I  New Keynesian Model for Indian Economy

Abstract

In this study I develop a New Keynesian Model to outfit the Indian economy and thereby to explain the nature of Indian domestic inflation, a key instrument for inflation targeting central banks. The Indian economy is an emerging market economy and primarily comprises of two sectors, namely, formal and informal and they are asymmetric in nature to each other. The formal sector shows sluggish prices and rigid wages and imperfections in the markets while informal sector characterizes the complete flexibility in prices and wages and perfections in markets. Thus, Indian economy comprises of a very typical mixture of Keynesian and Classical markets. The study shows that when Reserve Bank of India conducts monetary policy in such an environment then nominal and real effects (in short run) are observed in informal and formal sector markets, respectively. The main interest of this study is to study the nature of Indian domestic inflation and thereby to study the real variables of the economy. The New Keynesian Phillips Curve reveals that the degree of stickiness in prices in formal sector markets has a deep impact on the domestic inflation as informal sector markets are frictionless and have complete price flexibility (zero stickiness). Thus, degree of stickiness in prices in formal sector markets plays a major role to determine the domestic inflation and enables the monetary policy to stabilize formal sector output. Compactly, monetary policy affects the real variables of the economy in formal sector in short run while nominal variables (price and wage level) in informal sector. Thus, the study reveals that monetary policy in India has a very poor control on real variables of the economy in short run due to presence of huge informal sector.

JEL Classification: E12, E31, E32, E50, E51, E52, E58, E63, F41

Keywords: New Keynesian Model, Informal Sector, Sticky Prices, Domestic Inflation, New Keynesian Phillips Curve, Dynamic IS Curve, Taylor Rule, India
I believe myself to be writing a book on economic theory which will largely revolutionize – not I suppose, at once but in the course of next ten years\(^1\).

\textit{John Maynard Keynes}

\section{Resurgence of Keynesianism\(^2\)}

The New Classical School of thought comprises of earthshaking talents like Thomas Sargent, Neil Wallace, Edward Prescott and Robert Barro led by Robert Lucas, has exposed the very serious theoretical flaws and inconsistencies in the orthodox Keynesian theory in 1970s. The outclass orthodox Keynesian model, the only game in the town till 1960s in terms of macroeconomic policy, was totally failed to pass the empirical test of 1970s as the vital pillar of orthodox Keynesianism, the Phillips Curve, collapsed. A number of seminal papers in a row of Robert Lucas in 1970s pushed the celebrated Keynesian policy regime in a corner under pitiable conditions. That was a humiliated defeat of Keynesians.

The attack of Sweetwater Economists\(^3\) was not left unanswered. A mighty gang of superb people like Gregory Mankiw and Lawrence Summers (Harvard); Olivier Blanchard (IMF and at MIT), Stanley Fischer (Bank of Israel, formally at World Bank and at MIT); Michael Woodford (Columbia, formally at Princeton), Bruce Greenwald, Edmund Phelps and Joseph Stiglitz (Columbia); Jordi Gali (Pompeu Fabra, formally at New York and at Columbia); Ben Bernanke (Federal Reserve System and formally at Princeton); Laurence Ball (Johns Hopkins); George Akerlof, Janet Yellen and David Romer (Berkeley); Robert Hall and John Taylor (Stanford); Dennis Snower (Kiel) and Assar Lindbeck (Stockholm) who are the best in their profession on the green planet counterattacked on the ideas of New Classical School of thought and reestablished the Keynesian magic. This school of thought is labeled as New Keynesian Economics and a new term New Keynesian\(^4\) was emerged in the literature. Though, the New Keynesian Models are dissimilar in many aspects with their distant cousins of 1960s but still hold

\begin{footnotesize}
\begin{enumerate}
\item The letter of John Maynard Keynes reads written on New Year’s Day, 1935 to his friend George Bernard Shaw for his forthcoming masterpiece “The General Theory’.
\item Sweetwater Economists are referred as New Classical Economists.
\item Parkein and Bade in their book “Modern Macroeconomics” firstly used the term New Keynesian published by Oxford: Philip Allan in 1982.
\end{enumerate}
\end{footnotesize}
the main ideas of Keynesian revolution\(^5\). Summarily, the New Keynesian Economics is school of thought in modern Macroeconomics that evolved from the ideas of the father of modern Macroeconomics John Maynard Keynes in response to the New Classical School of thought.

The Saltwater Economists\(^6\) rigorously and robustly explain the non-neutrality of money and market imperfections. Sticky prices and wages make money non-neutral and a market imperfection explains this behavior of prices and wages. Thus, non-neutrality of money (sticky prices and wages) and market imperfections (absence of continuous market clearing) are the hallmarks of New Keynesian theory.


In this study I develop a New Keynesian Model to outfit the Indian economy and thereby to target the domestic price inflation, a key instrument for inflation targeting central banks. The Indian economy is an emerging market economy and it has all the characteristics of an emerging market economy. Indian economy, primarily, comprises of two sectors, namely, formal and informal and they are asymmetric in nature to each other. The formal sector shows sluggish prices and rigid wages and imperfections in the markets while informal sector of Indian economy characterizes the complete flexibility in prices and wages and perfections in markets. Thus, Indian economy comprises of Keynesian markets in the formal sector and Classical markets in the informal

\(^5\) "The General Theory of Employment, Interest and Money" a world shattering book written by John Maynard Keynes right after the great depression which published in February, 1936 is referred as the Keynesian revolution.

\(^6\) Saltwater Economists are referred as New Keynesian Economists.
To represent New Keynesian and New Classical ideas in a single model is a herculean task but I model them together in a single model to represent the behavior of Indian economy. Handling the typical mixture of sticky and flexible prices is a crucial property of my model.

2 Informality in Indian Economy

The structure of emerging market economies is somewhat differ than that of advance economies due to existence of large informal sector. The structure of goods, labour and credit markets are pretty dissimilar in formal and informal sectors of the economy as agents have different endowments and constraints. In the advance economies the relative size of informal sector is much smaller to that of formal sector; therefore, it is reasonable to ignore the informal sector in advanced economies as it has negligible impact on the aggregates. But in the emerging market economies where the informal sector is relatively large and plays an important role in the economy then neglecting the informal sector would not be justified; Schneider et al. (2010). Informal sector plays a major role in employment generation, especially for the developing world; Agenor and Montiel (1996); Harris-White and Sinha (2007); Marjit and Kar (2011) and Dutta et al. (2011). The informal sector is always complex to deal with as most of the activities of this sector are gone unrecorded.

Unorganised or informal sector constitutes a pivotal part of the Indian economy. More than 90 per cent of workforce and about 50 per cent of the national product are accounted for by the informal economy. A high proportion of socially and economically underprivileged sections of society are concentrated in the informal economic activities. The high level of growth of the Indian economy during the past two decades is accompanied by increasing informalisation. There are indications of growing interlinkages between informal and formal economic activities. There has been new dynamism of the informal economy in terms of output, employment and earnings. Faster and inclusive growth needs special attention to informal economy. Sustaining high levels of growth are also intertwined with improving domestic demand of those engaged in...
in informal economy, and addressing the needs of the sector in terms of credit, skills, technology, marketing and infrastructure, NSC\(^7\) (2012).

A number of studies have been conducted to trace out the effects of informality on the economy. Some of them are as under:

Batini et al. (2010) explore the costs and benefits of informality associated with the informal sector lying outside the tax regime in a two-sector New Keynesian model. The informal sector is more labour intensive, has a lower labour productivity, is untaxed and has a classical labour market. The formal sector bears all the taxation costs, produces all the government services and capital goods, and wages are determined by a real wage norm.

Batini et al. (2011) construct a two-sector, formal-informal new Keynesian closed-economy model. The informal sector is more labour intensive, is untaxed, has a classical labour market, faces high credit constraints in financing investment and is less visible in terms of observed output.

Bridji and Charpe (2011) develop a model of an economy with dual labour markets to understand the dynamics of the informal sector over the business cycle, as well as to analyze the implication of duality for the volatility of output and the persistence of employment. The informal labour market is competitive while the formal labour market is characterized by search costs. The size of each labour market segment depends on labour demand by firms as well as participation decisions of households. Authors show that the informal sector plays the role of a buffer, expanding in periods of recessions and shrinking when recovery sets in. Authors also show that workers switching between the two labour market increases the volatility of output. Finally, labour market segmentation modifies the properties of the search model, as the competitive labour market segment reduces the volatility of employment, unless transition costs are high.

Castillo and Montoro (2009) analyze the effects of informal labor markets on the dynamics of inflation and on the transmission of aggregate demand and supply shocks. In doing so, authors incorporate the informal sector in a modified New Keynesian model with labor market frictions as in the Diamond-Mortensen-
Pissarides model. Authors show that the informal economy generates a “buffer” effects that diminish the pressure of demand shocks on aggregate wages and inflation.

Gabriel et al. (2011) develop a closed-economy DSGE model of the Indian economy and estimate it by Bayesian Maximum Likelihood methods using Dynare. Authors build up in stages to a model with a number of features important for emerging economies in general and the Indian economy in particular: a large proportion of credit-constrained consumers, a financial accelerator facing domestic firms seeking to finance their investment and an informal sector.

Goyal (2007) represents an optimizing model of a small open emerging market economy (SOEME) with dualistic labour markets and two types of consumers, delivers a tractable model for monetary policy.

Goyal (2008) develops a simplified version of a typical dynamic stochastic open economy general equilibrium models used to analyze optimal monetary policy. Author outlines the chief modifications when dualism in labour and in consumption is introduced to adapt the model to a small open emerging market such as India. The implications of specific labour markets, and the structure of Indian inflation and its measurement are examined.

Haider et al. (2012) develop an open economy dynamic stochastic general equilibrium (DSGE) model based on New-Keynesian micro-foundations. Alongside standard features of emerging economies, such as a combination of producer and local currency pricing for exporters, foreign capital inflow in terms of foreign direct investment and oil imports. Authors also incorporate informal labor and production sectors. This customization intensifies the exposure of a developing economy to internal and external shocks in a manner consistent with the stylized facts of Business Cycle Fluctuations.

3 Households

The Indian economy has relatively very large informal sector as the lion’s share of Indian workforce works in this sector to contribute around half of its national product. In such an informal economic environment this study studies
the nature of domestic inflation and thereby studies the real variables of the economy i.e. output and employment. The related issues have been framed in an Open Economy New Keynesian Dynamic Stochastic General Equilibrium Model with micro-foundations.

The world economy is modeled as a continuum of small open economies with identical preferences, technology, and market structure, indexed by a unit interval \([0,1]\), so as it does not have any impact of policy decisions of any economy as in Gali and Monacelli (2005). Again, the home economy is divided into two sectors, namely, formal and informal following Conesa, et al. (2002); Ihrig and Moe (2004); Batini et al. (2010) and Batini et al. (2011). Each sector of home economy is populated by continuum of households and spreads on a unit mass \([0,1]\) with population size \(\gamma : (1 - \gamma) \equiv formal sector : informal sector\), moreover, each of the sectors consumes/produces continuum of differentiated goods as her population size.

The home economy is inhabited by an infinitely lived representative household who derives its utility from additively separable utility function comprises of consumption and leisure (negative utility from working/production) as \(U(C_t, L_t)\) and wishes to maximize the utility following Walsh (2003) and Woodford (2003) as:

\[
Max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)
\]

The CRRA\(^9\) (period) utility is given by as in Gali (2008):

\[
Max E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\varepsilon} - L_t^{1+\nu}}{1 - \varepsilon - \frac{1 + \nu}{1 + \nu}} \right)
\]

Where

\(^{8}\) Government spending (consumption spending and capital spending), money and habit formation are excluded from the model to keep it simple.

\(^{9}\) Constant Relative Risk Aversion Utility.
Macroeconomic Goals and Inflation Targeting in India

\[ U(C_t, L_t) = \left( \frac{C_t^{1-\varepsilon} - L_t^{1+\nu}}{1 - \varepsilon - 1 + \nu} \right) \]

Subject to

\[ C_t P_t + E_t \{ Q_{t,t+1} B_{t+1} \} = B_t + L_t W_t - T_t \quad (1.1.2) \]

Where \( C_t, L_t \) are consumption and labour supply indices, respectively. \( U(\cdot) \) is utility function. \( \varepsilon \) and \( \nu \) are intertemporal substitutability (elasticities of substitution) of consumption and that of labour supply between periods. \( \beta \) is discount factor, \( E_0 \) is expectational operator, \( P_t, W_t \) are general price level and nominal general wage level, \( T_t \) government transfer minus distorted tax, \( B_t \) represents the quantity of one period, nominally riskless discounted bonds purchased in the period \( t \) and maturing in the period \( t + 1 \). \( B_{t+1} \) is nominal payoff in the period \( t + 1 \) of the portfolio held at the end of the period \( t \). Each bond pay one unit of money at maturity and its price is \( Q_t \) and \( Q_t = \frac{1}{1+i_t} \), where, \( i_t \) is nominal interest rate. \( Q_{t,t+1} \) is the stochastic discount factor for one period ahead nominal payoffs relevant to domestic household, moreover, it is assumed as in Gali (2008) that households have access to a complete set of contingent claims, traded internationally.

Economy wide total expenditure\(^{10}\) \( C_t P_t \) of the domestic households on the consumption and economy wide total nominal wage income\(^{11}\) \( L_t W_t \) of the domestic households can be given as:

\[ C_t P_t = \int_0^1 [C_{HF,t}(i)][P_{HF,t}(i)]di \]

\[ + \int_0^1 \int_0^1 [C_{HF,t}(i)][P_{HF,t}(i)]di dj + \int_0^1 \int_0^1 [C_{j,t}(i)][P_{j,t}(i)]di dj \quad (1.1.3) \]

\(^{10}\) (A.1.1) to (A.1.5) in Appendix A make (1.1.3).

\(^{11}\) (A.1.7) to (A.1.9) in Appendix A make (1.1.4).
The domestic composite consumption aggregator $C_t$ can be given following Dixit and Stiglitz (1977) as:

$$C_t = \left[ (1 - \alpha) \frac{1}{\vartheta_a}(C_{H,t}) \frac{\vartheta_a - 1}{\vartheta_a} + (\alpha) \frac{1}{\vartheta_a}(C_{F,t}) \frac{\vartheta_a - 1}{\vartheta_a - 1} \right]$$

(1.1.5)

Where $C_{H,t}$ and $C_{F,t}$ are indices of domestic consumption of domestically and foreign produced goods, respectively and $\vartheta_a$ is intratemporal substitutability (elasticity of substitution) of consumption between domestically and foreign produced goods. $\alpha$ is degree of openness while $1 - \alpha$ is home biasness. The analogous CES aggregator of domestically produced goods $C_{H,t}$ can be given as:

$$C_{H,t} = \left[ (\gamma) \frac{1}{\vartheta_b}(C_{HF,t}) \frac{\vartheta_b - 1}{\vartheta_b} + (1 - \gamma) \frac{1}{\vartheta_b}(C_{HI,t}) \frac{\vartheta_b - 1}{\vartheta_b - 1} \right]$$

(1.1.6)

Where $C_{HF,t}$ and $C_{HI,t}$ are indices of domestic consumption of domestic formal and domestic informal sectors produced goods, respectively and $\vartheta_b$ is intratemporal elasticity of substitution of consumption between formal and informal sector produced goods. $\gamma$ and $1 - \gamma$ are share of domestic consumption of formal and informal sector produced goods, respectively. The CES function of domestic consumption of domestic formal sector produced goods $C_{HF,t}$ can be given as following Woodford (2003):

$$C_{HF,t} = \left[ \frac{1}{\vartheta_c} \left[ C_{HF,t}(i) \frac{\vartheta_c - 1}{\vartheta_c} \right] \right]^{\vartheta_c - 1}$$

(1.1.7)
Where \( C_{HF,t}(i) \) is the quantity of good \( i \) produced in the domestic formal sector and domestically consumed in period \( t \) and \( \vartheta_c \) is intratemporal elasticity of substitution of consumption between varieties of domestic formal sector produced goods. The CES function of domestic consumption of domestic formal sector produced goods \( C_{HF,t} \) can be given as:

\[
C_{HF,t} = \left[ \int_0^1 \left[ C_{HF,t}(i) \right]^{\frac{\vartheta_c}{\vartheta_c-1}} \right]^{\frac{\vartheta_c}{\vartheta_c-1}}
\]

(1.1.8)

Where \( C_{HF,t}(i) \) is the quantity of good \( i \) produced in the domestic informal sector and domestically consumed in period \( t \) and \( \vartheta_c \) is intratemporal elasticity of substitution of consumption between varieties of informal sector produced goods. The CES function of domestic consumption of foreign produced goods \( C_{F,t} \) can be given as:

\[
C_{F,t} = \left[ \int_0^1 \left[ C_{F,t}(i) \right]^{\frac{\vartheta_d}{\vartheta_d-1}} \right]^{\frac{\vartheta_d}{\vartheta_d-1}}
\]

