the Reynolds number 175 and shown in Fig.3.6(c). The peaks of the shear stresses and also oscillatory pattern are believed to cause severe damage to the arterial lumen which in turn help in detecting the aggregation sites of platelets [Fry (1969)]. This oscillatory pattern of wall shear stress appears at the Reynolds number 175 which is the limit of physiological range for the 75% blockage and may have several consequences in circulatory system.

The time history of streamwise velocity component at $z = 18$ (where $z$ is distance from the peak of the first constriction) for the Reynolds number 175 for 75% blockage has been shown in Fig.3.7. The oscillation with frequency 1.1 is noticed. The same frequency of oscillation in the streamwise velocity component is noted between two consecutive constrictions. No oscillation is noticed in the upstream of the first constriction.

The pattern of streamlines are shown in Fig.3.8(a), 3.8(b) for the Reynolds number 175. The streamlines are oscillatory along the axial direction and a large and stronger recirculating region is formed after the last constriction. These figures (Fig.3.8(a) and Fig.3.8(b)) clearly exhibit the formation of separating bubbles between consecutive constrictions.

Lastly, we discuss the flow structures created by single, two, three and four constrictions separately. Fig.3.9(a) shows the streamline pattern for single
constriction. A large separating bubble is created downstream of the single constriction as depicted in Fig.3.9(a). As anticipated, two recirculating regions (first one is formed between the constrictions) are formed for the two consecutive constrictions (see Fig.3.9(b)). Three and four recirculating zones (Fig.3.9(c) and Fig.3.9(d)) are formed for the three and four consecutive constrictions respectively. But the recirculating region created after the last constriction is larger and stronger indicating the main disturbances created by the multiple constrictions in the downstream of the last constriction.

3.5 Summary and concluding remarks

The axisymmetric unsteady flow of a viscous Newtonian incompressible fluid through a tube with multiple constrictions has been investigated in view of analysis of the blood flow in human cardiovascular system. For this, a stable two-stage numerical algorithm is developed for this axisymmetric flow. The numerical code is semi-implicit and for its verification, comparisons are made with both experimental and numerical investigations. Based on the predicted results the following observations are made.

(i) The disturbances due to multiple constrictions are mainly created in the downstream of the last constriction.

(ii) The centreline flow velocity diminishes downstream from its value at the maximum height of the constriction and further increases upstream towards the next constriction.

(iii) The axial flow velocity decreases in the presence of constriction.

(iv) The wall shear stress is affected by the severity of the constriction in such
a way that it is maximum at the site of the maximum constriction.

(v) Flow unsteadiness depends on the area reduction of the constrictions and on the Reynolds number and it is anticipated that the flow remains two-dimensional for $Re_{\lambda} < 180$.

(vi) The flow becomes periodic but stable at the Reynolds number 175 for the 75% area reduction and the frequency of streamwise velocity component is 1.1. The wall shear stress, pressure and streamlines oscillate. This result seems to be of great physiological interest and likely to have important medical applications.
Fig. 3.1: Geometry of the tube with four symmetric constrictions.
Fig. 3.2: Arrangement of dependent variables in a typical MAC cell
Fig. 3.3(a): Comparison of $u$-velocity for single constriction at $z=0$ for $Re=500$.

Fig. 3.3(b): Velocity comparisons for a single constriction at $z=2.5$ for $Re=500$. 
Fig. 3.4(a): Pressure distribution along the wall in a tube with four symmetric constrictions of same heights for 75% area reduction at Re=50.

Fig 3.4(b): Pressure distribution along the wall in a tube with four symmetric constrictions of same height at Re=175 for 75% area reduction.
Fig. 3.5(a): Centreline $u$-velocity for four symmetric constrictions of same height for 75% area reduction at Re=50.

Fig. 3.5(b): Centreline $u$-velocity for four symmetric constrictions of same height at Re=175 for 75% area reduction.
Fig. 3.6(a): Wall shear stress distribution in a tube with four symmetric constrictions of same height for 75% area reduction.

Fig 3.6(b): Wall shear stresses in a tube with four constrictions of same heights and of decreasing heights at Re=100 for 50% area reduction.
Fig. 3.6(c): Wall shear stress distribution in a tube with four symmetric constrictions of same height at $Re = 175$ for 75% area reduction.

Fig. 3.7: Time history of streamwise velocity component at $Re=175$ in a tube with four constrictions of same height after the last constriction for 75% area reduction.
Fig. 3.8(a): Streamlines at time $t=235.94$ at $Re=175$ for 75% area reduction.

Fig. 3.8(b): Streamlines at time $t=378.08$ at $Re=175$ for 75% area reduction.
Fig 3.9(a): Streamlines at time $t=250.15$ at $Re=150$ for single constriction in a tube for 75% area reduction.

Fig 3.9(b): Streamlines at time $t=222.04$ at $Re=150$ for double constrictions in a tube for 75% area reduction.
Fig. 3.9(c): Streamlines for three constrictions in a tube at Re=150 for 75% area reduction.

Fig. 3.9(d): Streamlines for four symmetric constrictions at Re=150 for 75% area reduction.