Chapter 4

7-cordial Labeling

4.1 Introduction

The previous chapter was devoted to the 5-cordial labeling of some standard graphs and operations on graphs. Here we consider $A =< Z_7, +_7 >$, that is additive group of modulo 7. This chapter is aimed to discuss 7-cordiality theme. We contribute nine new results in the context of 7-cordial labeling.

4.2 Some Existing Results

Youssef{19} proved

- The complete graph K_n is 4-cordial $\Longleftrightarrow n \leq 6$.
- The complete bipartite graph $K_{m,n}$ is 4-cordial $\Longleftrightarrow m \; or \; n \not\equiv 2(mod\, 4)$.
- The graph C_n^2 is 4-cordial $\Longleftrightarrow n \not\equiv 2(mod\, 4)$.

4.3 7-cordial Labeling of Some Standard Graphs

Theorem 4.3.1 *The fan f_n is 7-cordial.*

Proof: Let $G = f_n$ be the fan. Let $v_1, v_2, ..., v_n$ be the path vertices of f_n and v_0 be the apex vertex. We note that $|V(G)| = n + 1$ and $|E(G)| = 2n - 1$.

To define 7-cordial labeling $f : V(G) \rightarrow< Z_7, +_7 >$ we consider the following cases.

Case 1: $n \equiv 0, 1, 2, 6$(mod 7)

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 0$(mod 7);

$f(v_i) = 1; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 3 (\text{mod } 7); \quad 1 \leq i \leq n.$

<u>Case 2</u>: $n \equiv 3 (\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 0 (\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 3 (\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 6;$
$f(v_n) = 4.$

<u>Case 3</u>: $n \equiv 4 (\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 0 (\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 3 (\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 5;$
$f(v_{n-1}) = 1;$
$f(v_n) = 3.$

<u>Case 4</u>: $n \equiv 5 (\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 0 (\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 5 (\text{mod } 7);$

$f(v_i) = 4; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 3(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 1;$
$f(v_{n-1}) = 6;$
$f(v_n) = 3.$

The labeling pattern illustrated here encompasses all possible layout's of vertices. The graph under scrutiny within each possibility conforms to the vertex conditions and edge conditions for 7-cordial labeling as presented in *Table 4.3.1.* In other words, the fan f_n admits 7-cordial labeling.

Where $n = 7a + b$, $a, b \epsilon N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) + 1 = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
1	$v_f(0) = v_f(1) + 1 = v_f(2) = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) + 1 = e_f(6) + 1$
2	$v_f(0) = v_f(1) + 1 = v_f(2) = v_f(3) + 1$ $= v_f(4) = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3) + 1$ $= e_f(4) = e_f(5) + 1 = e_f(6) + 1$
3	$v_f(0) = v_f(1) + 1 = v_f(2) = v_f(3) + 1$ $= v_f(4) = v_f(5) + 1 = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) + 1 = e_f(6)$
4	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) + 1 = v_f(5) = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
5	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) + 1 = v_f(6)$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) + 1 = e_f(6)$
6	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3)$ $= e_f(4) = e_f(5) + 1 = e_f(6)$

Table 4.3.1: 7-cordial labeling of fan f_n.

Illustratoin 4.3.1 *The fan f_9 and its 7-cordial labeling is shown in* Figure 4.3.1.

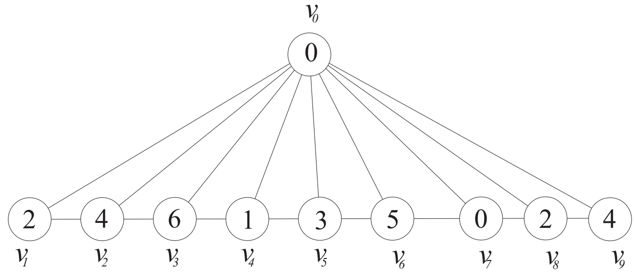

Figure 4.3.1: 7-cordial labeling of the fan f_9.

