Chapter 2

Basic Terminology and Preliminaries

2.1 Introduction

This chapter is intended to provide all the fundamental terminology and notations which are needed for the present work.

2.2 Basic Definitions

Definition 2.2.1 *A* graph $G = (V(G), E(G))$ *consists of two sets,* $V(G) = \{v_1, v_2, \ldots\}$ *called* vertex set *of* G *and* $E(G) = \{e_1, e_2, \ldots\}$ *called* edge set *of* G. *Sometimes we denote vertex set of* G *as* V *and edge set of* G *as* E. *Elements of* $V(G)$ *and* $E(G)$ *are called* vertices *and* edges *respectively.*

Definition 2.2.2 *The number of vertices in a given graph is called* order of the graph. *The order of a graph* G *is denoted by* p *or* $|V(G)|$.

Definition 2.2.3 *The number of edges in a given graph is called* size of the graph. *The size of a graph* G *is denoted by* q *or* $|E(G)|$.

Definition 2.2.4 *An edge of a graph that joins a vertex to itself is called a* loop. *A loop is an edge* $e = v_i v_i$.

Definition 2.2.5 *If two vertices of a graph are joined by more than one edge then these edges are called* multiple edges.

Definition 2.2.6 *A graph which has neither loops nor parallel edges is called a* simple graph.

Definition 2.2.7 *If two vertices of a graph are joined by an edge then these vertices are called* adjacent vertices.

Definition 2.2.8 *Two vertices of a graph which are adjacent are said to be* neighbours. *The set of all neighbours of a vertex v of G is called the* neighbourhood set *of v. It is denoted by $N(v)$ or $N[v]$ and they are respectively known as open and closed neighbourhood set.*

$N(v) = \{u \in V(G)/u$ adjacent to v and $u \neq v\}$.
$N[v] = N(v) \cup \{v\}$.

Definition 2.2.9 *If two or more edges of a graph have a common vertex then these edges are called* incident edges.

Definition 2.2.10 Degree *of a vertex v of any graph G is defined as the number of edges incident on v, counting twice the number of loops. It is denoted by $deg(v)$ or $d(v)$.*

Definition 2.2.11 *A* walk *is defined as a finite alternating sequence of vertices and edges of the form $v_i e_j v_{i+1} e_{j+1} \ldots e_k v_m$ which begins and ends with vertices such that each edge in the sequence is incident on the vertex preceding and succeeding it in the sequence. A walk from v_0 to v_n is denoted as $v_0 - v_n$ walk. A walk $v_0 - v_0$ is called a* closed walk.

Definition 2.2.12 *A walk is called a* trail *if no edge is repeated.*

Definition 2.2.13 *A walk in which no vertex is repeated is called a* path. *A path with n vertices is denoted as P_n. A path from v_0 to v_n is denoted as $v_0 - v_n$ path.*

Definition 2.2.14 *A closed path is called a* cycle. *A cycle with n vertices is denoted as C_n.*

Definition 2.2.15 *A graph $G = (V(G), E(G))$ is said to be* connected *if there is a path between every pair of vertices of G. A graph which is not connected is called a* disconnected graph.

Definition 2.2.16 Tadpole $T(n, l)$ *is a graph in which path P_l is attached to any one vertex of cycle C_n. Truszczyń called it* dragon.

It is easy to observe that $|V(T(n, l))| = n + l$ and $|E(T(n, l))| = n + l$.

Definition 2.2.17 *A closed trail which covers all the edges of given graph is called an* Eulerian trail. *A graph which has an Eulerian trail is called an* Eulerian graph.

Definition 2.2.18 *A graph which does not contain any cycle is known as* acyclic graph.

Definition 2.2.19 *A connected acyclic graph is called a* tree.

Definition 2.2.20 *Let G and H be two graphs. Then H is said to be a* subgraph *of G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. Here G is called* supergraph *of H.*

Definition 2.2.21 *A simple, connected graph is said to be* complete *if every pair of vertices of $G = (V(G), E(G))$ is connected by an edge. A complete graph on n vertices is denoted by K_n.*

Definition 2.2.22 *A graph $G = (V(G), E(G))$ is said to be* bipartite *if the vertex set can be partitioned into two disjoint subsets V_1 and V_2 such that for every edge $e_i = v_i v_j \in E(G)$, $v_i \in V_1$ and $v_j \in V_2$.*

Definition 2.2.23 *A* complete bipartite graph *is a simple bipartite graph such that two vertices are adjacent if and only if they are in different partite sets. If partite sets V_1 and V_2 are having m and n vertices respectively then the related complete bipartite graph is denoted by $K_{m,n}$ and V_1 is called* m-vertices part *and V_2 is called* n-vertices part *of $K_{m,n}$.*

Definition 2.2.24 *A complete bipartite graph $K_{1,n}$ is known as* star graph.

Definition 2.2.25 *A graph $G = (V(G), E(G))$ is called* n-partite graph *if the vertex set V can be partitioned into n nonempty sets $V_1, V_2, \ldots, V_n$ such that every edge of G joins the vertices from different subsets. It is often called a* multipartite graph.