(1.1.9)

Where \( C_{F,t} \) is the index of domestic consumption of country \( j \) produced goods and \( \vartheta_d \) is intratemporal elasticity of substitution of consumption of goods produced in different countries of the world. The CES function of domestic consumption of country \( j \) produced goods \( C_{j,t} \) can be given as:

\[
C_{j,t} = \left[ \int_0^1 \left[ C_{j,t}(i) \right]^{\frac{\vartheta_c}{\vartheta_c-1}} \right]^{\frac{\vartheta_c}{\vartheta_c-1}}
\]

(1.1.10)

Where \( C_{j,t}(i) \) is the quantity of good \( i \) produced in country \( j \) and domestically consumed in period \( t \) and \( \vartheta_c \) is intratemporal elasticity of substitution of consumption between varieties of country \( j \) produced goods. The corresponding
consumption based price indices of (1.1.5) to (1.1.10) are given by (1.1.11) to (1.1.16), respectively as under\(^{12}\) following Benigno and Benigno (2003) and Benigno (2004):

\[
P_t = \left[ (1 - \alpha)(P_{H,t})^{1-\theta_a} + (\alpha)(P_{F,t})^{1-\theta_a} \right]^{\frac{1}{1-\theta_a}} \quad (1.1.11)
\]

\[
P_{H,t} = \left[ \gamma(P_{HF,t})^{1-\theta_b} + (1 - \gamma)(P_{HL,t})^{1-\theta_b} \right]^{\frac{1}{1-\theta_b}} \quad (1.1.12)
\]

\[
P_{HF,t} = \left[ \int_0^1 [P_{HF,t}(i)]^{1-\theta_c} di \right]^{\frac{1}{1-\theta_c}} \quad (1.1.13)
\]

\[
P_{HL,t} = \left[ \int_0^1 [P_{HL,t}(i)]^{1-\theta_c} di \right]^{\frac{1}{1-\theta_c}} \quad (1.1.14)
\]

\[
P_{F,t} = \left[ \int_0^1 [P_{F,t}(j)]^{1-\theta_d} dj \right]^{\frac{1}{1-\theta_d}} \quad (1.1.15)
\]

\[
P_{j,t} = \left[ \int_0^1 [P_{j,t}(i)]^{1-\theta_e} di \right]^{\frac{1}{1-\theta_e}} \quad (1.1.16)
\]

Optimal allocation of goods derives the following demand functions in each category for given level of expenditure as\(^{13}\):

\[
C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_a} C_t \quad (1.1.17)
\]

\(^{12}\) (A.1.12) in the Appendix A derives (1.1.13), (1.1.11) to (1.1.12) and (1.1.14) to (1.1.16) can, analogously, be derived.

\(^{13}\) (A.1.11) in Appendix A derives (1.1.21). (1.1.17) to (1.1.20) and (1.1.22) to (1.1.24) can, analogously, be derived.
The domestic labour supply aggregator \( L_t \), analogous to (1.1.5) can be given following Dixit and Stiglitz (1977) as:

\[
L_t = \left[ (\gamma)^{\frac{1}{\theta_e}} L_{HF,t} \right]^{\frac{\theta_e - 1}{\theta_e}} + (1 - \gamma)^{\frac{1}{\theta_e}} \left( L_{HI,t} \right)^{\frac{\theta_e - 1}{\theta_e}} \]  

(1.1.25)

Where \( L_{HF,t} \) and \( L_{HI,t} \) are indices of domestic labour supply in domestic formal and informal sectors, respectively and \( \theta_e \) is intratemporal elasticity of substitution of labour supply between formal and informal sectors. \( \gamma \) and \( 1 - \gamma \) are share of domestic labour supply in formal and informal sectors, respectively.
The CES function of labour supply in domestic formal sector \( L_{HF,t} \) can be given as under:

\[
L_{HF,t} = \left[ \int \frac{1}{\frac{\partial f}{\partial f^{-1}}} \left[ L_{HF,t}(i) \right] \frac{\partial f^{-1}}{\partial f} \, di \right]^{\frac{\partial f}{\partial f^{-1}}} \tag{1.1.26}
\]

Where \( [L_{HF,t}(i)] \) is the quantity of type \( i \) labour supplied in domestic formal sector in period \( t \) and \( \partial f \) is intratemporal elasticity of substitution between varieties of labour supplied to formal sector. The CES function of labour supply in domestic informal sector \( L_{HI,t} \) can be given as:

\[
L_{HI,t} = \left[ \int \frac{1}{\frac{\partial f}{\partial f^{-1}}} \left[ L_{HI,t}(i) \right] \frac{\partial f^{-1}}{\partial f} \, di \right]^{\frac{\partial f}{\partial f^{-1}}} \tag{1.1.27}
\]

Where \( [L_{HI,t}(i)] \) is the quantity of type \( i \) labour supplied in domestic informal sector in period \( t \) and \( \partial f \) is intratemporal elasticity of substitution between varieties of labour supplied to informal sector. The corresponding labour supply based wage indices of (1.1.25) to (1.1.27) are given by (1.1.28) to (1.1.30), respectively as under:\(^\text{14}\):

\[
W_t = \left[ \gamma \left( W_{HF,t} \right)^{1-\theta_e} + (1-\gamma) \left( W_{HI,t} \right)^{1-\theta_e} \right]^{\frac{1}{1-\theta_e}} \tag{1.1.28}
\]

\[
W_{HF,t} = \left[ \int \frac{1}{\left[ W_{HF,t}(i) \right]^{1-\partial f}} \, di \right]^{\frac{1}{1-\partial f}} \tag{1.1.29}
\]

\(^{14}\) Wage indices can, analogously, be derived as prices indices.
Optimal allocation of labour derives the following supply functions in each category for given level of wage income.

\[
L_{HF,t}(i) = \left( \frac{W_{HF,t}(i)}{W_{HF,t}} \right)^{-\theta_f} L_{HF,t}
\]  
(1.1.31)

\[
L_{HL,t}(i) = \left( \frac{W_{HL,t}(i)}{W_{HL,t}} \right)^{-\theta_f} L_{HL,t}
\]  
(1.1.32)

\[
L_{HF,t} = \gamma \left( \frac{W_{HF,t}}{W_t} \right)^{-\theta_f} L_t
\]  
(1.1.33)

\[
L_{HL,t} = (1 - \gamma) \left( \frac{W_{HL,t}}{W_t} \right)^{-\theta_f} L_t
\]  
(1.1.34)

### 3.1 Optimal Preferences of Households

(1.1.1) and (1.1.2) write the optimal consumption-saving decision\(^{15}\) (optimal inter-temporal consumption decision, the consumption Euler equation) and the optimal consumption-leisure decision\(^{16}\) (optimal consumption-labour supply decision), respectively, as:

\[
1 = E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+1}} \frac{\beta}{Q_t}
\]  
(1.1.35)

\[
\frac{W_t}{P_t} = C_t^\varepsilon L_t^\nu
\]  
(1.1.36)

\(^{15}\) (A.1.20) in the Appendix A makes (1.1.35).

\(^{16}\) (A.1.21) in the Appendix A makes (1.1.36).
The log-linearization of (1.1.35) and (1.1.36) can be given by (1.1.37) and (1.1.38), respectively, as:

\[ c_t = E_t c_{t+1} + \frac{1}{\varepsilon} (\pi_t - i_t + E_t \pi_{t+1}) \]  

\[ \varepsilon c_t + \nu l_t = w_t - p_t \]  

Where small letter is the logarithm (with natural base) value of her corresponding capital letter and hereinafter the very same methodology is used throughout the text.

4 International Economic Environment

4.1 Terms of Trade

Bilateral terms of trade between domestic economy and country \( j \) is defined as price of country \( j \)'s goods in terms of home goods:

\[ S_{f,t} \equiv \frac{P_{f,t}}{P_{H,t}} \]  

(1.1.39)

The effective terms of trade are thus given by:

\[ S_t = \frac{P_{f,t}}{P_{H,t}} \]  

(1.1.40)

\[ S_t = \left( \int_0^1 [S_{f,j}]^{1-\theta} d\theta \right)^{\frac{1}{1-\theta}} \]

---

17 (A.1.24) in the Appendix A makes (1.1.37).

18 (A.1.25) in the Appendix A makes (1.1.38).
Macroeconomic Goals and Inflation Targeting in India

Log linearization makes:

\[ S_t = \frac{P_{F,t}}{P_{H,t}} = \left( \int_0^1 [S_{j,t}]^{1-\theta_a} \, dj \right)^{\frac{1}{1-\theta_a}} \]

4.2 The CPI Inflation

CPI index is given by (1.1.11) as:

\[ P_t = [(1 - \alpha)(P_{H,t})^{1-\theta_a} + (\alpha)(P_{F,t})^{1-\theta_a}]^{\frac{1}{1-\theta_a}} \]

Log linearization makes:

\[ p_t = (1 - \alpha)p_{H,t} + \alpha p_{F,t} \] (1.1.42)

(1.1.41) and (1.1.42) make:

\[ p_t = (1 - \alpha)p_{H,t} + \alpha(p_{H,t} + s_t) \]

\[ p_t = p_{H,t} - \alpha p_{H,t} + \alpha p_{H,t} + \alpha s_t \]

\[ p_t = p_{H,t} + \alpha s_t \] (1.1.43)

CPI inflation is given by:

\[ \pi_t = p_{t+1} - p_t \]

Plugging (1.1.43)

\[ \pi_t = p_{H,t+1} + \alpha s_{t+1} - (p_{H,t} + \alpha s_t) \]
\[ \pi_t = p_{H,t+1} - p_{H,t} + \alpha s_{t+1} - \alpha s_t \]

**4.3 Domestic inflation**

Domestic inflation is given by:

\[ \pi_{H,t} = p_{H,t+1} - p_{H,t} \quad (1.144) \]

Plugging (1.144) and \( \Delta s_t = s_{t+1} - s_t \)

\[ \pi_t = \pi_{H,t} + \alpha \Delta s_t \quad (1.145) \]

Where \( \pi_t \) and \( \pi_{H,t} \), respectively, represent CPI and domestic inflation.

**4.4 Real exchange rate**

Assume that the law of one price holds for individual goods at all times (both for import and export prices), implying that:

\[ p_{j,t}(i) = \epsilon_{j,t}[p_{j,t}^i(i)] \forall i, j \in [0,1] \quad (1.146) \]

Where \( \epsilon_{j,t} \) bilateral nominal exchange rate is defined as prices of country \( j \) currency in terms of domestic currency. \([p_{j,t}^i(i)]\) is the price of country \( j \)’s good \( i \) expressed in terms of own currency i.e. in the currency of country \( j \) itself.

Plugging (1.146) in (1.16)

\[ p_{j,t} = \left[ \int_0^1 (\epsilon_{j,t}[p_{j,t}^i(i)])^{1-\varphi^c} \, di \right]^{1-\varphi^c} \]
Macroeconomic Goals and Inflation Targeting in India

PhD Thesis

\[ P_{j,t} = \left( \epsilon_{j,t} \right)^{1-\theta_c} \left[ \int_0^1 \left( \left[ P_{j,t}^j (i) \right] \right)^{1-\theta_c} \, di \right]^{\frac{1}{1-\theta_c}} \]

\[ P_{j,t} = \epsilon_{j,t} \left[ \int_0^1 \left( \left[ P_{j,t}^j (i) \right] \right)^{1-\theta_c} \, di \right]^{\frac{1}{1-\theta_c}} \]

\[ P_{j,t} = \epsilon_{j,t} P_{j,t}^j \quad (1.1.47) \]

Where \( P_{j,t}^j \) is domestic prices index of country \( j \) and can be given, analogous to (1.1.16), as in her respective domestic prices index:

\[ P_{j,t}^j = \left[ \int_0^1 \left[ P_{j,t}^j (i) \right]^{1-\theta_c} \, di \right]^{\frac{1}{1-\theta_c}} \]

Inserting (1.1.47) in (1.1.15) to make:

\[ P_{F,t} = \left[ \int_0^1 \left[ \epsilon_{j,t} P_{j,t}^j \right]^{1-\theta_d} \, dj \right]^{\frac{1}{1-\theta_d}} \]

Log linearization makes:

\[ P_{F,t} = \int_0^1 \left( \epsilon_{j,t} + P_{j,t}^j \right) \, dj \]

\[ P_{F,t} = \int_0^1 \epsilon_{j,t} \, dj + \int_0^1 P_{j,t}^j \, dj \]

[18]
\[ p_{F,t} = e_t + p^W_t \]  \hfill (1.1.48)

Where \( e_t \) is effective nominal exchange rate and \( p^W_t \) is world price level and can be given as:

\[ e_t = \int_0^1 e_{j,t} \, dj \]  \hfill (1.1.49)

\[ p^W_t = \int_0^1 p^j_{t} \, dj \]  \hfill (1.1.50)

Plugging (1.1.48) in (1.1.41)

\[ s_t = e_t + p^W_t - p_{H,t} \]  \hfill (1.1.51)

For the world as a whole there is neither distinction between domestic and CPI price level nor between their corresponding inflation rates. \( p^j_{j,t} \) and \( p^j_t \) are the domestic and CPI price indices of the generic country \( j \).

\[ p^W_t = \int_0^1 p^j_t \, dj \]  \hfill (1.1.52)

The bilateral real exchange rate between home and country \( j \) is defined as the ratio of two countries CPI and both are express in terms of domestic currency.

\[ Q_{j,t} = \frac{\epsilon_{j,t} p^j_t}{P_t} \]  \hfill (1.1.53)

Log linearization:

\[ \log Q_{j,t} = \log \frac{\epsilon_{j,t} p^j_t}{P_t} \]
\[ \log Q_{j,t} = \log \epsilon_{j,t} + \log P_t^j - \log P_t \]

\[ q_{j,t} = e_{j,t} + p_t^j - p_t \quad (1.154) \]

\[ q_t = \int_0^1 q_{j,t} \, dj \quad (1.155) \]

Plugging (1.154) in (1.155)

\[ q_t = \int_0^1 (e_{j,t} + p_t^j - p_t) \, dj \]

\[ q_t = \int_0^1 e_{j,t} \, dj + \int_0^1 p_t^j \, dj - \int_0^1 p_t \, dj \]

\[ p_t = \int_0^1 p_t \, dj \quad (1.156) \]

Plugging (1.149), (1.152) and (1.156) make:

\[ q_t = e_t + p_t^W - p_t \]

Plugging (1.151)

\[ q_t = s_t + p_{H,t} - p_t \]

Plugging (1.143)

\[ q_t = s_t + p_{H,t} - p_{H,t} - \alpha s_t \]

\[ q_t = s_t(1 - \alpha) \quad (1.157) \]

Thus, (1.157), relates terms of trade to real exchange rate.
4.5 International risk sharing

Under the assumption of complete international financial markets and perfect capital mobility, the expected nominal return from risk free bonds, in terms of domestic currency, must be the same as the expected domestic currency return from foreign bonds. With this relationship, we can equate the intertemporal optimality conditions for the domestic and foreign households’ optimization problem. In the Appendix A, (A.1.19) and molding (A.1.19) for generic country $j$ both of them produce\(^{19}\):

\[
C_t = \varphi_j (c_{t}^j) (Q_{j,t})^{\frac{1}{\bar{r}}}
\]

(1.158)

Log linearization:

\[
c_t = c_t^j + \frac{1}{\epsilon} q_{j,t}
\]

Integrating over $j$ yields:

\[
\int_{0}^{1} c_t d j = \int_{0}^{1} c_t^j d j + \frac{1}{\epsilon} \int_{0}^{1} q_{j,t} d j
\]

\[
c_t = \int_{0}^{1} c_t d j
\]

(1.159)

\[
c_t^W = \int_{0}^{1} c_t^j d j
\]

(1.160)

Plugging (1.155), (1.159) and (1.160) to make:

\[
c_t = c_t^W + \frac{1}{\epsilon} q_t
\]

Plugging (1.157)

\(^{19}\) (A.1.26) in the Appendix A makes (1.158).
\[ c_t = c_t^W + \left( \frac{1 - \alpha}{\varepsilon} \right) s_t \]  \hspace{1cm} (1.1.61)

5 Firms

Indian economy comprises of two sectors of production\(^{20}\) i.e. domestic formal sector and domestic informal sector. Again domestic formal sector of production is made of three types of firms: final goods producing firms, intermediate goods producing firms and importers. The final goods producing firms buy the domestic intermediate varieties produced by domestic intermediate goods producing firms and assemble them as domestically produced final goods. These firms sell a portion of their goods in the domestic formal sector goods market and export the rest. Importers\(^{21}\) on the other hand purchase foreign produced goods at world market prices and sell them in the domestic formal sector goods market which are finally consumed by the domestic consumers. The domestic informal sector production is comprises of two types of firms: final goods producing firms and intermediate goods producing firms. In this sector there are no exporting or importing firms. The final goods producing informal sector firms work in a very similar pattern as that of the final goods producing formal sector firms. But the point of deviation is that they buy only the domestic informal sector intermediate varieties produced by domestic informal sector intermediate goods producing firms and assemble them as domestically produced final goods. The whole of final goods produced in informal sector are consumed by the domestic consumers.