Theorem 4.3.2 *The friendship graph F_n is 7-cordial.*

Proof: Let $G = F_n$ be the friendship graph. Let $v_1, v_2, ..., v_{2n}$ be partition vertices of n triangles consecutively of F_n and v_0 be the central vertex. We note that $|V(G)| = 2n+1$ and $|E(G)| = 3n$.

To define 7-cordial labeling $f : V(G) \rightarrow < Z_7, +_7 >$ we consider the following cases.

Case 1: $n \equiv 0, 1, 4 (\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq 2n.$

Case 2: $n \equiv 2(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq 2n-1$
$f(v_{2n}) = 2.$

Case 3: $n \equiv 3(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq 2n-3$

$f(v_{2n-2}) = 2;$
$f(v_{2n-1}) = 4;$
$f(v_{2n}) = 6.$

<u>Case 4</u>: $n \equiv 5(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq 2n-1$
$f(v_{2n}) = 6.$

<u>Case 5</u>: $n \equiv 6(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq 2n-2$
$f(v_{2n-1}) = 2;$
$f(v_{2n}) = 4.$

The labeling pattern illustrated here encompasses all possible layout's of vertices. The graph under scrutiny within each possibility conforms to the vertex conditions and edge conditions for 7-cordial labeling as presented in *Table 4.3.2*. In other words, the friendship graph F_n admits 7-cordial labeling.

Where $n = 7a + b$, $a, b \epsilon N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
1	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3)$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3)$ $= e_f(4) = e_f(5) + 1 = e_f(6) + 1$
2	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) + 1 = v_f(5) = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6) + 1$
3	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) = v_f(6)$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) + 1 = e_f(3)$ $= e_f(4) = e_f(5) + 1 = e_f(6) + 1$
4	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) + 1 = e_f(3) + 1$ $= e_f(4) = e_f(5) = e_f(6)$
5	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3)$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6)$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4) + 1 = e_f(5) + 1 = e_f(6)$
6	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3) + 1$ $= e_f(4) = e_f(5) = e_f(6)$

Table 4.3.2: 7-cordial labeling of friendship graph F_n.

Illustratoin 4.3.2 *The friendship graph F_7 and its 7-cordial labeling is shown in* Figure 4.3.2.

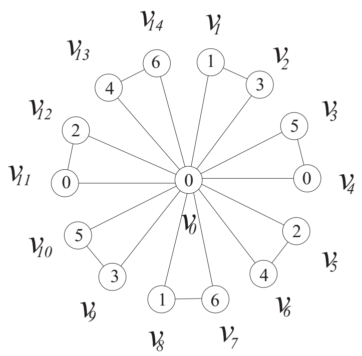

Figure 4.3.2: 7-cordial labeling of the friendship graph F_7.

Theorem 4.3.3 *The ladder L_n is 7-cordial.*

Proof: Let $G = L_n$ be the ladder. Let $v_1, v_2, ..., v_n$ be vertices of one path P_n and $v_1^{'}, v_2^{'}, ..., v_n^{'}$ are vertices of another path $p_n^{'}$. We note that $|V(G)| = 2n$ and $|E(G)| = 3n - 2$.

To define 7-cordial labeling $f : V(G) \rightarrow < Z_7, +_7 >$ we consider the following cases.

Case 1: $n \equiv 0, 1, 2, 6 (\text{mod } 7)$

$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7) \quad 1 \leq i \leq n.$

$f(v_i^{'}) = 0; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 0(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 4(\text{mod } 7); \quad 1 \leq i \leq n.$

Case 2: $n \equiv 3(\text{mod } 7)$

$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 6;$
$f(v_n) = 3.$

$f(v_i^{'}) = 0; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 0(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 4(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}^{'}) = 4;$
$f(v_n^{'}) = 2.$

Case 3: $n \equiv 4(\text{mod } 7)$

$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 2;$
$f(v_{n-1}) = 5;$
$f(v_n) = 0.$

$f(v_i^{'}) = 0; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 0(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 4(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}^{'}) = 4;$
$f(v_{n-1}^{'}) = 3;$
$f(v_n^{'}) = 6.$

Case 4: $n \equiv 5(\text{mod } 7)$

$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n) = 3.$

$f(v_i^{'}) = 0; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 0(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 4(\text{mod } 7); \quad 1 \leq i \leq n.$

The labeling pattern illustrated here encompasses all possible layout's of vertices. The graph under scrutiny within each possibility conforms to the vertex conditions and edge conditions for 7-cordial labeling as presented in *Table 4.3.3*. In other words, the ladder L_n admits 7-cordial labeling.