Definition 2.2.26 *Let G and H be two graphs such that $V(G) \cap V(H) = \phi$. Then* join of G and H *is denoted by $G + H$. It is the graph with $V(G + H) = V(G) \cup V(H)$, $E(G + H) = E(G) \cup E(H) \cup J$, where $J = \{uv/u \in V(G), v \in V(H)\}$.*

Definition 2.2.27 *Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. Then* cartesian product *of G_1 and G_2 which is denoted by $G_1 \times G_2$, is the graph with vertex set $V = V_1 \times V_2$ consisting of vertices $u = (u_1, u_2)$, $v = (v_1, v_2)$ such that u and v are adjacent in $G_1 \times G_2$ whenever ($u_1 = v_1$ and u_2 adjacent to v_2) or (u_1 adjacent to v_1 and $u_2 = v_2$).*

Definition 2.2.28 *Let $G = (V(G), E(G))$ be a graph and $G_1, G_2, \ldots, G_n$, $n \geq 2$ be n copies of graph G. Then the graph obtained by adding an edge from G_i to G_{i+1} (for $i = 1, 2, \ldots, n-1$) is called* path union *of G.*

Definition 2.2.29 *The* one point *union of m cycles of length n denoted as $C_n^{(m)}$ is the graph obtained by identifying one vertex of each cycle.*

Definition 2.2.30 Duplication *of a vertex v_k of graph G produces a new graph G_1 by adding a vertex v'_k with $N(v'_k) = N(v_k)$.*

In other words a vertex v'_k is said to be duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v'_k also.

Definition 2.2.31 Duplication *of an edge $e = uv$ by a new vertex w in graph G produces a new graph G_1 such that $N(w)$=$\{u, v\}$.*

Definition 2.2.32 *A* vertex switching *G_v of a graph G is obtained by taking a vertex v of G, removing all the edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.*

Definition 2.2.33 *Let u and v be two distinct vertices of a graph G. A new graph G_1 is constructed by* identifying(fusing) *two vertices u and v by a single vertex x is such that every edge which was incident with either u or v in G is now incident with x in G_1.*

Definition 2.2.34 *Let $G = (V(G), E(G))$ be a graph. Let $e = uv$ be an edge of G and w is not a vertex of G. The edge e is* subdivided *when it is replaced by edges $e' = uw$ and $e'' = wv$.*

Definition 2.2.35 *Let $G = (V(G), E(G))$ be a graph. If every edge of graph G is subdivided then the resulting graph is called* barycentric subdivision *of G.*

In other words barycentric subdivision is the graph obtained by inserting a vertex of degree 2 into every edge of the original graph. The barycentric subdivision of any graph G is denoted by $S(G)$. It is easy to observe that $|V(S(G))| = |V(G)| + |E(G)|$ and $|E(S(G))| = 2|E(G)|$.

Definition 2.2.36 *Let $G = (V(G), E(G))$ be a graph. A graph H is called a* supersubdivision *of G if H is obtained from G by replacing every edge e_i of G by a complete bipartite graph K_{2,m_i} for some m_i, $1 \leq i \leq q$ in such a way that the ends of each e_i are merged with the two vertices of 2-vertices part of K_{2,m_i} after removing the edge e_i from graph G.*

Definition 2.2.37 *A supersubdivision H of G is said to be an* arbitrary supersubdivision *of G if every edge of G is replaced by an arbitrary $K_{2,m}$ where m may vary for each edge arbitrarily.*

Definition 2.2.38 *The* corona *$G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as a graph obtained by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2 and attach one copy of G_2 at every vertex of G_1.*

Definition 2.2.39 *An* armed crown *is a graph in which path P_m is attached at each vertex of cycle C_n. This graph is denoted by $C_n \odot P_m$.*

Definition 2.2.40 *The cartesian product of two paths is known as* grid graph *which is denoted by $P_m \times P_n$. In particular the graph $L_n = P_n \times P_2$ is known as* ladder graph.

Definition 2.2.41 *The one-point union of n copies of cycle C_3 is known as* friendship graph. *It is denoted by F_n. That is $F_n = C_3^{(n)}$.*

Definition 2.2.42 *The* wheel graph W_n *is join of the graphs C_n and K_1. i.e. $W_n = C_n + K_1$. Here vertices corresponding to C_n are called* rim vertices *and C_n is called* rim *of W_n while the vertex corresponds to K_1 is called* apex vertex.

Definition 2.2.43 *A* helm $H_n, n \geq 3$ *is the graph obtained from the wheel W_n by adding a pendant edge at each vertex on the wheel's rim.*

Definition 2.2.44 *A* closed helm CH_n *is the graph obtained by taking a helm H_n and by adding edges between the pendant vertices.*

Definition 2.2.45 *A* web graph *is the graph obtained by joining the pendant vertices of a helm to form a cycle and then adding a single pendant edge to each vertex of this outer cycle.*

Definition 2.2.46 *A* triangular snake *is the graph obtained from a path $v_1, v_2, \ldots v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, \ldots, n-1$.*

Definition 2.2.47 *The* shadow graph *of a connected graph G is constructed by taking two copies of G say G' and G''. Join each vertex u' in G' to the neighbours of the corresponding vertex u'' in G''. The graph obtained is denoted as $D_2(G)$.*

Definition 2.2.48 *The* middle graph*, $M(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.*

Definition 2.2.49 *The* total graph*, $T(G)$ of graph G is the graph whose vertex set is $V(G) \cup E(G)$ in which two vertices are adjacent whenever they are either adjacent or incident in G.*

Definition 2.2.50 *A graph obtained by replacing each vertex of star graph $K_{1,n}$ by a graph G is called* star of G. *We denote it as G^*. We name* central graph *in G^* is the graph which replaces central vertex of graph $K_{1,n}$.*

2.3 Conclusion

This chapter provides basic definitions and terminology required for the advancement of the topic. For all other standard terminology and notations we refer to Harrary, West, Gross and Yellen, Clark and Holton.

The next chapter is focused on 5-cordial labeling of graphs.