The CES aggregator of domestically produced goods \(Y_t\), analogous to (1.1.6), can be given as following Dixit and Stiglitz (1977):

\[
Y_t = \left[ \gamma \frac{1}{\partial_b \left( Y_{HF,t} \right)} \frac{\partial b - 1}{\partial b} + \left( 1 - \gamma \right) \frac{1}{\partial_b \left( Y_{HI,t} \right)} \frac{\partial b - 1}{\partial b} \right] \frac{\phi_b}{\phi_{b-1}} \]  \hspace{1cm} (1.2.1)

\(^{20}\) Firms (production) strictly follow(s) the assumptions of households (consumption).
\(^{21}\) Importers are kept out of study as model is defined to target domestic inflation only.
Where $Y_{HF,t}$ and $Y_{HI,t}$ are indices of production of domestic formal and informal sector produced goods, respectively and $\theta_b$ is intratemporal elasticity of substitution of production between formal and informal sector produced goods. $\gamma$ and $1 - \gamma$ are share of domestic production of formal and informal sector produced goods, respectively. The CES function of production of domestic formal sector produced goods $Y_{HF,t}$, analogous to (1.1.7) can be given as:

$$Y_{HF,t} = \left[ \frac{1}{0} \left[ Y_{HF,t}(i) \right] \frac{\theta_{c-1}}{\theta_{c}} \right]^{\frac{\theta_{c}}{\theta_{c-1}}}$$

(1.2.2)

Where $[Y_{HF,t}(i)]$ is the quantity of good $i$ produced in the domestic formal sector in period $t$ and $\theta_c$ is intratemporal elasticity of substitution of production between varieties of formal sector produced goods. The CES function of production of domestic informal sector produced goods $Y_{HI,t}$, analogous to (1.1.8) can be given as:

$$Y_{HI,t} = \left[ \frac{1}{0} \left[ Y_{HI,t}(i) \right] \frac{\theta_{c-1}}{\theta_{c}} \right]^{\frac{\theta_{c}}{\theta_{c-1}}}$$

(1.2.3)

Where $[Y_{HI,t}(i)]$ is the quantity of good $i$ produced in the domestic informal sector in period $t$ and $\theta_c$ is intratemporal elasticity of substitution of production between varieties of informal sector produced goods. The corresponding consumption based price indices of (1.2.1) to (1.2.3) are given by (1.2.4) to (1.2.6), analogous to (1.1.12) to (1.1.14) as:

$$P_{H,t} = \left[ \gamma (P_{HF,t})^{1-\theta_b} + (1 - \gamma) (P_{HI,t})^{1-\theta_b} \right]^{\frac{1}{1-\theta_b}}$$

(1.2.4)
Intermediate goods producing firms of both formal and informal sector work in very similar fashion, they both use only labour as the key input to produce the intermediate goods. The capital stock is treated as fixed and investment is set to zero in short run following McCallum and Nelson (1999), who argue that, for most monetary policy and business-cycle analyses, fluctuations in the stock of capital do not play a major role. Walsh (2010) is, also, of the same view that capital stock be ignored in the short run as variation in capital stock does not have any significant effect on output.

(1.2.7) to (1.2.10) are given, analogously, to (1.1.19) to (1.1.22).
The intermediate goods producing firms use the following Cobb-Douglas technology to produce \( i \) type of good.

\[
[Y_{HI,t}(i)] = A_{HI,t}[L_{HI,t}(i)]^{1-\phi} (K_{HI,t})^\phi
\]  

(1.2.11)

Where \( [Y_{HI,t}(i)] \) is the quantity of type \( i \) good produced in the informal sector. \( A_{HI,t} \) is the state of technology used, evenly, throughout in the informal sector and \( K_{HI,t} \) stands for the capital stock\(^{23}\) used. In the short run capital stock remains unchanged, i.e. \( \phi = 0 \), therefore:

\[
[Y_{HI,t}(i)] = A_{HI,t}[L_{HI,t}(i)]^{1-0} (K_{HI,t})^0
\]

(1.2.12)

\[
[Y_{HI,t}(i)] = A_{HI,t}[L_{HI,t}(i)] \forall i \in [0,1]
\]

(1.2.13)

(1.2.13) tells a story that intermediate goods producing firms use a linear technology to produce the intermediate goods in the informal sector. Formal sector counterparts of informal sector (1.2.14) and (1.2.15) can be given, analogous to (1.2.12) and (1.2.13), as:

\[
[Y_{HF,t}(i)] = A_{HF,t}[L_{HF,t}(i)] \forall i \in [0,1]
\]

(1.2.14)

\[
[Y_{HF,t}] = A_{HF,t}[L_{HF,t}]
\]

(1.2.15)

Where \( [Y_{HF,t}(i)] \) is the quantity of type \( i \) good produced in the formal sector. \( A_{HF,t} \) is the state of technology used, evenly, throughout in the informal sector.

Domestic formal sector firms set prices staggering à la Calvo (1983). Each domestic formal sector firm may reset its price only with probability \( (1 - \theta_{HF} \) in any given period, independent of the time elapsed since the last adjustment.

\(^{23}\) Capital stock are assumed to be constant in the short run, it follows zero investment.
Thus, each period a measure \(1 - \theta_{HF}\) of domestic formal sector firms reset their prices, while a fraction \(\theta_{HF}\) keep their prices unchanged. As a result, the average duration of a price is given by \(\frac{1}{1-\theta_{HF}}\). In this context, \(\theta_{HF}\) becomes a natural index of price stickiness in the domestic formal sector, Gali (2008). The domestic formal sector price dynamics (inflation) can be given as \(^{24}\):

\[
\pi_{HF,t} = (1 - \theta_{HF})(p_{HF,t}^*-p_{HF,t-1})
\]  

(1.2.16)

Where \(p_{HF,t}^*\) is the optimal price set by domestic formal sector firms who are able to re-optimize in the period \(t\). \(^{24}\) makes it clear that domestic formal sector inflation results from the fact that firms in this sector re-optimizing in any given period choose a price that differs from the sector’s average price in the previous period.

5.1 Optimal Price Setting

A domestic formal sector representative firm who re-optimizing in period \(t\) will choose the price \(p_{HF,t}^*\) that maximizes the current market value of the profits generated while that price remains effective and the formal sector representative firm’s profit maximization problem can be given as:

\[
\max_{\{p_{HF,t}^\prime\}} \sum_{k=0}^{\infty} (\theta_{HF})^{k} E_{t}[Q_{t,t+k}\left(p_{HF,t}^*\right)\left(Y_{HF,t+k|t}(i)\right) - \left(TC_{HF,t+k+1|t}(i)\right)\left(Y_{HF,t+k|t}(i)\right)]
\]  

(1.2.17)

Subject to

\[
\left(Y_{HF,t+k|t}(i)\right) = \left(\frac{p_{HF,t}^*}{p_{HF,t+k}}\right)^{-\theta_{c}} C_{HF,t+k}
\]

\(^{24}\) (A.2.3) in the Appendix A makes (1.2.16).
\[ P_{HF,t}^* = \left( \frac{\vartheta_c}{\vartheta_c - 1} \right) E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \left[ \beta^k \left( P_{HF,t+k} \right)^{\vartheta_c} \left( C_{HF,t+k} \right)^{1-\vartheta_c} \left( MC_{HF,t+k|t}^B \right)^{1-\varepsilon} \left( MC_{HF,t+k|t}^N \right)^{\varepsilon} \right] \tag{1.2.18} \]

Where, \( E_t \left( TC_{HF,t+k|t}^N(i) \right) \) is expected future nominal total cost, for time \( t + k \), to produce domestic formal sector good \((i)\) and \( (MC_{HF,t+k|t}^B) \) is corresponding expected future nominal marginal cost. \( P_{HF,t}^* \) is optimal price\(^{25}\) of the formal sector firms and it is given as a weighted average of future real marginal cost of the formal sector.

\[ P_{HF,t}^* = \left( \frac{\vartheta_c}{\vartheta_c - 1} \right) MC_{HF,t|t}^N = \mathcal{M}_{HF} \cdot MC_{HF,t|t}^N \tag{1.2.19} \]

Where, \( \mathcal{M}_{HF} = \left( \frac{\vartheta_c}{\vartheta_c - 1} \right) \) and \((1.2.19)\) gives that \( \mathcal{M}_{HF} \) is a desired or frictionless markup\(^{26}\) for formal sector firms.

Linearization of \((1.2.18)\) around steady state makes:

\[ p_{HF,t}^* = (1 - \theta_{HF} \beta) E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \left[ (mc_{HF,t+k|t}^R - mc_{HF,t+k|t}^R) + p_{HF,t+k} \right] \tag{1.2.20} \]

\[ \mu_{HF} \equiv -mc_{HF}^R \tag{1.2.21} \]

Plugging \((1.2.21)\) in \((1.2.20)\)

\[ p_{HF,t} = \mu_{HF} + (1 - \theta_{HF} \beta) E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \left( mc_{HF,t+k|t}^R + p_{HF,t+k} \right) \tag{1.2.22} \]

---

\(^{25}\) (A.2.9) in the Appendix A makes (1.2.18).

\(^{26}\) (A.2.11) in the Appendix A makes (1.2.19).
(1.2.22) shows that domestic formal sector firms resetting their prices will choose a price that corresponds to the desired markup over a weighted average of their current and expected future real marginal costs, with the weights being proportional to the probability of the price remaining effective at each horizon $(\theta_{HF})^k$.

5.2 New Keynesian Phillips Curve for Domestic Formal Sector

Formal sector nominal wage is defined as:

$$MC_{HF,t}(i)MPL_{HF,t}(i) = W_{HF,t}$$  \hspace{1cm} (1.2.23)

Where $MPL_{HF,t}$ is marginal production of labour and $MC_{HF,t}$ nominal marginal cost.

$$\frac{MC_{HF,t}(i)}{P_{HF,t}} MPL_{HF,t}(i) = \frac{W_{HF,t}}{P_{HF,t}}$$

$$MC^R_{HF,t}(i) = \frac{MC_{HF,t}(i)}{P_{HF,t}}$$  \hspace{1cm} (1.2.24)

Plugging (1.2.24) in (1.2.23) makes:

$$MC^R_{HF,t}(i)MPL_{HF,t}(i) = \frac{W_{HF,t}}{P_{HF,t}}$$

Log linearization makes:

$$mc^R_{HF,t}(i) = w_{HF,t} - p_{HF,t} - mpl_{HF,t}(i)$$  \hspace{1cm} (1.2.25)

Rewriting (1.2.14)

$$[Y_{HF,t}(i)] = A_{HF,t}[L_{HF,t}(i)]$$
\[
\frac{d}{dt}[Y_{HF,t}(t)] = \frac{d}{dt}(A_{HF,t}[L_{HF,t}(i)])
\]

\[
MPL_{HF,t}(i) = A_{HF,t}
\]

Log linearization makes:

\[
mpl_{HF,t}(i) = a_{HF,t}
\] (1.2.26)

Substituting (1.2.26) in (1.2.25)

\[
mc^R_{HF,t}(i) = w_{HF,t} - p_{HF,t} - a_{HF,t}
\] (1.2.27)

(1.2.27) shows that \(mc^R_{HF,t}\) is independent of production level and uniform across all firms of the domestic formal sector, thus (1.2.27) becomes:

\[
mc^R_{HF,t} = w_{HF,t} - p_{HF,t} - a_{HF,t}
\] (1.2.28)

A similar expression to (1.2.27) for period \(t + k\) is given as:

\[
mc^R_{HF,t+k|t}(i) = w_{HF,t+k} - p_{HF,t+k} - a_{HF,t+k}
\] (1.2.29)

A similar expression to (1.2.28) for period \(t + k\) is given as:

\[
mc^R_{HF,t+k} = w_{HF,t+k} - p_{HF,t} - a_{HF,t+k}
\] (1.2.30)

(1.2.29) and (1.2.30) make:

\[
mc^R_{HF,t+k|t}(i) = mc^R_{HF,t+k}
\] (1.2.31)

Rewriting (1.2.20)
\[ p_{HF,t}^* = (1 - \theta_{HF}\beta)E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k [(mc_{HF,t+k|t}^R - mc_{HF}^R) + p_{HF,t+k}] \]

\[ p_{HF,t}^* \equiv p_{HF,t+k|t} \quad (1.2.32) \]

Using (1.2.32)

\[ p_{HF,t}^* - p_{HF,t-1} = (1 - \theta_{HF}\beta)E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k [(mc_{HF,t+k|t}^R - mc_{HF}^R) + p_{HF,t+k} - p_{HF,t-1}] \]

Inserting (1.2.31)

\[ p_{HF,t}^* - p_{HF,t-1} = (1 - \theta_{HF}\beta)E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k [(mc_{HF,t+k|t}^R(i) - mc_{HF}^R) + p_{HF,t+k} - p_{HF,t-1}] \]

\[ \tilde{mc}_{HF,t+k}^R = mc_{HF,t+k|t}^R(i) - mc_{HF}^R \quad (1.2.33) \]

Plugging (1.2.33)

\[ p_{HF,t}^* - p_{HF,t-1} = (1 - \theta_{HF}\beta)E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k [\tilde{mc}_{HF,t+k}^R + p_{HF,t+k} - p_{HF,t-1}] \]
\[ p_{HF,t} - p_{HF,t-1} \]
\[ = (1 - \theta_{HF}\beta) E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \bar{m}_{HF,t+k}^R \]
\[ + (1 - \theta_{HF}\beta) E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \left[p_{HF,t+k} - p_{HF,t-1}\right] \]
\[ = E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \pi_{HF,t+k} \]

(1.2.34)

Plugging (1.2.34)

\[ p_{HF,t} - p_{HF,t-1} = (1 - \theta_{HF}\beta) E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \bar{m}_{HF,t+k}^R + E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \pi_{HF,t+k} \]

Rewriting the expression by taking \( k = 0 \) out from the summation operator as:

\[ p_{HF,t} - p_{HF,t-1} \]
\[ = (1 - \theta_{HF}\beta) E_t \sum_{k=1}^{\infty} (\theta_{HF})^k \beta^k \bar{m}_{HF,t+k}^R \]
\[ + (1 - \theta_{HF}\beta)(\theta_{HF})^0 \beta^0 \bar{m}_{HF,t+0}^R + E_t \sum_{k=1}^{\infty} (\theta_{HF})^k \beta^k \pi_{HF,t+k} \]
\[ + (\theta_{HF})^0 \beta^0 \pi_{HF,t+0} \]

\[ p_{HF,t}^* - p_{HF,t-1} \]
\[ = (1 - \theta_{HF}\beta) E_t \sum_{k=1}^{\infty} (\theta_{HF})^k \beta^k \bar{m}_{HF,t+k}^R + E_t \sum_{k=1}^{\infty} (\theta_{HF})^k \beta^k \pi_{HF,t+k} \]
\[ + (1 - \theta_{HF}\beta)\bar{m}_{HF,t}^R + \pi_{HF,t} \]
\[ p^*_{HF,t} - p^*_{HF,t-1} \]
\[ = (\theta_{HF})\beta \left[ (1 - \theta_{HF}\beta)E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \bar{m}c_{HF,t+k+1} \right. \]
\[ + E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \pi_{HF,t+k+1} \left. \right] + (1 - \theta_{HF}\beta)\bar{m}c_{HF,t} + \pi_{HF,t} \]
\[ p^*_{HF,t+1} - p^*_{HF,t-1+1} \]
\[ = \left[ (1 - \theta_{HF}\beta)E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \bar{m}c_{HF,t+k+1} \right. \]
\[ + E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \pi_{HF,t+k+1} \left. \right] \quad (1.2.35) \]

Plugging (1.2.35)

\[ p^*_{HF,t} - p^*_{HF,t-1} = (\theta_{HF})\beta( p^*_{HF,t+1} - p^*_{HF,t}) + (1 - \theta_{HF}\beta)\bar{m}c_{HF,t} + \pi_{HF,t} \]