Where $n = 7a + b$, $a, b \epsilon N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) + 1 = e_f(6)$
1	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) + 1 = e_f(6) + 1$
2	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3) + 1$ $= e_f(4) = e_f(5) = e_f(6) + 1$
3	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) + 1 = v_f(6)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
4	$v_f(0) = v_f(1) + 1 = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) + 1 = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) + 1 = e_f(6)$
5	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3)$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
6	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) + 1 = e_f(6)$

Table 4.3.3: 7-cordial labeling of ladder L_n.

Illustratoin 4.3.3 *The ladder L_9 and its 7-cordial labeling is shown in* Figure 4.3.3.

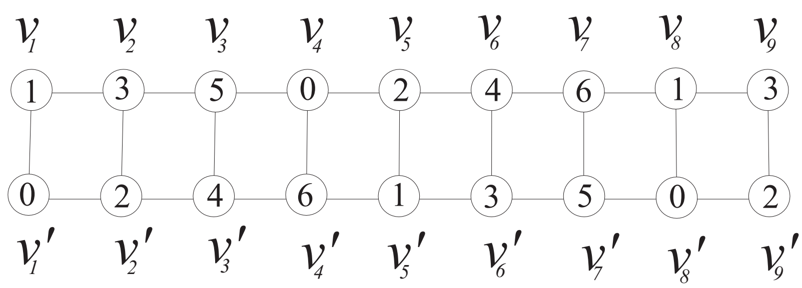

Figure 4.3.3: 7-cordial labeling of the ladder L_9.

Theorem 4.3.4 *The double fan Df_n is 7-cordial.*

Proof: Let $G = Df_n$ be the double fan. Let $v_1, v_2, ..., v_n$ be the path vertices of Df_n and v_0 and $v_0^{'}$ be two apex vertices. We note that $|V(G)| = n + 2$ and $|E(G)| = 3n - 1$.

To define 7-cordial labeling $f : V(G) \rightarrow < Z_7, +_7 >$ we consider the following cases.

Case 1: $n \equiv 0, 1, 2, 6 (\text{mod } 7)$

$f(v_0) = 0;$
$f(v_0^{'}) = 6;$
$f(v_i) = 0; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3 (\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n.$

Case 2: $n \equiv 3 (\text{mod } 7)$

$f(v_0) = 0;$
$f(v_0^{'}) = 6;$
$f(v_i) = 0; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3 (\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 5;$
$f(v_n) = 3.$

Case 3: $n \equiv 4 (\text{mod } 7)$

$f(v_0) = 0;$
$f(v_0^{'}) = 6;$
$f(v_i) = 0; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3 (\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 5;$
$f(v_{n-1}) = 3;$
$f(v_n) = 4.$

<u>Case 4</u>: $n \equiv 5 (\text{mod } 7)$

$f(v_0) = 0;$
$f(v_0^{'}) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-11$
$f(v_{n-10}) = 6;$
$f(v_{n-9}) = 2;$
$f(v_{n-8}) = 5;$
$f(v_{n-7}) = 4;$
$f(v_{n-6}) = 3;$
$f(v_{n-5}) = 1;$
$f(v_{n-4}) = 5;$
$f(v_{n-3}) = 2;$
$f(v_{n-2}) = 3;$
$f(v_{n-1}) = 4;$
$f(v_n) = 6.$

The labeling pattern illustrated here encompasses all possible layout's of vertices. The graph under scrutiny within each possibility conforms to the vertex conditions and edge conditions for 7-cordial labeling as presented in *Table 4.3.4*. In other words, the double fan Df_n admits 7-cordial labeling.

