Chapter 7

Concluding remarks

Graph theory becomes the leading choice of research in mathematics subject for many researchers in these days. The credit of such popularity to research in this area goes to the rapid growth and the applications of graph theory in many directions since last several decades. Graph theory has a great potential for applications in discrete mathematics, theoretical computer science, engineering sciences etc.

The results written in the thesis are based on the parameters related to the edges of the graph. Various graph operations like vertex removal, edge removal and edge addition are considered. In particularly, the effects of these operations on the edge parameters of the graph are observed.

In chapter 2, edge domination of the graph is considered and the effects of vertex removal, edge removal and edge addition operations on the edge domination number are observed. Further there is a possibility to find the necessary and sufficient condition while applying the graph operations vertex removal, edge removal/addition on the graph and the edge domination number does not change.

In chapter 2, it is proved that the weak edge dominating set of the graph is an edge dominating set of the corresponding dual hypergraph of the graph. The further scope of research is to find the relation between the minimal weak edge dominating set of the graph and the minimal dominating set of the corresponding dual hypergraph of the graph also the relation between weak edge domination number of the graph and the domination number of the corresponding dual hypergraph.
In chapter 3, the definition of edge H-dominating set arises a hope to define a stronger concept of edge H-domination by considering the edges incident to both end vertices of an edge.

Graph theory plays an important role in computer applications for the development of graph algorithms. There are many algorithms used to solve problems that are modeled in the form of graphs. Some of the algorithms are as follows.

- Algorithms to find the shortest path in a network.
- Algorithms to find the number of dominating sets in a graph and point out the dominating set(s) with minimum cardinality.
- Algorithms to find a minimum spanning tree and so on.

The algorithm is written to find an edge H-dominating set of the graph in chapter 3. A question arises that ‘Is it possible to find a minimum edge H-dominating set by an algorithm with less number of iterations?’ To write an algorithm which gives a weak edge dominating set of the graph is an interesting problem also how to find a minimum weak edge dominating set of the graph by an algorithm is a further scope of research. A new concept called edge stability of the graph is defined in chapter 6. An open problem from chapter 6 is to develop an algorithm which gives an edge stable set of the graph also it point out the maximum edge stable set.

In chapter 4, the edge cover of the graph is considered and a strong edge cover is defined. We have calculated the number of minimum edge covers of such graph in which the edge covering number decreases while removal of each vertex from the graph. The further scope of research is to find some method by which the number of all minimum edge covers of any graph are obtained also the number of all minimum edge dominating sets of any graph are obtained. Further a possibility is to find the necessary and sufficient condition for ‘Edge addition/removal and the edge covering number does not change’. We have defined the total transversal of the graph and observed the effect of vertex removal operation on the total transversal number of the graph [23]. The total transversal is a vertex cover of the graph such that the subgraph induced by its vertices does not have isolated vertices. It is interesting to consider an edge dominating set without isolated
edges (total edge dominating set), edge cover without isolated edges (total edge cover) and so on. Also to observe the effects of removal of a vertex(edge) on the corresponding total edge parameters are further scope of research.

In chapter 5, it is interesting to consider vertex removal operation on the graph. To observe the effects of this operation on the edge independence number and the strong edge independence number of the graph are further scope of research from this chapter.

We have considered the secure domination concept on the set of vertices of the graph and introduced a new concept called secure independence in graph [25]. It is interesting to define a secure edge set as ’an edge set $F$ with property $p$ is a secure edge set if interchanging an edge of $F$ with the neighbor edge from outside of $F$ doesn’t break property $p$’ and so on. Also consider the cardinality of extreme secure edge set called secure edge number of the graph. The effects of vertex removal and edge removal operations on the secure edge number of the graph are further scope of research.