(1.2.16) writes as:

\[ E_t\pi_{HF,t+1} = (1 - \theta_{HF})( p^*_{HF,t+1} - p^*_{HF,t}) \]

\[ ( p^*_{HF,t+1} - p^*_{HF,t}) = \frac{E_t\pi_{HF,t+1}}{(1 - \theta_{HF})} \quad (1.2.36) \]

Inserting (1.2.36)

\[ p^*_{HF,t} - p^*_{HF,t-1} = (\theta_{HF})\beta \frac{E_t\pi_{HF,t+1}}{(1 - \theta_{HF})} + (1 - \theta_{HF}\beta)\bar{m}c_{HF,t} + \pi_{HF,t} \]

Substituting (1.2.16)
\[
\frac{\pi_{HF,t}}{(1 - \theta_{HF})} = (\theta_{HF})\beta \frac{E_t \pi_{HF,t+1}}{(1 - \theta_{HF})} + (1 - \theta_{HF})\beta \hat{m}c^R_{HF,t} + \pi_{HF,t}
\]

\[
\frac{\pi_{HF,t}}{(1 - \theta_{HF})} - \pi_{HF,t} = (\theta_{HF})\beta \frac{E_t \pi_{HF,t+1}}{(1 - \theta_{HF})} + (1 - \theta_{HF})\beta \hat{m}c^R_{HF,t}
\]

\[
\theta_{HF} \frac{\pi_{HF,t}}{1 - \theta_{HF}} = (\theta_{HF})\beta \frac{E_t \pi_{HF,t+1}}{(1 - \theta_{HF})} + (1 - \theta_{HF})\beta \hat{m}c^R_{HF,t}
\]

\[
\pi_{HF,t} = \beta E_t \pi_{HF,t+1} + \frac{(1 - \theta_{HF})(1 - \theta_{HF})\beta}{\theta_{HF}} \hat{m}c^R_{HF,t}
\]

\[
\lambda_{HF} = \frac{(1 - \theta_{HF})(1 - \theta_{HF}\beta)}{\theta_{HF}} \tag{1.2.37}
\]

Plugging (1.2.37)

\[
\pi_{HF,t} = \beta E_t \pi_{HF,t+1} + \lambda_{HF} \hat{m}c^R_{HF,t} \tag{1.2.38}
\]

(1.2.38) is a New Keynesian Phillips Curve for domestic formal sector. New Keynesian Phillips Curve constitutes one of the key building blocks of New Keynesian Model. Informal sector counterparts to formal sector (1.2.38) and (1.2.37) are, analogously, given by (1.2.39) and (1.2.40), respectively.

\[
\pi_{HI,t} = \beta E_t \pi_{HI,t+1} + \lambda_{HI} \hat{m}c^R_{HI,t} \tag{1.2.39}
\]

\[
\lambda_{HI} = \frac{(1 - \theta_{HI})(1 - \theta_{HI}\beta)}{\theta_{HI}} \tag{1.2.40}
\]

6 Equilibrium Dynamics

6.1 Equilibrium in Domestic Goods Market

Market clearing for the domestic goods market requires:
\[ Y_t = Y_{HF,t} + Y_{HL,t} = C_{HF,t} + C_{HL,t} + \int_0^1 C_{H,t}^j \, dj \] (13.1)

Where \( C_{H,t}^j \) is the consumption of domestically produced goods in the country \( j \) and \( \int_0^1 C_{H,t}^j \, dj \) is consumption across the globe for domestically produced goods. Plugging (A.3.3), (A.3.6) and (A.3.17) from Appendix A in (1.3.1) make as:

\[
Y_t = (1 - \alpha) C_t \left( \frac{P_{HF,t}}{P_t} \right)^{-\theta_a} \left( \frac{P_{HL,t}}{P_{H,t}} \right)^{-\theta_b} C_t \\
+ \left( 1 - \alpha(1 - \gamma) \left( \frac{P_{HL,t}}{P_{H,t}} \right)^{-\theta_b} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_a} C_t \right) \\
+ \int_0^1 \alpha \left( \frac{P_{H,t}}{\epsilon_{j,t} P_{F,t}^j} \right)^{-\theta_d} \left( \frac{P_{F,t}^j}{P_t^j} \right)^{-\theta_a} C_t^j \, dj
\]

\[
Y_t = (1 - \alpha) C_t \left( \frac{P_{HF,t}}{P_t} \right)^{-\theta_a} \left( \gamma \left( \frac{P_{HF,t}}{P_{H,t}} \right)^{-\theta_b} + (1 - \gamma) \left( \frac{P_{HL,t}}{P_{H,t}} \right)^{-\theta_b} \right) \\
+ \alpha \int_0^1 C_t^j \left( P_{H,t} \right)^{-\theta_d} \left( \epsilon_{j,t} P_{F,t}^j \right)^{-\theta_d} \left( P_{F,t}^j \right)^{-\theta_a} \left( P_{F,t}^j \right)^{\theta_a} \, dj
\]

\[
Y_t = (1 - \alpha) C_t \left( \frac{P_{HF,t}}{P_t} \right)^{-\theta_a} \left( \gamma \left( \frac{P_{HF,t}}{P_{H,t}} \right)^{-\theta_b} + (1 - \gamma) \left( \frac{P_{HL,t}}{P_{H,t}} \right)^{-\theta_b} \right) \\
+ \alpha \int_0^1 C_t^j \left( P_{F,t}^j \right)^{-\theta_d} \left( \epsilon_{j,t} P_{F,t}^j \right)^{-\theta_d} \left( P_{F,t}^j \right)^{-\theta_a} \left( P_{F,t}^j \right)^{\theta_a} \, dj
\]

\[
Y_t = (1 - \alpha) C_t \left( \frac{P_{HF,t}}{P_t} \right)^{-\theta_a} \left( \gamma \left( \frac{P_{HF,t}}{P_{H,t}} \right)^{-\theta_b} + (1 - \gamma) \left( \frac{P_{HL,t}}{P_{H,t}} \right)^{-\theta_b} \right) \\
+ \alpha \int_0^1 C_t^j \left( P_{F,t}^j \right)^{-\theta_d} \left( \epsilon_{j,t} P_{F,t}^j \right)^{-\theta_d} \left( P_{F,t}^j \right)^{-\theta_a} \left( P_{F,t}^j \right)^{\theta_a} \left( \frac{P_{F,t}^j}{P_t^j} \right)^{\theta_a} \, dj
\]
\[ Y_t = (1 - \alpha) C_t \left( \frac{P_{H,t}^I}{P_t} \right)^{-\theta_a} \left( \gamma \left( \frac{P_{HF,t}^I}{P_{H,t}} \right)^{-\theta_b} + (1 - \gamma) \left( \frac{P_{HI,t}^I}{P_{H,t}} \right)^{-\theta_b} \right) \]

\[
+ \alpha \int_0^1 C_t^j \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_d} \left( \frac{P_{j,t}^I}{P_{j,t}} \right)^{\theta_d-\theta_a} \left( \frac{\epsilon_{j,t} P_{j,t}^I}{P_t} \right)^{\theta_d-\theta_a} \left( \frac{1}{P_t} \right)^{-\theta_a} \left[ Q_{j,t} \right]^{\theta_a} \, dj
\]

Inserting (1.1.53)

\[ Y_t = (1 - \alpha) C_t \left( \frac{P_{H,t}^I}{P_t} \right)^{-\theta_a} \left( \gamma \left( \frac{P_{HF,t}^I}{P_{H,t}} \right)^{-\theta_b} + (1 - \gamma) \left( \frac{P_{HI,t}^I}{P_{H,t}} \right)^{-\theta_b} \right) \]

\[
+ \alpha \int_0^1 C_t^j \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_d} \left( \frac{P_{j,t}^I}{P_{j,t}} \right)^{\theta_d-\theta_a} \left( \epsilon_{j,t} P_{j,t}^I \right)^{\theta_d-\theta_a} \left( \frac{1}{P_t} \right)^{-\theta_a} \left[ Q_{j,t} \right]^{\theta_a} \, dj
\]

\[ Y_t = (1 - \alpha) C_t \left( \frac{P_{H,t}^I}{P_t} \right)^{-\theta_a} \left( \gamma \left( \frac{P_{HF,t}^I}{P_{H,t}} \right)^{-\theta_b} + (1 - \gamma) \left( \frac{P_{HI,t}^I}{P_{H,t}} \right)^{-\theta_b} \right) \]

\[
+ \alpha \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_a} \int_0^1 C_t^j \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_d} \left( \frac{1}{P_{j,t}^I} \right)^{-\theta_a} \left( \epsilon_{j,t} P_{j,t}^I \right)^{\theta_d-\theta_a} \left[ Q_{j,t} \right]^{\theta_a} \, dj
\]

\[ Y_t = \left( \frac{P_{H,t}^I}{P_t} \right)^{-\theta_a} \left[ (1 - \alpha) C_t \left( \gamma \left( \frac{P_{HF,t}^I}{P_{H,t}} \right)^{-\theta_b} + (1 - \gamma) \left( \frac{P_{HI,t}^I}{P_{H,t}} \right)^{-\theta_b} \right) \right] \]

\[
+ \alpha \int_0^1 C_t^j \left( \frac{\epsilon_{j,t} P_{j,t}^I}{P_t} \right)^{\theta_d-\theta_a} \left[ Q_{j,t} \right]^{\theta_a} \, dj
\]

Plugging (1.1.58) by assuming \( (Q_{j,t})^{-\gamma} C_t \equiv C_t^j \)
\[
Y_t = \left( \frac{P_{H,t}}{P_t} \right)^{-\vartheta_a} \left[ (1 - \alpha) C_t \left( \gamma \left( \frac{P_{HF,t}}{P_{H,t}} \right)^{-\vartheta_b} + (1 - \gamma) \left( \frac{P_{HL,t}}{P_{H,t}} \right)^{-\vartheta_b} \right) \right.
\]
\[
+ \alpha \int_0^1 \left( \frac{e_{j,t} P_{F,t}}{P_{H,t}} \right)^{\vartheta_a - \vartheta_a} \left[ Q_{j,t} \right]^\vartheta_a \left( \frac{1}{\varepsilon} \right) \tau C_t \, dj \]
\]
\[
Y_t = \left( \frac{P_{H,t}}{P_t} \right)^{-\vartheta_a} \left[ (1 - \alpha) C_t \left( \gamma \left( \frac{P_{HF,t}}{P_{H,t}} \right)^{-\vartheta_b} + (1 - \gamma) \left( \frac{P_{HL,t}}{P_{H,t}} \right)^{-\vartheta_b} \right) \right.
\]
\[
+ \alpha C_t \int_0^1 \left( \frac{e_{j,t} P_{F,t}}{P_{H,t}} \right)^{\vartheta_a - \vartheta_a} \left[ Q_{j,t} \right]^\vartheta_a \left( \frac{1}{\varepsilon} \right) \tau d j \]
\]
\[
Y_t = C_t \left( \frac{P_{H,t}}{P_t} \right)^{-\vartheta_a} \left[ (1 - \alpha) \left( \gamma \left( \frac{P_{HF,t}}{P_{H,t}} \right)^{-\vartheta_b} + (1 - \gamma) \left( \frac{P_{HL,t}}{P_{H,t}} \right)^{-\vartheta_b} \right) \right.
\]
\[
+ \alpha \int_0^1 \left( S_{j,t} S_{j,t} \right)^{\vartheta_a - \vartheta_a} \left[ Q_{j,t} \right]^\vartheta_a \left( \frac{1}{\varepsilon} \right) \tau d j \]
\]

Plugging (1.1.39) and \( S_t^j = \frac{e_{j,t} P_{F,t}}{P_{H,t}} \)

\[
Y_t = C_t \left( \frac{P_{H,t}}{P_t} \right)^{-\vartheta_a} \left[ (1 - \alpha) \left( \gamma \left( \frac{P_{HF,t}}{P_{H,t}} \right)^{-\vartheta_b} + (1 - \gamma) \left( \frac{P_{HL,t}}{P_{H,t}} \right)^{-\vartheta_b} \right) \right.
\]
\[
+ \alpha \int_0^1 \left( S_{j,t} S_{j,t} \right)^{\vartheta_a - \vartheta_a} \left[ Q_{j,t} \right]^\vartheta_a \left( \frac{1}{\varepsilon} \right) \tau d j \]
\]

Log Linearization of (1.3.2) at steady state \( P_{HF,t} = P_{HL,t} = P_{H,t} = P_t = P \)

\[
y_t = c_t + \alpha \vartheta_d s_t + \alpha \left( \vartheta_a - \frac{1}{\varepsilon} \right) q_t \]
\]

(1.3.3)
Substituting (1.1.57) in (1.3.3)

\[ y_t = c_t + \alpha s_t \left[ \frac{\theta_d + \left( \frac{\theta_a - 1}{\varepsilon} \right) (1 - \alpha)}{\varepsilon} \right] \]

Plugging (1.3.4) in the expression makes:

\[ y_t = c_t + \alpha \varepsilon s_t \cdot \frac{\omega}{\varepsilon} \]

Following (1.3.5) the total output of a generic country \( j \) can be written as:

\[ y_t^j = c_t^j + \alpha \frac{\omega}{\varepsilon} s_t^j \]

The total output of the world can be written as:

\[ y_t^w = \int_0^1 y_t^j \, dj = \int_0^1 \left( c_t^j + \alpha \frac{\omega}{\varepsilon} s_t^j \right) \, dj \]

\[ y_t^w = \int_0^1 \left( c_t^j \right) \, dj + \alpha \frac{\omega}{\varepsilon} \int_0^1 \left( s_t^j \right) \, dj \]
Plugging (1.1.60)

\[ y_t^w = c_t^w + \alpha \frac{\omega}{\varepsilon} \int_{0}^{1} (s_t^j) \, dj \]

\[ \int_{0}^{1} (s_t^j) \, dj = 0 \quad (1.3.7) \]

Inserting (1.3.7)

\[ y_t^w = c_t^w \quad (1.3.8) \]

The total output of the world is equal to the total consumption of the world.

Inserting (1.1.61) in (1.3.5)

\[ y_t = c_t^w + \frac{(1 - \alpha)}{\varepsilon} s_t + \alpha s_t \frac{\omega}{\varepsilon} \]

\[ y_t = c_t^w + \frac{s_t}{\varepsilon} (1 - \alpha + \alpha \omega) \]

Inserting (1.3.8)

\[ y_t = y_t^w + \frac{s_t}{\varepsilon} (1 - \alpha + \alpha \omega) \]

\[ y_t = y_t^w + \frac{s_t}{\varepsilon} \frac{1}{(1 - \alpha + \alpha \omega)} \]

Assume
\[ \varepsilon_a = \frac{\varepsilon}{(1 - \alpha + \alpha \omega)} \]  

(1.3.9)

Plugging (1.3.9)

\[ y_t = y_t^w + \frac{s_t}{\varepsilon_a} \]  

(1.3.10)

Substituting (1.3.5) in the consumption Euler (1.1.37) make:

\[ y_t - \alpha s_t \frac{\omega}{\varepsilon} = E_t c_{t+1} + \frac{1}{\varepsilon} (r - i_t + E_t \pi_{t+1}) \]

(1.3.5) makes:

\[ y_{t+1} - \alpha s_{t+1} \frac{\omega}{\varepsilon} = c_{t+1} \]  

(1.3.11)

Inserting (1.3.11)

\[ y_t - \alpha s_t \frac{\omega}{\varepsilon} = E_t \left[ y_{t+1} - \alpha s_{t+1} \frac{\omega}{\varepsilon} \right] + \frac{1}{\varepsilon} (r - i_t + E_t \pi_{t+1}) \]

\[ y_t - \alpha s_t \frac{\omega}{\varepsilon} = E_t y_{t+1} - \frac{\omega}{\varepsilon} \alpha E_t s_{t+1} + \frac{1}{\varepsilon} (r - i_t + E_t \pi_{t+1}) \]

\[ y_t = E_t y_{t+1} - \frac{\omega}{\varepsilon} \alpha E_t s_{t+1} + \alpha s_t \frac{\omega}{\varepsilon} + \frac{1}{\varepsilon} (r - i_t + E_t \pi_{t+1}) \]

\[ y_t = E_t y_{t+1} + \frac{1}{\varepsilon} (r - i_t + E_t \pi_{t+1}) - \frac{\omega}{\varepsilon} \alpha E_t \Delta s_{t+1} \]  

(1.3.12)

(1.3.12) is an IS Equation and can be represented in some other forms as follows:

(1.1.45) makes:
\[
\pi_{t+1} = \pi_{H,t+1} + \alpha \Delta s_{t+1} \tag{1.3.13}
\]

Substituting (1.3.13) in (1.3.12)

\[
y_t = E_t y_{t+1} + \frac{1}{\varepsilon} \left( r - i_t + E_t [\pi_{H,t+1} + \alpha \Delta s_{t+1}] \right) - \frac{\omega}{\varepsilon} \alpha E_t \Delta s_{t+1}
\]

\[
y_t = E_t y_{t+1} + \frac{1}{\varepsilon} \left( r - i_t + E_t \pi_{H,t+1} + E_t \alpha \Delta s_{t+1} \right) - \frac{\omega}{\varepsilon} \alpha E_t \Delta s_{t+1}
\]

\[
y_t = E_t y_{t+1} + \frac{1}{\varepsilon} \left( r - i_t + E_t \pi_{H,t+1} + \frac{1}{\varepsilon} (E_t \alpha \Delta s_{t+1}) \right) - \frac{\omega}{\varepsilon} \alpha E_t \Delta s_{t+1}
\]

\[
y_t = E_t y_{t+1} + \frac{1}{\varepsilon} \left( r - i_t + E_t \pi_{H,t+1} \right) - \left( \frac{\omega - 1}{\varepsilon} \right) \alpha E_t \Delta s_{t+1}
\]

Assume

\[
(\omega - 1) = \Theta \tag{1.3.14}
\]

Plugging (1.3.14) to make:

\[
y_t = E_t y_{t+1} + \frac{1}{\varepsilon} \left( r - i_t + E_t \pi_{H,t+1} \right) - \frac{\Theta}{\varepsilon} \alpha E_t \Delta s_{t+1} \tag{1.3.15}
\]

(1.3.15) is an another version of IS equation.