Where $n = 7a + b$, $a, b \epsilon N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
1	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6)$	$e_f(0) = e_f(1) = e_f(2) + 1 = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) + 1 = e_f(6) + 1$
2	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3)$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) + 1 = e_f(6) + 1$
3	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3)$ $= v_f(4) + 1 = v_f(5) = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) + 1 = e_f(6) + 1$
4	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3)$ $= v_f(4) = v_f(5) = v_f(6)$	$e_f(0) = e_f(1) = e_f(2) + 1 = e_f(3)$ $= e_f(4) = e_f(5) + 1 = e_f(6) + 1$
5	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) = v_f(6)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
6	$v_f(0) = v_f(1) + 1 = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3) + 1$ $= e_f(4) = e_f(5) + 1 = e_f(6) + 1$

Table 4.3.4: 7-cordial labeling of double fan Df_n.

Illustratoin 4.3.4 *The double fan Df_{12} and its 7-cordial labeling is shown in* Figure 4.3.4.

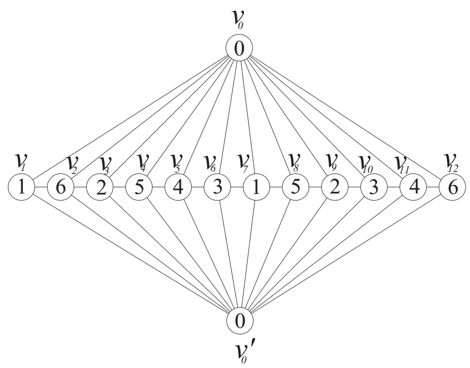

Figure 4.3.4: 7-cordial labeling of the double fan Df_{12}.

Theorem 4.3.5 *The wheel W_n is 7-cordial.*

Proof: Let $G = W_n$ be the wheel. Let $v_1, v_2, ..., v_n$ be the rim vertices of the wheel W_n and v_0 be the apex vertex. We note that $|V(G)| = n + 1$ and $|E(G)| = 2n$.

To define 7-cordial labeling $f : V(G) \rightarrow < Z_7, +_7 >$ we consider the following cases.

<u>Case 1</u>: $n \equiv 0, 1 (\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$

$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n.$

Case 2: $n \equiv 2, 5(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 6;$
$f(v_n) = 1.$

Case 3: $n \equiv 3, 5(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 2;$
$f(v_n) = 4.$

Case 4: $n \equiv 4(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$

$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 2;$
$f(v_n) = 5.$

<u>Case 5</u>: $n \equiv 6(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 2;$
$f(v_{n-1}) = 4;$
$f(v_n) = 6.$

The labeling pattern illustrated here encompasses all possible layout's of vertices. The graph under scrutiny within each possibility conforms to the vertex conditions and edge conditions for 7-cordial labeling as presented in *Table 4.3.5*. In other words, the wheel W_n admits 7-cordial labeling.

Where $n = 7a + b$, $a, b \epsilon N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
1	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) + 1 = e_f(6) + 1$
2	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) = e_f(6)$
3	$v_f(0) = v_f(1) = v_f(2) = v_f(3) + 1$ $= v_f(4) = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
4	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) + 1 = v_f(5) = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4) + 1 = e_f(5) = e_f(6) + 1$
5	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3) + 1$ $= e_f(4) = e_f(5) = e_f(6) + 1$
6	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) = v_f(6)$	$e_f(0) = e_f(1) = e_f(2) + 1 = e_f(3)$ $= e_f(4) = e_f(5) + 1 = e_f(6)$

Table 4.3.5: 7-cordial labeling of wheel W_n.

Illustratoin 4.3.5 *The wheel W_{11} and its 7-cordial labeling is shown in* Figure 4.3.5.

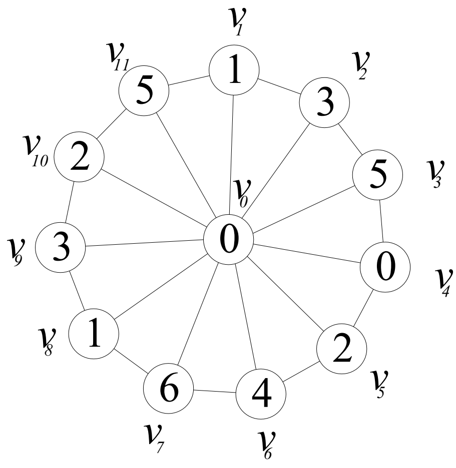

Figure 4.3.5: 7-cordial labeling of the wheel W_{11}.

