INTRODUCTION

The study of the problems of vibration of different elastic bodies is of great interest to the scientists and engineers from the theoretical point of view as well as on account of their uses in different mechanical devices. Owing to the importance of the subject a few problems of vibration of practical interest have been studied in this thesis.

The subject matter of the thesis is divided into six chapters. A brief report of the problems discussed in this thesis is given in the following paragraphs.

The first chapter contains four problems regarding transverse vibrations of rotating elastic bodies composed of homogeneous materials. Such type of vibrations can be seen in turbine and helicopter blades, propellers and more recently in the rods used as antenna elements of artificial communication satellites, etc. The first problem of this chapter is concerned with the transverse vibration of a homogeneous solid thin rod in the form of a pyramid with square base. The second problem deals with the transverse vibration of some thin solid rods of revolution formed by revolving an area in the form of a parabola of fourth order, second order and a semi-cubical parabola about their axes. In the third problem we have discussed the transverse vibration of an isotropic beam of variable cross-section. In the first two problems, the rods considered were thin in the sense that their lateral dimensions were small compared to their lengths but the rod involved in the third problem was not so thin. Rather its lateral dimensions were comparable to its length. For this reason, in this problem we had to take into account the effects of rotatory
inertia and shearing forces on the rotating rod. Taking these effects into consideration Huang (1959) also discussed the problem of transverse vibration of a non-rotating isotropic prismatic bar. Here we have employed the approximate method of Ritz and Galerkin in solving out the problem. In all the above three problems each of the rods considered is assumed to be rotating steadily about an axis passing through one end and perpendicular to the length of the rod. In the fourth problem we have discussed the transverse vibration of a semi-circular disc rotating uniformly about its diameter. Applying energy method an approximate result for the gravest mode of vibration is obtained. The applications of such type of vibrations can be experienced in such as turbines, gyroscopes and high speed cameras. More recently, rotating discs have been suggested for use on devices that are planned for soft landings of space-crafts. On impact such devices would convert the kinetic energy of translation of the vehicle to that of rotation of discs, thereby providing a "cushoning effect" for soft landings. In attempt to explain the failures of the turbine discs, the landing vibrations of circular discs rotating uniformly about its normal axis have been studied by Lamb and Southwell (1921).

There are many problems in which the differential equations involved do not yield exact solutions. Consequently, we do not get exact values of the fundamental frequency of vibration and have to satisfy ourselves by getting a fairly approximate value which can serve our purpose to a great extent. To get such an approximate value of the fundamental frequency of vibration we have employed in the first three problems of this chapter the well-known Rayleigh-Ritz method and that of Southwell. For the sake of convenience a brief description of the methods is given below.
By Rayleigh-Ritz method we get an upper limit of the fundamental frequency of vibration of an elastic solid body. Southwell (1936) deduced a very convenient method which helps us to determine easily the lower limit of the same frequency mentioned above. By taking the mean of these two limits we get a fairly approximate value of the fundamental frequency of vibration. (Here lies the advantage of the procedure of obtaining a nearly correct value of the fundamental frequency which is very helpful to the designers of mechanical devices).

In the second chapter there are only two problems dealing with the transverse vibrations of plates of moderate thickness. In the first problem, the plate considered is an isotropic circular one and of variable density. The law of variation being given to be a linear function of the distance from the centre of the plate. A series solution of the differential equation of motion of the plate is obtained from which we get, by applying boundary conditions, an approximate value of the fundamental frequency of vibration. In the second problem the plate considered is an orthotropic rectangular one with a concentrated mass placed on it. Using Dirac's delta function and also Fourier Finite Sine Transform the angular frequencies including the gravest one are obtained. A numerical result is obtained for the gravest mode of vibration for the case of a square plate.

The third chapter consists of two problems dealing with the transverse vibrations of plates resting on elastic foundations. In the first problem the plates considered are anisotropic and they are of (i) rectangular and (ii) elliptic boundaries. In the first case, an exact solution is obtained and in the second case an approximate solution is found by using Galerkin's method. The second problem ... of this chapter.
deals with the transverse vibration of an isotropic circular plate resting on elastic foundation. An exact solution is obtained for the above plate with the boundary condition that the plate is clamped at the rim. The foundation model on which the plate is assumed to be resting is of Pasternak-type. In this type of foundation models, Pasternak assumed the existence of shear interactions between the spring elements. The shear interactions were accomplished by connecting the ends of the springs to a beam or plate consisting of incompressible vertical elements which deform only by transverse shear. For this reason, this type of models is considered as a natural development to that of Winkler type.

The fourth chapter contains two problems dealing with the longitudinal vibrations of rods of variable cross-sections composed of homogeneous materials. In the first problem the rod considered is a spindl shaped thin isotropic solid body which is formed by the revolution of an area bounded by a cosine curve about the axis of the rod. Such an assumption for the shape of the rod is made, for it gives an elegantly simple result of the governing differential equation. An exact solution is obtained and some numerical values for the gravest mode of vibration are calculated for different boundary conditions. The rods involved in the second problem are non-homogeneous. The cross-section and the co-efficient of elasticity are both assumed to vary as powers of binomial expressions. The indices in these two variable quantities are not necessarily the same. Exact solutions are obtained corresponding to different values of the indices of the binomial expressions of the variables mentioned above. Vodicka (1963) discussed the problem of longitudinal vibration of isotropic rods of variable cross-sections.
In this paper, the cross-sections of the rods were assumed to vary as powers of binomial expressions but the elastic modulus was supposed to be constant.

The fifth chapter also contains two problems which deal with the torsional vibrations of thin solid rods. The solid body involved in the first problem of this chapter is an isotropic bar with a metallic circular disc placed at the middle of the bar. This problem has been solved by using Dirac's delta function and Laplace's Transformation Technique. Some numerical values of the gravest frequency of vibration are calculated. The second problem of this chapter is an attempt to consider the torsional vibration of a non-homogeneous uniform rod. The coefficient of elasticity is assumed to vary linearly as the distance from one end of the rod. The problem has been solved by using Perturbation Technique. Some numerical results are calculated showing the ratios of the frequency of vibration corresponding to perturbed cases to those of unperturbed cases.

To all these problems of elastic solids two problems of membranes have been added and discussed in the last chapter.

The sixth or the last chapter contains two problems dealing with the transverse vibrations of circular membranes of variable density. Membranes are widely used in musical instruments as well as in some other appliances meant for producing and receiving of sounds. In the first problem of this chapter the membrane considered is a composite one consisting of two concentric regions. The inner circular portion is composed of homogeneous material but the outer annulus region is composed of material whose density varies inversely as the square of the distance from the centre. Exact solutions are obtained for both the regions and applying the boundary conditions and maintaining the continuity at the common boundary an approximate numerical result
is obtained for the fundamental frequency. Vodicka (1962) investigated the problem of transverse vibration of a composite circular membrane. In his paper he considered the membrane to be composed of a number of piece-wise homogeneous concentric annulus regions. In the second problem of this chapter the law of variation of density of the material of the circular membrane is assumed to be \( e^{-\epsilon r} \), where \( r \) is the distance from the centre and \( \epsilon \) is a small constant. This problem has been solved by Perturbation Technique and an approximate result for the fundamental frequency is obtained. A similar problem has been solved by the same method jointly by Sen Gupta and Ghosh (1964). In their problem the law of variation of the density of the material of the membrane has been taken to be \( e^{-\epsilon r^2} \), \( r \) denoting the distance from the centre.