Rewriting (1.3.10)

\[
y_t = y_t^w + \frac{s_t}{\varepsilon \alpha}
\]

\[
\varepsilon \alpha (y_t - y_t^w) = s_t \tag{1.3.16}
\]
\[ \epsilon_t (y_{t+1} - y_{t+1}^w) = s_{t+1} \quad (1.3.17) \]

\[ \Delta s_{t+1} = s_{t+1} - s_t \quad (1.3.18) \]

Plugging (1.3.17) and (1.3.16) in (1.3.18)

\[ \Delta s_{t+1} = \epsilon_t (y_{t+1} - y_{t+1}^w) - \epsilon_t (y_t - y_t^w) \]

\[ \Delta s_{t+1} = \epsilon_t ((y_{t+1} - y_{t+1}^w) - (y_t - y_t^w)) \quad (1.3.19) \]

Plugging (1.3.19) in (1.3.15)

\[ y_t = E_t y_{t+1} + \frac{1}{\epsilon} (r - i_t + E_t \pi_{H,t+1}) - \frac{\Theta}{\epsilon} \alpha E_t [\epsilon_t ((y_{t+1} - y_{t+1}^w) - (y_t - y_t^w))] \]

\[ y_t = E_t y_{t+1} + \frac{1}{\epsilon} (r - i_t + E_t \pi_{H,t+1}) - \frac{\Theta \alpha \epsilon}{\epsilon} E_t [y_{t+1} - y_{t+1}^w - y_t + y_t^w] \]

\[ y_t = E_t y_{t+1} - \frac{\Theta \alpha \epsilon}{\epsilon} E_t y_{t+1} + \frac{1}{\epsilon} (r - i_t + E_t \pi_{H,t+1}) + \frac{\Theta \alpha \epsilon}{\epsilon} E_t [y_{t+1}^w - y_{t+1}^w + y_t] \]

\[ \Delta y_{t+1}^w = y_{t+1}^w - y_t^w \quad (1.3.20) \]

Plugging (1.3.20)

\[ y_t = \left( 1 - \frac{\Theta \alpha \epsilon}{\epsilon} \right) E_t y_{t+1} + \frac{1}{\epsilon} (r - i_t + E_t \pi_{H,t+1}) + \frac{\Theta \alpha \epsilon}{\epsilon} E_t [\Delta y_{t+1}^w + y_t] \]

\[ y_t = \left( 1 - \frac{\Theta \alpha \epsilon}{\epsilon} \right) E_t y_{t+1} + \frac{1}{\epsilon} (r - i_t + E_t \pi_{H,t+1}) + \frac{\Theta \alpha \epsilon}{\epsilon} E_t y_{t+1}^w + \frac{\Theta \alpha \epsilon}{\epsilon} y_t \]

\[ \left( 1 - \frac{\Theta \alpha \epsilon}{\epsilon} \right) y_t = \left( 1 - \frac{\Theta \alpha \epsilon}{\epsilon} \right) E_t y_{t+1} + \frac{1}{\epsilon} (r - i_t + E_t \pi_{H,t+1}) + \frac{\Theta \alpha \epsilon}{\epsilon} E_t \Delta y_{t+1}^w \]
\[ y_t = \left(1 - \frac{\Theta \alpha \varepsilon}{\varepsilon}\right) E_t y_{t+1} + \frac{1}{\varepsilon} \left(1 - \frac{\Theta \alpha \varepsilon}{\varepsilon}\right) (r - i_t + E_t \pi_{H,t+1}) + \frac{\Theta \alpha \varepsilon}{\varepsilon} E_t \Delta y^W_{t+1} \]

\[ y_t = E_t y_{t+1} + \frac{1}{\varepsilon} \left(1 - \frac{\Theta \alpha \varepsilon}{\varepsilon}\right) (r - i_t + E_t \pi_{H,t+1}) + \frac{\Theta \alpha \varepsilon}{\varepsilon} E_t \Delta y^W_{t+1} \]

\[ y_t = E_t y_{t+1} + \frac{1}{\varepsilon - \Theta \alpha \varepsilon} \left(1 - \frac{\Theta \alpha \varepsilon}{\varepsilon - \Theta \alpha \varepsilon}\right) (r - i_t + E_t \pi_{H,t+1}) + \frac{\Theta \alpha \varepsilon}{\varepsilon - \Theta \alpha \varepsilon} E_t \Delta y^W_{t+1} \]

\[ y_t = E_t y_{t+1} + \frac{1}{\varepsilon - \Theta \alpha \varepsilon} \left(1 - \frac{\Theta \alpha \varepsilon}{\varepsilon - \Theta \alpha \varepsilon}\right) (r - i_t + E_t \pi_{H,t+1}) + \frac{\Theta \alpha \varepsilon}{\varepsilon - \Theta \alpha \varepsilon} E_t \Delta y^W_{t+1} \]

Plugging (1.3.9) and (1.3.14)

\[ y_t = E_t y_{t+1} + \frac{1}{\varepsilon - (\omega - 1)\alpha} \frac{\varepsilon}{(1 - \alpha + \alpha \omega)} (r - i_t + E_t \pi_{H,t+1}) \]

\[ + \frac{(\omega - 1)\alpha}{\varepsilon - (\omega - 1)\alpha} \frac{\varepsilon}{(1 - \alpha + \alpha \omega)} E_t \Delta y^W_{t+1} \]

\[ y_t = E_t y_{t+1} + \frac{1}{\varepsilon - (\omega - 1)\alpha} \frac{\varepsilon}{(1 - \alpha + \alpha \omega)} (r - i_t + E_t \pi_{H,t+1}) \]

\[ + \frac{\alpha \varepsilon (\omega - 1)}{\varepsilon - (\omega - 1)\alpha} \frac{\varepsilon}{(1 - \alpha + \alpha \omega)} E_t \Delta y^W_{t+1} \]


\[ y_t = E_t y_{t+1} + \frac{1}{\varepsilon(1 - \alpha + \alpha \omega) - (\omega - 1)\alpha \varepsilon} \left( r - i_t + E_t \pi_{H,t+1} \right) \]

\[ + \frac{\alpha \varepsilon(\omega - 1)}{(1 - \alpha + \alpha \omega)} E_t \Delta y^w_{t+1} \]

\[ y_t = E_t y_{t+1} + \frac{1}{\varepsilon(1 - \alpha + \alpha \omega)} \left( r - i_t + E_t \pi_{H,t+1} \right) + \frac{\alpha \varepsilon(\omega - 1)}{(1 - \alpha + \alpha \omega)} E_t \Delta y^w_{t+1} \]

\[ y_t = E_t y_{t+1} + \frac{1 - \alpha + \alpha \omega}{\varepsilon} \left( r - i_t + E_t \pi_{H,t+1} \right) + \frac{\alpha \varepsilon(\omega - 1)}{(1 - \alpha + \alpha \omega)} E_t \Delta y^w_{t+1} \]

\[ y_t = E_t y_{t+1} + \frac{1 - \alpha + \alpha \omega}{\varepsilon} \left( r - i_t + E_t \pi_{H,t+1} \right) + \alpha \varepsilon(\omega - 1) E_t \Delta y^w_{t+1} \]

Plugging (1.3.9) and (1.3.14)

\[ y_t = E_t y_{t+1} + \frac{1}{\varepsilon} \left( r - i_t + E_t \pi_{H,t+1} \right) + \alpha \varepsilon(\omega - 1) E_t \Delta y^w_{t+1} \]

(1.3.21) represents another format of an IS equation.

6.2 The trade balance

The net trade balance is given in terms of domestic output and expressed as fraction of steady state output as:

\[ NX_t = \frac{Y_t - \frac{P_t}{P_{H,t}} C_t}{Y} \]
Macroeconomic Goals and Inflation Targeting in India

Where, $NX_t$ is net trade balance in time $t$ and $Y$ is defined as domestic output at steady state. Log linearization of (1.3.22) makes:

$$nx_t = y_t - c_t - p_t + p_{H,t}$$  \hspace{1cm} (1.3.23)

Substituting (1.1.43)

$$nx_t = y_t - c_t - p_{H,t} - \alpha s_t + p_{H,t}$$

$$nx_t = y_t - c_t - \alpha s_t$$

Inserting (1.3.5)

$$nx_t = \alpha s_t \left( \frac{\omega}{\varepsilon} - 1 \right)$$  \hspace{1cm} (1.3.24)

(1.2.14) writes as:

$$[L_{HF,t}(i)] = \frac{[Y_{HF,t}(i)]}{A_{HF,t}}$$

$$\int_0^1 [L_{HF,t}(i)] \, di = \int_0^1 \frac{[Y_{HF,t}(i)]}{A_{HF,t}} \, di$$

$$L_{HF,t} = \int_0^1 [L_{HF,t}(i)] \, di$$  \hspace{1cm} (1.3.25)

Plugging (1.3.25) and (1.2.9) in the expression to make:
\[
L_{HF,t} = \frac{1}{A_{HF,t}} \int_0^1 \left( \frac{P_{HF,t}(i)}{P_{HF,t}} \right)^{-\varphi_c} Y_{HF,t} \, di \tag{1.3.26}
\]

(1.3.27) analogous to (1.3.26) writes as:

\[
L_{HI,t} = \frac{1}{A_{HI,t}} \int_0^1 \left( \frac{P_{HI,t}(i)}{P_{HI,t}} \right)^{-\varphi_c} Y_{HI,t} \, di \tag{1.3.27}
\]

\[L_t = L_{HF,t} + L_{HI,t} \tag{1.3.28}\]

Using (1.3.26), (1.3.27) and (1.3.28)

\[
L_t = \frac{1}{A_{HF,t}} \int_0^1 \left( \frac{P_{HF,t}(i)}{P_{HF,t}} \right)^{-\varphi_c} Y_{HF,t} \, di + \frac{1}{A_{HI,t}} \int_0^1 \left( \frac{P_{HI,t}(i)}{P_{HI,t}} \right)^{-\varphi_c} Y_{HI,t} \, di \tag{1.3.29}
\]

Log linearization of (1.3.29) at steady state \([P_{HF,t}(i)] = [P_{HI,t}(i)] = P_{HF,t} = P_{HI,t} = P_t = P\)

\[l_t = y_{HF,t} - a_{HF,t} + y_{HI,t} - a_{HI,t} \]

\[y_t = y_{HF,t} + y_{HI,t} \tag{1.3.30}\]

Assume

\[a_t = a_{HF,t} + a_{HI,t} \tag{1.3.31}\]

Plugging (1.3.30) and (1.3.31) in the expression to make:

\[y_t = l_t + a_t \tag{1.3.32}\]
Domestic inflation is given by

$$\pi_{H,t} = \gamma \pi_{HF,t} + (1 - \gamma) \pi_{HI,t}$$  \hfill (1.3.33)

Plugging (1.2.38) and (1.2.39) in (1.3.33)

$$\pi_{H,t} = \gamma [\beta E_t \pi_{HF,t+1} + \lambda_{HF} \bar{m}_H \bar{c}_{HF,t}] + (1 - \gamma) \beta E_t \pi_{HI,t+1} + (1 - \gamma) \lambda_{HI} \bar{m}_H \bar{c}_{HI,t}$$

$$\pi_{H,t} = \gamma [\beta E_t \pi_{HF,t+1} + \gamma \lambda_{HF} \bar{m}_H \bar{c}_{HF,t} + (1 - \gamma) \beta E_t \pi_{HI,t+1} + (1 - \gamma) \lambda_{HI} \bar{m}_H \bar{c}_{HI,t}]$$

$$\pi_{H,t} = \beta [\gamma E_t \pi_{HF,t+1} + (1 - \gamma) E_t \pi_{HI,t+1}] + \gamma \lambda_{HF} \bar{m}_H \bar{c}_{HF,t} + (1 - \gamma) \lambda_{HI} \bar{m}_H \bar{c}_{HI,t}$$

The domestic informal sector has perfect Classical markets i.e. it has complete flexibility in prices (no stickiness in prices) and perfections in markets; it follows $\theta_{HI} = 0$ and in turn it $(\theta_{HI})$ makes:

$$\bar{m}_H \bar{c}_{HI,t} = 0$$  \hfill (1.3.34)

Rewriting the domestic inflation as:

$$E_t \pi_{H,t+1} = \gamma E_t \pi_{HF,t+1} + (1 - \gamma) E_t \pi_{HI,t+1}$$  \hfill (1.3.35)

Plugging (1.3.34), (1.3.35) and at steady state $\bar{m}c_{HF,t} = \bar{m}c_{t}$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \gamma \lambda_{HF} \bar{m}_H \bar{c}_{H,t}$$

(1.3.36), analogous to (1.2.28) can be given as:

$$mc_t = w_t - p_{H,t} - a_t$$  \hfill (1.3.37)

Rewriting (1.3.37)
\[ mc_t^R = w_t - p_t + p_t - p_{H,t} - a_t \]

Plugging (1.1.38) and (1.1.43)

\[ mc_t^R = \epsilon c_t + \nu l_t + \alpha s_t - a_t \]

Inserting (1.1.61) and (1.3.32)

\[ mc_t^R = \epsilon (c_t^w + \frac{1 - \alpha}{\epsilon} s_t) + \nu (y_t - a_t) + \alpha s_t - a_t \]

\[ mc_t^R = \epsilon c_t^w + s_t - \alpha s_t + \nu y_t - \nu a_t + \alpha s_t - a_t \]

\[ mc_t^R = \epsilon c_t^w + s_t + \nu y_t - \nu a_t - a_t \]

\[ mc_t^R = \epsilon c_t^w + s_t + \nu y_t - a_t(\nu + 1) \]

Substituting (1.3.8)

\[ mc_t^R = \epsilon y_t^w + s_t + \nu y_t - a_t(\nu + 1) \]

Plugging (1.3.16)

\[ mc_t^R = \epsilon y_t^w + s_t + \nu y_t - a_t(\nu + 1) \]

\[ mc_t^R = \epsilon y_t^w + \epsilon_\alpha (y_t - y_t^w) + \nu y_t - a_t(\nu + 1) \]

\[ mc_t^R = \epsilon y_t^w + \epsilon_\alpha y_t - \epsilon_\alpha y_t^w + \nu y_t - a_t(\nu + 1) \]

\[ mc_t^R = \epsilon y_t^w - \epsilon_\alpha y_t^w + \nu y_t + \epsilon_\alpha y_t - a_t(\nu + 1) \]

\[ mc_t^R = y_t^w(\epsilon - \epsilon_\alpha) + y_t(\nu + \epsilon_\alpha) - a_t(\nu + 1) \] \hspace{1cm} (1.3.38)
When prices are completely flexible \((1.3.39)\), analogous to \((1.2.21)\) can be given as:

\[
m_c^R = -\mu
\]  \((1.3.39)\)

Natural level of output is defined as \(y^n_t\) and using \((1.3.39)\) a flexible prices version of \((1.3.38)\) can be given as:

\[
m_c^R = -\mu = y^w_t (\varepsilon - \varepsilon_a) + y^n_t (v + \varepsilon_a) - a_t (v + 1)
\]

\[
y^n_t (v + \varepsilon_a) = -\mu - y^w_t (\varepsilon - \varepsilon_a) + a_t (v + 1)
\]

\[
y^n_t = a_t \frac{(v + 1)}{(v + \varepsilon_a)} - \frac{\mu}{(v + \varepsilon_a)} - y^w_t \frac{(\varepsilon - \varepsilon_a)}{(v + \varepsilon_a)}
\]  \((1.3.40)\)