Theorem 4.3.6 *The double wheel DW_n is 7-cordial.*

Proof: Let $G = DW_n$ be the double wheel. Let v_0 be the central vertex.

Let $v_1, v_2, ..., v_n$ be inner rim vertices of DW_n and $v_1^{'}, v_2^{'}, ...v_n^{'}$ be outer rim vertices of DW_n. We note that $|V(G)| = 2n + 1$ and $|E(G)| = 4n$.

To define 7-cordial labeling $f : V(G) \rightarrow < Z_7, +_7 >$ we consider the following cases.

Case 1: $n \equiv 0(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$

$f(v_i^{'}) = 6; \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n.$

<u>Case 2</u>: $n \equiv 1 (\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3 (\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n.$

$f(v_i^{'}) = 0; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3 (\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n^{'}) = 6.$

<u>Case 3</u>: $n \equiv 2 (\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3 (\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n) = 6.$

$f(v_i^{'}) = 0; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3 (\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n-2$

$f(v'_{n-1}) = 3;$
$f(v'_n) = 4.$

<u>Case 4</u>: $n \equiv 3 (\text{mod } 7)$

$f(v_0) = 6;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 0;$
$f(v_{n-1}) = 5;$
$f(v_n) = 2.$

$f(v'_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v'_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v'_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v'_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v'_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v'_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v'_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v'_n) = 4.$

<u>Case 5</u>: $n \equiv 4 (\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 2;$
$f(v_n) = 5.$

$f(v'_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v'_i) = 1; \quad i \equiv 1(\text{mod } 7);$

$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}^{'}) = 6;$
$f(v_{n-1}^{'}) = 3;$
$f(v_n^{'}) = 4.$

Case 6: $n \equiv 5(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 2;$
$f(v_{n-1}) = 5;$
$f(v_n) = 6.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-4$
$f(v_{n-3}^{'}) = 2;$
$f(v_{n-2}^{'}) = 4;$
$f(v_{n-1}^{'}) = 3;$
$f(v_n^{'}) = 5.$

Case 7: $n \equiv 6(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$

$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 2;$
$f(v_{n-1}) = 4;$
$f(v_n) = 6.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n.$

The labeling pattern illustrated here encompasses all possible layout's of vertices. The graph under scrutiny within each possibility conforms to the vertex conditions and edge conditions for 7-cordial labeling as presented in *Table 4.3.6*. In other words, the double wheel DW_n admits 7-cordial labeling.

Where $n = 7a + b$, $a, b \epsilon N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
1	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) = e_f(6)$
2	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3)$ $= v_f(4) = v_f(5) + 1 = v_f(6)$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1 = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) + 1 = e_f(6) + 1$
3	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) = v_f(6)$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1 = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
4	$v_f(0) + 1 = v_f(1) = v_f(2) + 1 = v_f(3)$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1 = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) = e_f(6) + 1$
5	$v_f(0) + 1 = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) + 1 = v_f(5) = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) + 1 = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
6	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3) + 1$ $= e_f(4) = e_f(5) = e_f(6) + 1$

Table 4.3.6: 7-cordial labeling of double wheel DW_n.

Illustratoin 4.3.6 *The double wheel DW_{10} and its 7-cordial labeling is shown in* Figure 4.3.6.

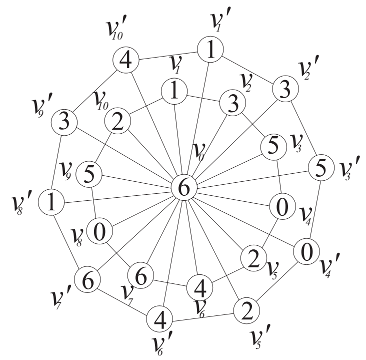

Figure 4.3.6: 7-cordial labeling of the double wheel DW_{10}.