Plugging \((1.3.9)\) and \((1.3.14)\)

\[
y^n_t = a_t \frac{(v + 1)}{(v + \varepsilon_a)} - \frac{\mu}{(v + \varepsilon_a)} - y^w_t \frac{(\varepsilon - \varepsilon_a)}{(v + \varepsilon_a)}
\]

\[
y^n_t = a_t \frac{(v + 1)}{(v + \varepsilon_a)} - \frac{\mu}{(v + \varepsilon_a)} - y^w_t \frac{(\varepsilon - \varepsilon_a) - \varepsilon}{1 + \alpha \Theta}
\]

\[
y^n_t = a_t \frac{(v + 1)}{(v + \varepsilon_a)} - \frac{\mu}{(v + \varepsilon_a)} - y^w_t \frac{\varepsilon - \varepsilon}{1 + \alpha \Theta}
\]

Again plugging \((1.3.9)\) and \((1.3.14)\)

\[
y^n_t = a_t \frac{(v + 1)}{(v + \varepsilon_a)} - \frac{\mu}{(v + \varepsilon_a)} - y^w_t \frac{\varepsilon - \varepsilon_a}{1 + \alpha \Theta}
\]  \((1.3.41)\)

Plugging \((1.3.42), (1.3.43)\) and \((1.3.44)\) in \((1.3.41)\)
\[ \Gamma_0 = -\frac{\mu}{(v + \varepsilon_\alpha)} \]  

\[ \Gamma_a = \frac{(v + 1)}{(v + \varepsilon_\alpha)} \]  

\[ \Gamma_w = -\frac{a\Theta\varepsilon_\alpha}{(v + \varepsilon_\alpha)} \]  

\[ y^n_t = \Gamma_0 + a_t \Gamma_a + y^w_t \Gamma_w \]  

7 New Keynesian Phillips Curve for Indian Economy

Domestic output gap can be defined as:

\[ \hat{y}_t = y_t - y^n_t \]  

\[ (1.3.38) \] and its flexible prices version make as:

\[ mc^R_t - mc^R = [y^w_t(\varepsilon - \varepsilon_\alpha) + y_t(v + \varepsilon_\alpha) - a_t(v + 1)] - [y^w_t(\varepsilon - \varepsilon_\alpha) + y^n_t(v + \varepsilon_\alpha) - a_t(v + 1)] \]

\[ mc^R_t - mc^R = [y_t(v + \varepsilon_\alpha)] - [y^n_t(v + \varepsilon_\alpha)] \]

\[ \tilde{mc}^R_t = mc^R_t - mc^R \]  

\[ (1.3.47) \]

Substituting \( (1.3.47) \)

\[ \tilde{mc}^R_t = (v + \varepsilon_\alpha)(y_t - y^n_t) \]  

\[ (1.3.48) \]

Inserting \( (1.3.46) \)
\[
\tilde{\mu}_t^R = (\nu + \varepsilon_\alpha) \tilde{\gamma}_t
\]

(1.3.49)

Plugging (1.3.49) in (1.3.36)

\[
\pi_{H,t} = \beta E_t\pi_{H,t+1} + \gamma \lambda_{HF}(\nu + \varepsilon_\alpha) \tilde{\gamma}_t
\]

\[
\rho = \gamma \lambda_{HF}(\nu + \varepsilon_\alpha)
\]

(1.3.50)

Plugging (1.3.50)

\[
\pi_{H,t} = \beta E_t\pi_{H,t+1} + \rho \tilde{\gamma}_t
\]

(1.3.51)

(1.3.51) is an open economy New Keynesian Phillips Curve for Indian Economy.

8 Dynamic IS Curve for Indian Economy

The real rate of interest can be defined as:

\[
i_t^R = i_t - E_t\pi_{H,t+1}
\]

(1.3.52)

The real rate of interest at its natural level can be defined as:

\[
i_t^{Rn} = i_t - E_t\pi_{H,t+1}
\]

(1.3.53)

Natural level of output, analogous to (1.3.21) can be given using (1.3.53) as:

\[
y_t^n = E_t y_{t+1}^n + \frac{1}{\varepsilon_\alpha} \left( r - (i_t^{Rn} + E_t\pi_{H,t+1}) + E_t\pi_{H,t+1} \right) + \alpha \Theta E_t \Delta y_{t+1}^w
\]

\[
y_t^n = E_t y_{t+1}^n + \frac{1}{\varepsilon_\alpha} \left( r - i_t^{Rn} \right) + \alpha \Theta E_t \Delta y_{t+1}^w
\]

(1.3.54)

(1.3.21) and (1.3.54) make as:
\[ y_t - y_t^n = \left[ E_t y_{t+1} + \frac{1}{\varepsilon_\alpha} \left( r - i_t + E_t \pi_{H,t+1} \right) + \alpha \Theta E_t \Delta y_{t+1}^w \right] \\
\quad \quad \quad - \left[ E_t y_{t+1}^n + \frac{1}{\varepsilon_\alpha} \left( r - i_t^R \right) + \alpha \Theta E_t \Delta y_{t+1}^w \right] \]

\[ y_t - y_t^n = \left[ E_t y_{t+1} + \frac{1}{\varepsilon_\alpha} \left( r - i_t + E_t \pi_{H,t+1} \right) \right] - \left[ E_t y_{t+1}^n + \frac{1}{\varepsilon_\alpha} \left( r - i_t^R \right) \right] \]

\[ E_t \bar{y}_{t+1} = E_t y_{t+1} - E_t y_{t+1}^n \quad (1.3.55) \]

Plugging (1.3.46) and (1.3.55), analogous to (1.3.46) make

\[ \bar{y}_t = \left[ E_t \bar{y}_{t+1} + \frac{1}{\varepsilon_\alpha} \left( i_t^R - i_t + E_t \pi_{H,t+1} \right) \right] \quad (1.3.56) \]

(1.3.56) is Dynamic IS curve for the Indian Economy. Open economy New Keynesian Phillips Curve (1.3.51), Dynamic IS Curve (1.3.56) and monetary policy rule (Taylor rule) (2.2.15), which is derived in the next study entitled “Inflation Targeting Model for Indian Economy”, are the key building blocks of the New Keynesian Model.

9 Conclusion

I have developed a New Keynesian Model for the Indian economy with heterogeneous sectors, namely, formal and informal. The formal sector shows sluggish prices and rigid wages and imperfections in the markets while informal sector of Indian economy characterizes the complete flexibility in prices and wages and perfections in markets. The formal sector comprises of Keynesian markets while informal sector is made of pure Classical markets. Thus, Indian economy comprises of a very typical mixture of Keynesian and Classical markets.

When monetary policy is conducted (variation in money supply) by the Reserve Bank of India in such an environment then only nominal effects are seen in the informal sector markets i.e. variations in price and wage levels; but real
effects are observed in the formal sector markets in short run i.e. monetary policy affects output and level of employment in short run.

This study is firstly and foremost targeted to study the nature of Indian domestic inflation and thereby to study the real variables of the economy. The aggregate supply equation, the New Keynesian Phillips Curve, establishes a relationship between domestic inflation and output (gap). The New Keynesian Phillips Curve reveals that the degree of stickiness in prices in formal sector markets has a deep impact on the domestic inflation as informal sector markets are frictionless and have complete price flexibility (zero stickiness). Thus, degree of stickiness in prices in formal sector markets plays a major role to determine the domestic inflation and enables the monetary policy to stabilize formal sector output and level of employment.

Summarily, monetary policy affects the real variables of the economy in formal sector in short run while nominal variables (price and wage level) in informal sector. Thus, monetary policy in India has a very poor control on real variables of the economy in short run due to presence of huge informal sector. This conclusion is based on pure theory and may have deviation from reality. This study provides opportunities for further empirical work.

10 Limitation of the Study

I have dropped the government, money, habit formation and corruption in public spending (consumption spending and capital spending) related issues in this study to keep it simple and easily traceable without diluting the main concern of the study. This study is based on pure theory and no empirical support is supplemented.

11 References


12 Appendix A

Derivation of expressions / equations for 1.3 Households

Consumption expenditures $C_{HF,t} P_{HF,t}$, $C_{HL,t} P_{HL,t}$, $C_{F,t} P_{F,t}$, $C_{H,t} P_{H,t}$ and $C_t P_t$ of the domestic households can be given as:

\[
\int_0^1 [C_{HF,t}(i)][P_{HF,t}(i)] di = C_{HF,t} P_{HF,t} \tag{A.1.1}
\]

\[
\int_0^1 [C_{HL,t}(i)][P_{HL,t}(i)] di = C_{HL,t} P_{HL,t} \tag{A.1.2}
\]

\[
\int_0^1 \int_0^1 [C_{j,t}(i)][P_{j,t}(i)] dj\, di = C_{F,t} P_{F,t} \tag{A.1.3}
\]

\[
C_{H,t} P_{H,t} = C_{HF,t} P_{HF,t} + C_{HL,t} P_{HL,t} \tag{A.1.4}
\]

\[
C_t P_t = C_{H,t} P_{H,t} + C_{F,t} P_{F,t} \tag{A.1.5}
\]

\[
C_t P_t = \int_0^1 [C_{HF,t}(i)][P_{HF,t}(i)] di + \int_0^1 [C_{HL,t}(i)][P_{HL,t}(i)] di + \int_0^1 \int_0^1 [C_{j,t}(i)][P_{j,t}(i)] dj \tag{A.1.6}
\]

Where (A.1.1) to (A.1.5) collapse to (A.1.6).

Nominal wage incomes $L_{HF,t} W_{HF,t}$, $L_{HL,t} W_{HL,t}$ and $L_t W_t$ of the domestic household can be given as:
\[
\begin{align*}
\int_{0}^{1} \left[ L_{HF,t}(i) \right] W_{HF,t}(i) di = L_{HF,t} W_{HF,t} \\
\int_{0}^{1} \left[ L_{HI,t}(i) \right] W_{HI,t}(i) di = L_{HI,t} W_{HI,t} \\
L_{t} W_{t} = L_{HF,t} W_{HF,t} + L_{HI,t} W_{HI,t}
\end{align*}
\]  

(A.1.7)  

(A.1.8)  

(A.1.9)

Where (A.1.7) to (A.1.9) collapse to (A.1.10).

\[
L_{t} W_{t} = \int_{0}^{1} \left[ L_{HF,t}(i) \right] W_{HF,t}(i) di + \int_{0}^{1} \left[ L_{HI,t}(i) \right] W_{HI,t}(i) di
\]

(A.1.10)

Plugging (1.1.7) in (A.1.1) makes

\[
P_{HF,t} \left[ \int_{0}^{1} \left[ C_{HF,t}(i) \right] \frac{\theta_{c} - 1}{\varphi_{c}} di \right] \frac{\varphi_{c}}{\varphi_{c} - 1} = \int_{0}^{1} \left[ C_{HF,t}(i) \right] P_{HF,t}(i) di
\]

\[
\min_{\left[ C_{HF,t}(i) \right]} P_{HF,t} \left[ \int_{0}^{1} \left[ C_{HF,t}(i) \right] \frac{\theta_{c} - 1}{\varphi_{c}} di \right] \frac{\varphi_{c}}{\varphi_{c} - 1} - \int_{0}^{1} \left[ C_{HF,t}(i) \right] P_{HF,t}(i) di
\]

First Order Condition with respect to \( C_{t}^{HF}(i) \) is given by

\[
P_{HF,t} \left[ \int_{0}^{1} \left[ C_{HF,t}(i) \right] \frac{\theta_{c} - 1}{\varphi_{c}} di \right] \frac{\varphi_{c}}{\varphi_{c} - 1} = \int_{0}^{1} \left[ C_{HF,t}(i) \right] P_{HF,t}(i) di
\]

\[
\min_{\left[ C_{HF,t}(i) \right]} P_{HF,t} \left[ \int_{0}^{1} \left[ C_{HF,t}(i) \right] \frac{\theta_{c} - 1}{\varphi_{c}} di \right] \frac{\varphi_{c}}{\varphi_{c} - 1} - \int_{0}^{1} \left[ C_{HF,t}(i) \right] P_{HF,t}(i) di = 0
\]
\[
[C_{HF,t}(i)]^{-1} = \frac{P_{HF,t}(i)}{P_{HF,t}} \left[ \int_0^1 [C_{HF,t}(i)]^{\theta_c-1} \frac{\partial c}{\partial c} \, di \right]^{-1} \]

\[
[C_{HF,t}(i)]^{\theta_c} = \left( \frac{P_{HF,t}(i)}{P_{HF,t}} \right)^{-\theta_c} \left[ \int_0^1 [C_{HF,t}(i)]^{\theta_c-1} \frac{\partial c}{\partial c} \, di \right]^{\theta_c} \]

\[
[C_{HF,t}(i)] = \left( \frac{P_{HF,t}(i)}{P_{HF,t}} \right)^{-\theta_c} \left[ \int_0^1 [C_{HF,t}(i)]^{\theta_c-1} \frac{\partial c}{\partial c} \, di \right]^{\theta_c} \]

Plugging (1.17) makes

\[
[C_{HF,t}(i)] = \left( \frac{P_{HF,t}(i)}{P_{HF,t}} \right)^{-\theta_c} C_{HF,t}
\]  

(1.17) to (1.20) and (1.22) to (1.24) can, analogously, be derived.

Plugging (A1.11) in (A1.1) makes

\[
C_{HF,t}P_{HF,t} = \int_0^1 \left( \frac{P_{HF,t}(i)}{P_{HF,t}} \right)^{-\theta_c} C_{HF,t} [P_{HF,t}(i)] \, di
\]

\[
C_{HF,t}P_{HF,t} = C_{HF,t} \left( \frac{1}{P_{HF,t}} \right)^{-\theta_c} \left[ \int_0^1 [P_{HF,t}(i)]^{-\theta_c}P_{HF,t}(i) \, di \right]
\]

\[
P_{HF,t} = \left( \frac{1}{P_{HF,t}} \right)^{\theta_c} \left[ \int_0^1 [P_{HF,t}(i)]^{1-\theta_c} \, di \right]
\]
\[
(P_{HF,t})^{1-\vartheta_c} = \frac{1}{0} \left[ P_{HF,t}(i) \right]^{1-\vartheta_c} \, di
\]

\[
P_{HF,t} = \left[ \frac{1}{0} \left[ P_{HF,t}(i) \right]^{1-\vartheta_c} \, di \right]^{1-\vartheta_c}
\]

(A.1.12)

(1.1.11) to (1.1.12) and (1.1.14) to (1.1.16) can, analogously, be derived.

Rewriting the (1.1.2)

\[
C_tP_t = B_t + L_t W_t - T_t - E_t\{Q_{t,t+1}B_{t+1}\}
\]

The left hand side of the equations shows the consumption expenditure in the period \(t\). The right hand side term \(B_t + L_t W_t - T_t\) represents the available gross income in the period \(t\), while \(E_t\{Q_{t,t+1}B_{t+1}\}\) represents the time \(t\) investment in the portfolio with the nominal payoffs \(B_{t+1}\) in the period \(t+1\). Thus whatever income left after tax and after portfolio investment is used in consumption. The intertemporal problem for the household with respect to the optimal one period portfolio purchase writes as:

\[
\max_{B_{t+1}} \left[ U(C_t, L_t) + E_t \beta U(C_{t+1}, L_{t+1}) \right]
\]

(A.1.13)

Subject to

\[
C_tP_t = B_t + L_t W_t - T_t - E_t \int V_{t,t+1}B_{t+1} \, d\tau
\]

(A.1.14)

\[
E_tC_{t+1}P_{t+1} = E_t \left( \int \xi_{t,t+1}B_{t+1} \, d\tau + L_{t+1}W_{t+1} - T_{t+1} - E_tV_{t,t+2}B_{t+2} \right)
\]

(A.1.15)
\(E_t \int V_{t,t+1} B_{t+1} \, d\tau\) is the market price of one period portfolio yielding random payoff \(B_{t+1}\). It has been integrated over all possible states of nature indexed by \(\tau\). \(V_{t,t+1}\) is the period \(t\) price of the Arrow security\(^1\). \(\xi_{t,t+1}\) is the probability that a given state of nature is realized in the period \(t + 1\). Equivalently, the price can be written as \(E_t \frac{V_{t,t+1}}{\xi_{t,t+1}} B_{t+1}\). Thus, the stochastic discount factor can be defined as:

\[
Q_{t,t+1} = \frac{V_{t,t+1}}{\xi_{t,t+1}}
\]  

(A.1.16)

Rewriting (A.1.14) and (A.1.15) as:

\[
C_t = \frac{B_t}{P_t} + L_t \frac{W_t}{P_t} - T_t \frac{1}{P_t} - E_t \int \frac{V_{t,t+1}}{P_t} B_{t+1} \, d\tau
\]  

(A.1.17)

\[
E_t C_{t+1} = E_t \left( \int \frac{\xi_{t,t+1}}{P_{t+1}} B_{t+1} \, d\tau + L_{t+1} \frac{W_{t+1}}{P_{t+1}} - T_{t+1} \frac{1}{P_{t+1}} - E_t \frac{V_{t,t+2}}{P_{t+1}} B_{t+2} \right)
\]  

(A.1.18)

Using (1.1.1) one period utility can be given as:

\[
\max_{B_{t+1}} E_0 \sum_{t=0}^{1} \beta^1 \left\{ \frac{C_t^{1-\epsilon}}{1 - \epsilon} - \frac{L_t^{1+\nu}}{1 + \nu} \right\}
\]

\[
\max_{B_{t+1}} \left[ \beta^0 \left( \frac{C_t^{1-\epsilon}}{1 - \epsilon} - \frac{L_t^{1+\nu}}{1 + \nu} \right) + E_t \beta^1 \left( \frac{C_{t+1}^{1-\epsilon}}{1 - \epsilon} - \frac{L_{t+1}^{1+\nu}}{1 + \nu} \right) \right]
\]

\[
\max_{B_{t+1}} \left[ \left( \frac{C_t^{1-\epsilon}}{1 - \epsilon} - \frac{L_t^{1+\nu}}{1 + \nu} \right) + E_t \beta \left( \frac{C_{t+1}^{1-\epsilon}}{1 - \epsilon} - \frac{L_{t+1}^{1+\nu}}{1 + \nu} \right) \right]
\]

Plugging the values of \(C_t\) and \(E_t C_{t+1}\) from (A.1.17) and (A.1.18) in the optimization problem.

\(^1\) Arrow security is a one period security that yields one unit of domestic currency if a specific state of nature is realized in period \(t + 1\) and zero otherwise.
Macroeconomic Goals and Inflation Targeting in India

\[
\begin{align*}
\max_{B_{t+1}} & \left\{ \left( \frac{B_t + L_t \frac{W_t}{P_t} - T_t \frac{1}{P_t} - E_t \int \frac{V_{t,t+1}}{P_t} B_{t+1} \, d\tau}{1 - \varepsilon} \right) \left( \frac{1}{1 + \varepsilon} \right) - L_{t+1}^1 \right\} \\
+ & E_t \beta \left\{ \left( \frac{E_t \left( \int \xi_{t,t+1} B_{t+1} \, d\tau + L_{t+1} \frac{W_{t+1}}{P_{t+1}} - T_{t+1} \frac{1}{P_{t+1}} - E_t \frac{V_{t,t+2}}{P_{t+1}} B_{t+2} \right)}{1 - \varepsilon} \right) \left( \frac{1}{1 + \varepsilon} \right) - L_{t+1}^1 \right\}
\end{align*}
\]

First Order Condition with respect to \(B_{t+1}\)

\(B_{t+1}: -\frac{1}{1 - \varepsilon} \left( \frac{B_t + L_t \frac{W_t}{P_t} - T_t \frac{1}{P_t} - E_t \int \frac{V_{t,t+1}}{P_t} B_{t+1} \, d\tau}{1 - \varepsilon} \right) \left( \frac{1}{1 + \varepsilon} \right) - \frac{V_{t,t+1}}{P_t} = 0
\)

\(B_{t+1}: -\frac{B_t + L_t \frac{W_t}{P_t} - T_t \frac{1}{P_t} - E_t \int \frac{V_{t,t+1}}{P_t} B_{t+1} \, d\tau}{1 - \varepsilon} \left( \frac{1}{1 + \varepsilon} \right) - \frac{V_{t,t+1}}{P_t} = 0
\)

Plugging the \(C_t\) and \(E_tC_{t+1}\) in the expression for their respective values as in (A.1.17) and (A.1.18):

\(B_{t+1}: -(C_t)^{-\varepsilon} \frac{V_{t,t+1}}{P_t} + \beta (E_tC_{t+1})^{-\varepsilon} \frac{\xi_{t,t+1}}{P_{t+1}} = 0\)
First Order Condition with respect to $L_t$

$$L_t: \frac{1}{1-\varepsilon} \left( \frac{B_t + L_t}{P_t} W_t - T_t \frac{1}{P_t} - E_t \int \frac{V_{t,t+1}}{P_t} B_{t+1} \, dt \right)^{1-\varepsilon} \frac{W_t}{P_t} - \frac{1 + \nu L_t^{1+\nu-1}}{1 + \nu} = 0$$

$$L_t: \frac{B_t}{P_t} + L_t \frac{W_t}{P_t} - T_t \frac{1}{P_t} - E_t \int \frac{V_{t,t+1}}{P_t} B_{t+1} \, dt \right)^{-\varepsilon} \frac{W_t}{P_t} - L_t^\nu = 0$$

Plugging the $C_t$ in the expression for its value as in (A.17)

$$L_t: C_t^{-\varepsilon} \frac{W_t}{P_t} - L_t^\nu = 0$$
Log linearization of (A.1.20).

Given that

\[ Q_t = \frac{1}{1 + i_t} \]

Taking logarithm

\[ \log Q_t = \log \left(\frac{1}{1 + i_t}\right) \]

\[ \log Q_t = -\log(1 + i_t) \]

\[ -\log Q_t = \log(1 + i_t) \approx i_t \]

\[ -\log Q_t \approx i_t \]

(A.1.22)

Given that

\[ \beta = \frac{1}{1 + r} \]

Taking logarithm

\[ \log \beta = \log \left(\frac{1}{1 + r}\right) \]
\[ \log \beta = -\log(1 + r) \]

\[ -\log \beta = \log(1 + r) \approx r \]

\[ -\log \beta \approx r \quad \text{(A.1.23)} \]

Rewriting (A.1.20)

\[
1 = \mathbb{E}_t \left\{ \frac{P_t}{P_{t+1}} \left( \frac{C_{t+1}}{C_t} \right)^{-\epsilon} \frac{\beta}{Q_t} \right\}
\]

\[
1 = \mathbb{E}_t \left\{ e^{-\epsilon \log \left( \frac{P_t}{P_{t+1}} \frac{C_{t+1}}{C_t} \right) - \log \frac{\beta}{Q_t}} \right\}
\]

\[
1 = \mathbb{E}_t \left\{ e^{-\epsilon \log C_{t+1} - \log C_t + \log \beta} \right\}
\]

Plugging (A.1.22) and (A.1.23)

\[
1 = \mathbb{E}_t \left\{ e^{-\epsilon (\log P_{t+1} - \log P_t) - \epsilon (\log C_{t+1} - \log C_t) + \log \beta - \log Q_t} \right\}
\]

Where small letter is the logarithm (with natural base) value of her corresponding capital letter and hereinafter the very same methodology is used throughout the Appendix.

\[
1 = \mathbb{E}_t \left\{ e^{-\pi_{t+1} - \epsilon \Delta c_{t+1} - r + i_t} \right\}
\]

Where \( \pi_{t+1} = \{p_{t+1} - p_t\} \) and \( \Delta c_{t+1} = c_{t+1} - c_t \)

In the steady state \( r = i - \pi - \epsilon \Delta c \) thus, the Euler equation around steady state becomes.
First order Taylor expansion yields:

\[ 1 = E_t \{ 1 - \pi_{t+1} - \pi - \varepsilon(\Delta c_{t+1} - \Delta c) - (r - r) + (i_t - i) \} \]

Plugging \( \Delta c = \kappa \)

\[ 1 = E_t \{ 1 - \pi_{t+1} + \pi - \varepsilon \Delta c_{t+1} + \varepsilon \kappa + i_t - i \} \]

\[ 1 = (1 + \pi - i + \varepsilon \kappa) + E_t (i_t - \pi_{t+1} - \varepsilon \Delta c_{t+1}) \]

\[ 1 = (1 + \pi - i + \varepsilon \kappa) + (i_t - E_t \pi_{t+1} - \varepsilon E_t \Delta c_{t+1}) \]

Plugging the steady state \( r = i - \pi - \varepsilon \Delta c = i - \pi - \varepsilon \kappa \)

\[ 1 = (1 - r) + (i_t - E_t \pi_{t+1} - \varepsilon E_t \Delta c_{t+1}) \]

Plugging \( \Delta c_{t+1} = c_{t+1} - c_t \)

\[ 0 = i_t - r - E_t \pi_{t+1} - \varepsilon E_t \Delta c_{t+1} + \varepsilon c_t \]

\[ c_t = E_t c_{t+1} + \frac{1}{\varepsilon} (r - i_t + E_t \pi_{t+1}) \quad (A.1.24) \]

Taking log of (A.1.21) for its log linearization.

\[ \log C_t^e L_t^\nu = \log \frac{W_t}{p_t} \]

\[ \varepsilon c_t + v l_t = w_t - p_t \quad (A.1.25) \]
Derivation of expressions/equations for 1.4 International Economic Environment

International Risk Sharing

(A.1.19) writes for generic country $j$, analogously, as:

$$1 = \beta E_t \left\{ \frac{1}{Q_{t,t+1}} \left( \frac{C_{t+1}^j C_t^j}{C_t} \right)^{-e} \frac{P_t}{P_{j,t+1}} \right\}$$

Equating (A.1.19) and molded (A.1.19) for generic country $j$ given above as:

$$\beta E_t \left\{ \frac{1}{Q_{t,t+1}} \left( \frac{C_{t+1}^j C_t^j}{C_t} \right)^{-e} \frac{P_t}{P_{j,t+1}} \right\} = \beta E_t \left\{ \frac{1}{Q_{t,t+1}} \left( \frac{C_{t+1}^j C_t^j}{C_t} \right)^{-e} \frac{P_{j,t}}{P_{j,t+1}} \right\}$$

$$1 = \frac{\beta E_t \left\{ \frac{1}{Q_{t,t+1}} \left( \frac{C_{t+1}^j C_t^j}{C_t} \right)^{-e} \frac{P_t}{P_{j,t+1}} \right\}}{\beta E_t \left\{ \frac{1}{Q_{t,t+1}} \left( \frac{C_{t+1}^j C_t^j}{C_t} \right)^{-e} \frac{P_{j,t}}{P_{j,t+1}} \right\}}$$

$$1 = \frac{\beta E_t \left\{ \left( \frac{C_{t+1}^j}{C_t} \right)^{-e} \frac{P_t}{P_{t+1}} \right\}}{\beta E_t \left\{ \left( \frac{C_{t+1}^j}{C_t} \right)^{-e} \frac{P_{j,t}}{P_{j,t+1}} \right\}}$$

Plugging (1.1.47)

$$1 = \frac{\beta E_t \left\{ \left( \frac{C_{t+1}^j}{C_t} \right)^{-e} \frac{P_t}{P_{t+1}} \right\}}{\beta E_t \left\{ \left( \frac{C_{t+1}^j}{C_t} \right)^{-e} \frac{P_{j,t}}{P_{j,t+1}} \right\}}$$
\[ 1 = E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\epsilon} \left( \frac{P_t}{P_{t+1}} \right) \right\} \]

\[ 1 = E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\epsilon} \left[ \frac{P_t \epsilon_{j,t+1} P_{j,t+1}^j}{P_{t+1} \epsilon_{j,t+1} P_{j,t+1}^j} \right] \right] \]

\[ 1 = E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\epsilon} \left[ \frac{P_t \epsilon_{j,t+1} P_{j,t+1}^j}{P_{t+1} \epsilon_{j,t+1} P_{j,t+1}^j} \right] \right] \]

Plugging (1.153)

\[ 1 = E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\epsilon} \frac{Q_{j,t+1}}{Q_{j,t}} \right] \]

\[ 1 = E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\epsilon} \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\epsilon} \frac{Q_{j,t+1}}{Q_{j,t}} \right] \right] \]

\[ (C_t)^{-\epsilon} = E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\epsilon} \left( \frac{Q_{j,t+1}}{Q_{j,t}} \right)^{\frac{1}{\epsilon}} \right] \]

\[ C_t = E_t \left[ \left( \frac{C_{t+1}}{C_t} \right) \left( \frac{Q_{j,t+1}}{Q_{j,t}} \right)^{\frac{1}{\epsilon}} \right] \]

\[ C_t = E_t \left[ \left( \frac{C_{t+1}}{C_t} \right) \left( \frac{Q_{j,t+1}}{Q_{j,t}} \right)^{\frac{1}{\epsilon}} \right] \]

\[ C_t = \phi_j C_t \left( \frac{Q_{j,t}}{Q_{j,t+1}} \right)^{\frac{1}{\epsilon}} \]  

(A.1.26)
Where

\[ \varphi_j = E_t \left[ \frac{C_{t+1}}{C_t} \left( \frac{Q_{j,t+1}}{Q_{j,t}} \right)^{\frac{1}{\sigma}} \right] \]

**Derivation of expressions / equations for 1.5 Firms**

Formal sector price dynamics (inflation)

Formal sector price index can be given as:

\[ P_{HF,t} = \left[ \theta_{HF} \left[ P_{HF,t-1} \right]^{1-\theta_c} + (1 - \theta_{HF}) \left[ P_{HF,t}^* \right]^{1-\theta_c} \right]^{1/(1-\theta_c)} \]

\[ P_{HF,t} \frac{1}{P_{HF,t-1}} = \left[ \theta_{HF} \left[ P_{HF,t-1} \right]^{1-\theta_c} + (1 - \theta_{HF}) \left[ P_{HF,t}^* \right]^{1-\theta_c} \right]^{1/(1-\theta_c)} \frac{1}{P_{HF,t-1}} \]

\[ \left( \frac{P_{HF,t}}{P_{HF,t-1}} \right)^{1-\theta_c} = \left[ \theta_{HF} \left[ P_{HF,t-1} \right]^{1-\theta_c} + (1 - \theta_{HF}) \left[ P_{HF,t}^* \right]^{1-\theta_c} \right] \]

\[ \left( \frac{P_{HF,t}}{P_{HF,t-1}} \right)^{1-\theta_c} = \theta_{HF} + (1 - \theta_{HF}) \left[ \frac{P_{HF,t}^*}{P_{HF,t-1}} \right]^{1-\theta_c} \]

Formal sector inflation is defined as:

\[ \Pi_{HF,t} = \frac{P_{HF,t}}{P_{HF,t-1}} \]

Plugging (A.2.2)
\[(\Pi_{HF,t})^{1-\theta_c} = \left[\theta_{HF} + (1 - \theta_{HF}) \left(\frac{P_{HF,t}^*}{P_{HF,t-1}}\right)^{1-\theta_c}\right]\]

Linearization around steady state makes:

\[
\pi_{HF,t} = (1 - \theta_{HF})(p_{HF,t}^* - p_{HF,t-1}) \tag{A.2.3}
\]

Optimal Price Setting

The stochastic discount factor for nominal payoffs in the period \(t + k\) is \(Q_{t,t+k}\) and can be given as:

\[
Q_{t,t+k} = \beta^k \left(\frac{C_{HF,t+k}}{C_{HF,t}}\right)^{-\varepsilon} \left(\frac{P_{HF,t}}{P_{HF,t+k}}\right) \tag{A.2.4}
\]

The representative firm’s profit maximization problem can be given as:

\[
\text{Max}_{\{p_{HF,t}^*\}} \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[Q_{t,t+k} \left(\frac{Y_{HF,t+k,t}(i)}{Y_{HF,t+k,t}(i)}\right) - \left(\frac{TC_{HF,t+k,t}(i)}{TC_{HF,t+k,t}(i)}\right)\right]\tag{A.2.5}
\]

Subject to

\[
\left(Y_{HF,t+k,t}(i)\right) = \left(\frac{P_{HF,t}}{P_{HF,t+k}}\right)^{-\theta_c} C_{HF,t+k} \tag{A.2.6}
\]

Plugging (A.2.4) and (A.2.6) in (A.2.5) make:
\[
\max_{\{P_{HF,t}\}} \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[ \beta^k \left( \frac{C_{HF,t+k}}{C_{HF,t}} \right)^{-\varphi} \left( \frac{P_{HF,t}}{P_{HF,t+k}} \right) \left( \frac{P_{HF,t}^*}{P_{HF,t+k}} \right)^{-\varphi_c} C_{HF,t+k} \right. \\
\left. - \left( TC_{HF,t+k|t}(i) \right) \left( \frac{P_{HF,t}^*}{P_{HF,t+k}} \right)^{-\varphi_c} C_{HF,t+k} \right] \\
\]

\[
P_{HF,t}^* \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[ \beta^k \left( \frac{C_{HF,t+k}}{C_{HF,t}} \right)^{-\varphi} \left( \frac{P_{HF,t}}{P_{HF,t+k}} \right) \left( 1 - \varphi_c \right) \left( \frac{P_{HF,t}^*}{P_{HF,t+k}} \right)^{-\varphi_c} C_{HF,t+k} \right] \\
+ \left( MC_{HF,t+k|t} \right) \left( \frac{P_{HF,t}^*}{P_{HF,t+k}} \right)^{-\varphi_c} C_{HF,t+k} = 0
\]