Theorem 4.3.7 *The helm H_n is 7-cordial.*

Proof: Let $G = H_n$ be the helm. Let v_0 be the apex vertex. Let $v_1, v_2, ..., v_n$ be rim vertices and $v_1', v_2', ...v_n'$ be pendant vertices of H_n. We note that $|V(G)| = 2n + 1$ and $|E(G)| = 3n$.

To define 7-cordial labeling $f : V(G) \rightarrow < Z_7, +_7 >$ we consider the following cases.

<u>Case 1</u>: $n \equiv 0(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n.$

$f(v_i') = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i') = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i') = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i') = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i') = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i') = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i') = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n.$

Case 2: $n \equiv 1(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7).$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7). \quad 1 \leq i \leq n.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n^{'}) = 2.$

Case 3: $n \equiv 2(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 6;$
$f(v_n) = 1.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}^{'}) = 5;$

$f(v_n^{'}) = 2.$

Case 4: $n \equiv 3 (\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}^{'}) = 6;$
$f(v_{n-1}^{'}) = 2;$
$f(v_n^{'}) = 4.$

Case 5: $n \equiv 4 (\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n) = 4.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$

$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}^{'}) = 2;$
$f(v_n^{'}) = 6.$

Case 6: $n \equiv 5(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 2;$
$f(v_n) = 4.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}^{'}) = 2;$
$f(v_{n-1}^{'}) = 5;$
$f(v_n^{'}) = 6.$

Case 7: $n \equiv 6(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 2;$
$f(v_{n-1}) = 4;$

$f(v_n) = 6.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}^{'}) = 2;$
$f(v_{n-1}^{'}) = 4;$
$f(v_n^{'}) = 6.$

The labeling pattern illustrated here encompasses all possible layout's of vertices. The graph under scrutiny within each possibility conforms to the vertex conditions and edge conditions for 7-cordial labeling as presented in *Table 4.3.7* . In other words, the helm H_n admits 7-cordial labeling.

Where $n = 7a + b$, $a, b \epsilon N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
1	$v_f(0) = v_f(1) = v_f(2) = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) + 1 = e_f(5) + 1 = e_f(6) + 1$
2	$v_f(0) = v_f(1) = v_f(2) = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
3	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) = e_f(6) + 1$
4	$v_f(0) + 1 = v_f(1) = v_f(2) + 1 = v_f(3)$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6) + 1$
5	$v_f(0) + 1 = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) + 1 = v_f(5) = v_f(6) + 1$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1 = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) + 1 = e_f(6) + 1$
6	$v_f(0) + 1 = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3)$ $= e_f(4) = e_f(5) + 1 = e_f(6)$

Table 4.3.7: 7-cordial labeling of helm H_n

Illustratoin 4.3.7 *The helm graph H_{12} and its 7-cordial labeling is shown in* Figure 4.3.7.

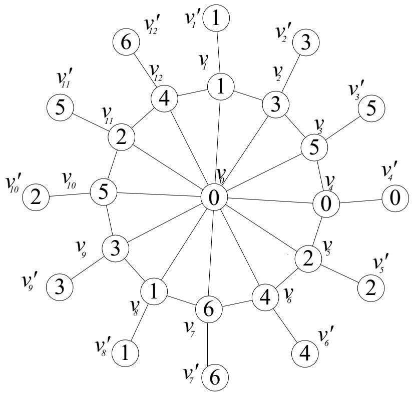

Figure 4.3.7: 7-cordial labeling of the helm H_{12}.

Theorem 4.3.8 *The closed helm CH_n is 7-cordial.*

Proof: Let $G = CH_n$ be the closed helm. Let v_0 be the central vertex.

Let $v_1, v_2, ..., v_n$ be rim vertices and $v_1^{'}, v_2^{'}, ..., v_n^{'}$ be pendant vertices of CH_n by joining them one can obtain a closed helm. We note that $|V(G)| = 2n+1$ and $|E(G)| = 4n$.

To define 7-cordial labeling $f : V(G) \to < Z_7, +_7 >$ we consider the following cases.

<u>Case 1</u>: $n \equiv 0 (\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$

$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n.$

<u>Case 2</u>: $n \equiv 1(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n^{'}) = 6.$

<u>Case 3</u>: $n \equiv 2(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n) = 6.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$

$f(v'_{n-1}) = 3;$
$f(v'_n) = 2.$

Case 4: $n \equiv 3(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 2;$
$f(v_n) = 5.$

$f(v'_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v'_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v'_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v'_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v'_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v'_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v'_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v'_{n-2}) = 3;$
$f(v'_{n-1}) = 4;$
$f(v'_n) = 6.$

Case 5: $n \equiv 4(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 2;$
$f(v_n) = 6.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}^{'}) = 5;$
$f(v_n^{'}) = 4.$

Case 6: $n \equiv 5(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 2;$
$f(v_n) = 4.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n^{'}) = 6.$

Case 7: $n \equiv 6(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$

$f(v_i) = 6; \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 2;$
$f(v_{n-1}) = 4;$
$f(v_n) = 6.$

$f(v_i^{'}) = 0; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3 (\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}^{'}) = 4;$
$f(v_n^{'}) = 6.$

The labeling pattern illustrated here encompasses all possible layout's of vertices. The graph under scrutiny within each possibility conforms to the vertex conditions and edge conditions for 7-cordial labeling as presented in *Table 4.3.8.* In other words, the closed helm CH_n admits 7-cordial labeling.

Where $n = 7a + b$, $a, b \epsilon N \cup \{0\}$.

b	**Vertex conditions**	**Edge conditions**
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
1	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) = e_f(6) + 1$
2	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) + 1 = e_f(6) + 1$
3	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) = v_f(6)$	$e_f(0) = e_f(1) + 1 = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) + 1 = e_f(6)$
4	$v_f(0) + 1 = v_f(1) = v_f(2) + 1 = v_f(3)$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) + 1 = e_f(6) + 1$
5	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3)$ $= v_f(4) + 1 = v_f(5) = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
6	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3)$ $= v_f(4) = v_f(5) = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3)$ $= e_f(4) = e_f(5) + 1 = e_f(6) + 1$

Table 4.3.8: 7-cordial labeling of closed helm CH_n

Illustratoin 4.3.8 *The closed helm CH_{13} and its 7-cordial labeling is shown in* Figure 4.3.8.

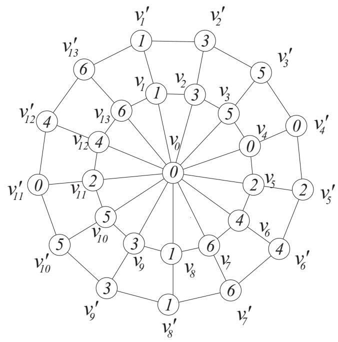

Figure 4.3.8: 7-cordial labeling of the closed helm CH_{13}.

Theorem 4.3.9 *The web graph $W(2,n)$ is 7-cordial.*

Proof: Let $G = W(2,n)$ be the web graph. Let v_0 be the central vertex. Let $v_1, v_2, ..., v_n$ be rim vertices and $v_1^{'}, v_2^{'}, ..., v_n^{'}$ be pendant vertices of $W(2,n)$ which are to be joined to form an outer cycle.

Let $v_1^{''}, v_2^{''}, ..., v_n^{''}$ be pendant vertices to obtain pendant edge to each vertex of the outer cycle. We note that $|V(G)| = 3n+1$ and $|E(G)| = 5n$.

To define 7-cordial labeling $f : V(G) \rightarrow < Z_7, +_7 >$ we consider the following cases.