By the definition of marginal cost

\[
TC_{HF,t+k|t}(i) = MC_{HF,t+k|t} \tag{A.2.7}
\]

Plugging (A.2.4) and (A.2.6)

\[
\left( Y_{HF,t+k|t}(i) \right) = \left( \frac{P_{HF,t}^*}{P_{HF,t+k}} \right)^{-\varphi_c} C_{HF,t+k}
\]

\[
P_{HF,t} \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[ Q_{t,k} \left( 1 - \varphi_c \right) \left( Y_{HF,t+k|t}(i) \right) \right. \\
\left. + \left( MC_{HF,t+k|t} \right) \left( \frac{P_{HF,t}^*}{P_{HF,t+k}} \right)^{-1} \frac{\varphi_c}{P_{HF,t+k}} \right] = 0
\]

\[
P_{HF,t}^* \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[ Q_{t,k} \left( Y_{HF,t+k|t}(i) \right) \left( 1 - \varphi_c \right) + \left( MC_{HF,t+k|t} \right) \left( \frac{\varphi_c}{P_{HF,t}} \right) \right] = 0
\]

\[
P_{HF,t}^* \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[ Q_{t,k} \left( Y_{HF,t+k|t}(i) \right) \left( P_{HF,t}^* \right) + \left( MC_{HF,t+k|t} \right) \left( \frac{\varphi_c}{P_{HF,t}} \right) \right] = 0
\]
Macroeconomic Goals and Inflation Targeting in India

\[ P_{HF,t}^* = \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left( Q_{t,t+k} \left( Y_{HF,t+k|t} (i) \right) \left( P_{HF,t}^* - \left( M C_{HF,t+k|t}^N \right) \left( \frac{\theta_c}{\theta_c} \right) \right) \right) = 0 \]

\[ P_{HF,t}^* = \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left( Q_{t,t+k} \left( Y_{HF,t+k|t} (i) \right) P_{HF,t}^* \right) \]

\[ = \left( \frac{\theta_c}{\theta_c} - 1 \right) \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left( Q_{t,t+k} \left( Y_{HF,t+k|t} (i) \right) \left( M C_{HF,t+k|t}^N \right) \right) \]

Again plugging (A.2.4) and (A.2.6) and solve for \( P_{HF,t}^* \)

\[ P_{HF,t}^* = \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[ \beta^k \left( C_{HF,t+k} \right) \left( P_{HF,t}^* \right) \left( P_{HF,t+k} \right) \right] \]

\[ = \left( \frac{\theta_c}{\theta_c} - 1 \right) \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[ \beta^k \left( C_{HF,t+k} \right) \left( P_{HF,t}^* \right) \left( P_{HF,t+k} \right) \right] \]

\[ P_{HF,t}^* = \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[ \beta^k \left( C_{HF,t+k} \right) \left( P_{HF,t}^* \right) \right] \]

\[ = \left( \frac{\theta_c}{\theta_c} - 1 \right) \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[ \beta^k \left( C_{HF,t+k} \right) \left( P_{HF,t}^* \right) \right] \]

\[ \left( P_{HF,t}^* \right)^{1-\theta_c} \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[ \beta^k \left( C_{HF,t+k} \right) \left( P_{HF,t}^* \right) \right] \]

\[ = \left( \frac{\theta_c}{\theta_c} - 1 \right) \left( P_{HF,t}^* \right)^{-\theta_c} \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[ \beta^k \left( C_{HF,t+k} \right) \left( P_{HF,t}^* \right) \right] \]

[70]
\[ MC_{HF,t+k|t}^R = \frac{MC_{HF,t+k|t}^N}{P_{HF,t+k}} \]  
\[ \text{(A.2.8)} \]

Plugging (A.2.8)

\[
P_{HF,t}^* \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[ \beta^k (C_{HF,t+k})^{1-\varepsilon} (P_{HF,t+k})^{\vartheta_{c-1}} \right]
= \left( \frac{\vartheta_{c}}{\vartheta_{c} - 1} \right) \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[ \beta^k (C_{HF,t+k})^{1-\varepsilon} (P_{HF,t+k})^{\vartheta_{c}} \left\{ MC_{HF,t+k|t}^R \right\} \right]
\]
\[ \text{(A.2.9)} \]

\[ P_{HF,t}^* = \left( \frac{\vartheta_{c}}{\vartheta_{c} - 1} \right) \frac{E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \left[ \beta^k (C_{HF,t+k})^{1-\varepsilon} (P_{HF,t+k})^{\vartheta_{c}} \left\{ MC_{HF,t+k|t}^R \right\} \right]}{E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \left[ \beta^k (C_{HF,t+k})^{1-\varepsilon} \left( \frac{P_{HF,t+k}}{P_{HF,t}} \right)^{\vartheta_{c-1}} \right]} \]  
\[ \text{(A.2.10)} \]

Divide (A.2.9) by \( P_{HF,t} \) to get the optimal real price as a weighted average of future real marginal cost.

\[
\frac{P_{HF,t}^*}{P_{HF,t}} = \left( \frac{\vartheta_{c}}{\vartheta_{c} - 1} \right) \frac{E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \left[ \beta^k (C_{HF,t+k})^{1-\varepsilon} \left( \frac{P_{HF,t+k}}{P_{HF,t}} \right)^{\vartheta_{c}} \left\{ MC_{HF,t+k|t}^R \right\} \right]}{E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \left[ \beta^k (C_{HF,t+k})^{1-\varepsilon} \left( \frac{P_{HF,t+k}}{P_{HF,t}} \right)^{\vartheta_{c-1}} \right]} \]
\[ \text{(A.2.10)} \]

For the flexible prices \( \theta_{HF} = 0 \). All the firms change their prices in every period. Therefore, solve the problem for one period.

\[
P_{HF,t}^* = \left( \frac{\vartheta_{c}}{\vartheta_{c} - 1} \right) \frac{\beta^0 (C_{HF,t})^{1-\varepsilon} (P_{HF,t})^{\vartheta_{c}} \left\{ MC_{HF,t|t}^R \right\}}{\beta^0 (C_{HF,t})^{1-\varepsilon} (P_{HF,t})^{\vartheta_{c-1}}} \]
\[ \text{[71]} \]
\[ P^*_{HF,t} = \left( \frac{\theta_c}{\partial_c - 1} \right) P_{HF,t} M C_{HF,t|t}^R \]

Plugging (A.2.8)

\[ P^*_{HF,t} = \left( \frac{\theta_c}{\partial_c - 1} \right) M C_{HF,t|t}^N \] (A.2.11)

This is a frictionless markup for formal sector firms.

Log linearization of (A.2.9) around steady state makes:

\[ p^*_{HF,t} = (1 - \theta_{HF} \beta) E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k (m c_{HF,t+k|t}^R - m c_H^R) + p_{HF,t+k} \] (A.2.12)

\[ \mu_{HF} \equiv -m c_H^R \] (A.2.13)

Plugging (A.2.13) in (A.2.12) makes

\[ p^*_{HF,t} = \mu_{HF} + (1 - \theta_{HF} \beta) E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k (m c_{HF,t+k|t}^R + p_{HF,t+k}) \] (A.2.14)

(A.2.13) is a desired markup for the formal sector firms.

**Derivation of expressions / equations for 1.6 Equilibrium Dynamics**

(1.1.21), (1.1.19) and (1.1.17) make the nested demand function as under:

\[ [c_{HF,t}(i)] = \gamma (1 - \alpha) \left( \frac{[P_{HF,t}(i)]}{P_{HF,t}} \right)^{-\theta_c} \left( \frac{P_{HF,t}}{P_t} \right)^{-\theta_b} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_a} C_t \] (A.3.1)

Plugging (A.3.1) in (1.1.7) makes:
Macroeconomic Goals and Inflation Targeting in India

\[ C_{HF,t} = \left[ \int_0^1 \gamma (1 - \alpha) \left( \frac{P_{HF,t}}{P_{H,t}} \right)^{-\theta_c} \left( \frac{P_{HF,t}}{P_t} \right)^{-\theta_b} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_a} C_t \right]^{\frac{\theta_c-1}{\theta_c}} \frac{\theta_c}{\theta_c-1} \, dt \]

\[ C_{HF,t} = \left[ \gamma (1 - \alpha) \left( \frac{P_{HF,t}}{P_{H,t}} \right)^{-\theta_b} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_a} C_t \left( \frac{1}{P_{HF,t}} \right)^{-\theta_c} \right]^{\frac{\theta_c-1}{\theta_c}} \frac{\theta_c}{\theta_c-1} \int_0^1 \left[ P_{HF,t}(i) \right]^{1-\theta_c} \, di \]

(1.1.13) makes:

\[ P_{HF,t}^{1-\theta_c} = \int_0^1 \left[ P_{HF,t}(i) \right]^{1-\theta_c} \, di \]  \hspace{1cm} (A.3.2)

Inserting (A.3.2) in the expression makes:

\[ C_{HF,t} = \left[ \gamma (1 - \alpha) \left( \frac{P_{HF,t}}{P_{H,t}} \right)^{-\theta_b} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_a} C_t \left( \frac{1}{P_{HF,t}} \right)^{-\theta_c} \right]^{\frac{\theta_c-1}{\theta_c}} \frac{\theta_c}{\theta_c-1} \left( P_{HF,t} \right)^{1-\theta_c} \]

\[ C_{HF,t} = \gamma (1 - \alpha) \left( \frac{P_{HF,t}}{P_{H,t}} \right)^{-\theta_b} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_a} C_t \]  \hspace{1cm} (A.3.3)

(1.1.22), (1.1.20) and (1.1.17) make the nested demand function as under:
\[ C_{H,t}(i) = (1 - \gamma)(1 - \alpha) \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\theta_c} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_b} \left( \frac{P_t}{P_{H,t}} \right)^{-\theta_a} C_t \]  

(A.3.4)

Plugging (A.3.4) in (1.1.8)

\[
C_{H,t} = \left[ \int_0^1 \left( (1 - \gamma)(1 - \alpha) \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\theta_c} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_b} \left( \frac{P_t}{P_{H,t}} \right)^{-\theta_a} C_t \right] \frac{\partial_{x^{-1}}}{\partial_{y^{-1}}} \frac{\partial_{c}}{\partial_{c^{-1}}} di \right] 
\]

\[
C_{H,t} = \left[ (1 - \alpha)(1 
- \gamma) \left( \frac{P_{H,t}}{P_{H,t}} \right)^{-\theta_b} \left( \frac{P_t}{P_{H,t}} \right)^{-\theta_a} C_t \left( \frac{1}{P_{H,t}} \right)^{-\theta_c} \frac{\partial_{c^{-1}}}{\partial_{c}} \frac{\partial_{c}}{\partial_{c^{-1}}} \int_0^1 \left[ \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\theta_c} \right] \frac{\partial_{c^{-1}}}{\partial_{c}} \frac{\partial_{c}}{\partial_{c^{-1}}} di \right] 
\]

(1.1.14) makes:

\[
\left( \frac{P_{H,t}}{P_t} \right)^{1-\theta_c} = \int_0^1 \left[ \frac{P_{H,t}(i)}{P_{H,t}} \right]^{1-\theta_c} di 
\]

(A.3.5)

Plugging (A.3.5) in the expression makes:
Demand functions for generic country $j$ analogous to (1.1.23), (1.1.24) and (1.1.18) are given as:

$$[C_{H,t}^j(i)] = \left[ \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\theta_c} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_a} C_t \right]^{-\frac{\phi_c}{\phi_c-1}}$$  \hspace{0.5cm} (A.3.7)

$$C_{H,t}^j = (1-\alpha)(1-\gamma)\left( \frac{P_{H,t}}{P_t} \right)^{-\theta_b} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_a} C_t$$  \hspace{0.5cm} (A.3.6)

$$C_{F,t}^j = \alpha \left( \frac{P_{F,t}^j}{P_t^j} \right)^{-\theta_a} C_t$$  \hspace{0.5cm} (A.3.9)

(A.3.7), (A.3.8) and (A.3.9) make the nested demand function as:

$$[c_{H,t}^j(i)] = \alpha \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\theta_c} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta_d} \left( \frac{P_{F,t}^j}{P_t^j} \right)^{-\theta_a} C_t$$  \hspace{0.5cm} (A.3.10)

(1.1.10), (1.1.9), (1.1.16) and (1.1.15) become for a generic country $j$ as:

$$C_{H,t}^j = \left[ \int_0^1 \left[ C_{H,t}^j(i) \right]^{-\frac{\phi_c-1}{\phi_c}} di \right]^{-\frac{\phi_c}{\phi_c-1}}$$  \hspace{0.5cm} (A.3.11)
\[ C_{F,t}^l = \left[ \int_0^1 \left[ C_{H,t}^l \right]^{\frac{\theta_d - 1}{\theta_d}} \right]^{\frac{\theta_d}{\theta_d - 1}} \]  

(A.3.12)

\[ P_{H,t}^l = \left[ \int_0^1 \left[ P_{H,t}^l(i) \right]^{1-\theta_c} di \right]^{\frac{1}{1-\theta_c}} \]  

(A.3.13)

\[ P_{F,t}^l = \left[ \int_0^1 \left[ P_{H,t}^l(i) \right]^{1-\theta_d} di \right]^{\frac{1}{1-\theta_d}} \]  

(A.3.14)

Plugging (A.3.10) in (A.3.11) makes:

\[
C_{H,t}^l = \left[ \int_0^1 \left[ \alpha \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\theta_c} \left( \frac{P_{H,t}}{\epsilon_{j,t} P_{F,t}} \right)^{-\theta_d} \left( \frac{P_{F,t}}{P_t} \right)^{-\theta_a} \left( \frac{P_{F,t}}{P_t} \right) \right]^{\frac{\theta_c - 1}{\theta_c}} C_t^{j \frac{\theta_c - 1}{\theta_c}} di \right]^{\frac{\theta_d}{\theta_d - 1}}
\]

\[
C_{H,t}^l = \left[ \alpha \left( \frac{P_{H,t}}{\epsilon_{j,t} P_{F,t}} \right)^{-\theta_d} \left( \frac{P_{F,t}}{P_t} \right)^{-\theta_a} \left( \frac{1}{P_t} \right)^{-\theta_c} C_t^{j \frac{\theta_c - 1}{\theta_c}} \right]^{\frac{\theta_d}{\theta_d - 1}} \left[ \int_0^1 \left[ P_{H,t}(i) \right]^{-\theta_c} di \right]^{\frac{\theta_c - 1}{\theta_c}}
\]

\[
C_{H,t}^j = \left[ \alpha \left( \frac{P_{H,t}}{\epsilon_{j,t} P_{F,t}} \right)^{-\theta_d} \left( \frac{P_{F,t}}{P_t} \right)^{-\theta_a} \left( \frac{1}{P_t} \right)^{-\theta_c} C_t^{j \frac{\theta_c - 1}{\theta_c}} \right]^{\frac{\theta_d}{\theta_d - 1}} \left[ \int_0^1 \left[ P_{H,t}(i) \right]^{1-\theta_c} di \right]^{\frac{\theta_c - 1}{\theta_c}}
\]

\[ P_{H,t}, \text{ analogous to (1.1.13), is given as:} \]

\[ P_{H,t} = \left[ \int_0^1 \left[ P_{H,t}(i) \right]^{1-\theta_c} di \right]^{\frac{1}{1-\theta_c}} \]  

(A.3.15)

(A.3.15) makes:
\[ [P_{H,t}]^{1-\varphi_c} = \int_0^1 [P_{H,t}(i)]^{1-\varphi_c} \, di \quad \text{(A.3.16)} \]

Plugging (A.3.16) in the expression makes:

\[
C_{H,t}^j = \alpha \left( \frac{P_{H,t}}{\varepsilon_{t,F,t} P_{F,t}^j} \right)^{-\varphi_d} \left( \frac{P_{F,t}^j}{P_t^j} \right)^{-\varphi_a} C_t^j \left[ P_{H,t} \right]^{1-\varphi_c} \left[ P_{H,t} \right]^{1-\varphi_c} \]

\[
C_{H,t}^j = \alpha \left( \frac{P_{H,t}}{\varepsilon_{t,F,t} P_{F,t}^j} \right)^{-\varphi_d} \left( \frac{P_{F,t}^j}{P_t^j} \right)^{-\varphi_a} C_t^j \quad \text{(A.3.17)}
\]