Case 1: $n \equiv 0(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \le i \le n.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$

$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n.$

$f(v_i^{''}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{''}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{''}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{''}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{''}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{''}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{''}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n.$

Case 2: $n \equiv 1(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n^{'}) = 6.$

$f(v_i^{''}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{''}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{''}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{''}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{''}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{''}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{''}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n^{''}) = 4.$

Case 3: $n \equiv 2(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 3;$
$f(v_n) = 4.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n^{'}) = 6.$

$f(v_i^{''}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{''}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{''}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{''}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{''}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{''}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{''}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}^{''}) = 5;$
$f(v_n^{''}) = 2.$

<u>Case 4</u>: $n \equiv 3(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 2;$

$f(v_n) = 5.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}^{'}) = 3;$
$f(v_{n-1}^{'}) = 4;$
$f(v_n^{'}) = 6.$

$f(v_i^{''}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{''}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{''}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{''}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{''}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{''}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{''}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}^{''}) = 5;$
$f(v_{n-1}^{''}) = 1;$
$f(v_n^{''}) = 0.$

Case 5: $n \equiv 4(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 2;$
$f(v_n) = 6.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$

$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n^{'}) = 4.$

$f(v_i^{''}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{''}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{''}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{''}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{''}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{''}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{''}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-4$
$f(v_{n-3}^{''}) = 6;$
$f(v_{n-2}^{''}) = 2;$
$f(v_{n-1}^{''}) = 5;$
$f(v_n^{''}) = 0.$

Case 6: $n \equiv 5(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}^{'}) = 2;$
$f(v_n^{'}) = 4.$

$f(v_i^{''}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{''}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{''}) = 2; \quad i \equiv 5(\text{mod } 7);$

$f(v_i^{''}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{''}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{''}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{''}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-5$
$f(v_{n-4}^{''}) = 6;$
$f(v_{n-3}^{''}) = 2;$
$f(v_{n-2}^{''}) = 6;$
$f(v_{n-1}^{''}) = 5;$
$f(v_n^{''}) = 4.$

<u>Case 7</u>: $n \equiv 6(\text{mod } 7)$

$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n.$

$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n^{'}) = 6.$

$f(v_i^{''}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{''}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{''}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{''}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{''}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{''}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{''}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-4$
$f(v_{n-3}^{''}) = 2;$
$f(v_{n-2}^{''}) = 6;$
$f(v_{n-1}^{''}) = 4;$
$f(v_n^{''}) = 4.$

The labeling pattern illustrated here encompasses all possible layout's of vertices. The graph under scrutiny within each possibility conforms to the vertex conditions and edge conditions for 7-cordial labeling as presented in *Table 4.3.9*. In other words, the web graph $W(2, n)$ admits 7-cordial labeling.

Where $n = 7a + b$, $a, b \epsilon N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6)$
1	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3) + 1$ $= v_f(4) = v_f(5) + 1 = v_f(6)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) + 1 = e_f(5) = e_f(6) + 1$
2	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) = v_f(6)$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1 = e_f(3)$ $= e_f(4) = e_f(5) + 1 = e_f(6) + 1$
3	$v_f(0) + 1 = v_f(1) = v_f(2) = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) + 1 = e_f(6) + 1$
4	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) + 1 = v_f(5) = v_f(6)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$ $= e_f(4) = e_f(5) = e_f(6) + 1$
5	$v_f(0) + 1 = v_f(1) + 1 = v_f(2) = v_f(3) + 1$ $= v_f(4) + 1 = v_f(5) = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) = e_f(6) + 1$
6	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$ $= v_f(4) = v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) = e_f(3) + 1$ $= e_f(4) + 1 = e_f(5) + 1 = e_f(6)$

Table 4.3.9: 7-cordial labeling of web graph $W(2, n)$.

Illustratoin 4.3.9 *The web graph $W(2, 10)$ and its 7-cordial labeling is shown in* Figure 4.3.9.

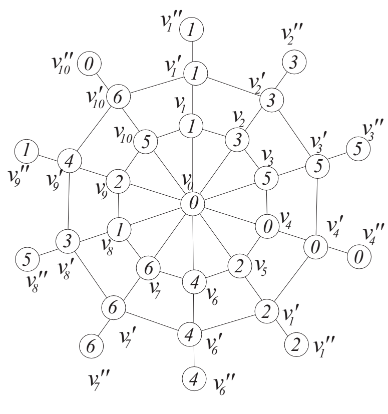

Figure 4.3.9: 7-cordial labeling of the web graph $W(2, 10)$.

4.4 Conclusion

In this chapter we have proved nine results related 7-cordial labeling of some standard graphs.