CHAPTER 11

OPTIMIZATION OF CUTTING CONDITIONS FOR SINGLE PASS FINE BORING OPERATION

11.1 INTRODUCTION TO OPTIMUM CUTTING CONDITIONS

Machining system have been often discussed and expanded upon in isolation to the rest of the structure of the system. But recent trends indicate that the subject is tending to be treated as a sub-system of the total production system involving several exogenous interactions. Production is a process of transformation of a set of input elements with a set of desired output elements for increasing utility of goods and services. The word process means the engineering methods or techniques for preparing technological transformation. Consider a set of input elements \( \{U_i\} \) in figure 11.1 which undergo a technological transformation to yield a set of output elements \( \{V_i\} \) in order to optimize an objective function \( B \) which not only includes input output variable but also a factor \( Q \) dependent on characteristics of the environmental conditions. The optimality will however be governed by a set of constraints involving parametric combinations of input and output variables.

Thus \( T \{U_i\} = V_i \)

in order that \( B = B \) optimum

\( = f (U_i, V_i, Q) \)

Subject to a set of constraints

\( C_j \{U_i, V_i\} \leq 0 \)
Environmental Factors (Q)

Input (U)

Technological Transformation

Objective Function (B)

Output (V)

FIGURE 11.1

FIGURE SHOWING OPTIMIZATION PARAMETERS
The criteria of optimality can be

(i) increased utility of goods and services.

(ii) enhanced productivity which is defined as the ratio between value of outputs and inputs.

Machining is a major manufacturing process and plays a key role in the creation of wealth. Modern manufacturing systems like fine boring operations would use high production time on machining operations. This trend is encouraging and places a great demand to optimize the machining operations for further economic gain.

Traditionally the optimization of machining operation involves the selection of economic cutting conditions such as feed and speed according to variety of economic criteria like maximise material removal rate and minimise production time. A realistic optimization study should also consider the many technological and practical constraints which limit feasible domain for the selection of optimum cutting conditions. It requires intricate mathematical analysis and depends on quantitatively reliable mathematical functions for machining performance such as tool life, power and surface finish.

Productivity can be increased by increasing material removal rate. Hence in this research an attempt is made to maximise material removal rate for fine boring operation. While increasing material removal rate it should be kept in mind that the maximum performance should be maintained. The machining time should be kept minimum.
Hence the objective function is taken as maximization of material removal rate and constraints are taken as maximum surface finish, minimum machining time and minimum tool life required, maximum power available and constraints of speed limits for chosen material and also feed limit for higher surface finish. First the mathematical model is formulated with speed and feed as variables. Then the problem is solved by graphical method.

11.2 OBJECTIVE FUNCTION

The quantity material removal per unit time in rotational operation is given by

\[
\text{M.R.R.} = \frac{1000 \times s \times V_c \times t}{1000} \text{ mm}^3/\text{min}
\]

Thus for a given depth of cut the material removal rate depends on feed and speed. It was experimentally found that in fine boring machine the depth of cut of 0.3 mm give maximum surface finish. Hence depth of cut \( t \) in this model is taken as 0.3 mm. The variables speed and feed are represented by \( x_1 \) and \( x_2 \). Since the diameter of component taken for study is 50 mm.
M.R.R. = \pi \times 50 \times x_1 \times x_2 \times 0.3

= 47.12 \times x_1 \times x_2

Hence objective function is maximise M.R.R. = 47.12 \times x_1 \times x_2

11.3 RESTRICTIONS

11.3.1 Restriction on cutting speed

Cutting speed vary with work material and tool material. For our case cast iron work material (The material of the component taken for study is cast iron) and carbide tip tool material the cutting speed range is between 80 to 120 m/min (value got from data book)

\[
\text{Cutting speed} = \frac{\pi DN}{1000}
\]

D - Dia of work piece, \quad N - rpm

\[
80 \leq \text{cutting speed} \leq 120
\]

\[
80 \leq \frac{\pi DN}{1000} \leq 120
\]

\[
\frac{1000 \times 80}{\pi D} \leq N \leq \frac{1000 \times 120}{\pi D}
\]

\[
\frac{1000 \times 80}{\pi \times 50} \leq N \leq \frac{1000 \times 120}{\pi \times 50}
\]

509 \leq N \leq 763
11.3.2 Restriction on feed

It was found experimentally that the feed $\leq 0.07$ mm/rev gives high surface finish. (Based on Experiments conducted by HMT Engineers)

Hence

\[ s \leq 0.07 \]
\[ x_2 \leq 0.07 \]

11.3.3 Restriction on cutting time

\[
\text{Machining time} = \frac{L}{N \times s}
\]

For our analysis we have taken a bore length of 70 mm.

\[ N \text{ - speed in rpm} \]
\[ s \text{ - feed rate of table in mm/rev.} \]

The maximum allowable time was found to be 2.33 minutes. (value decided by experiments for High Production rate).

\[ \text{Hence} \quad \frac{70}{x_1 \times x_2} \leq 2.33 \]
11.3.4 Restriction on surface finish

The performance of the machining is decided by surface finish. Maximum surface finish should always be maintained. In fine boring operation surface finish is a very important criteria. The surface finish is represented by surface roughness. In fine boring maximum surface finish or surface roughness was found to be 0.8 microns. The allowable surface roughness should be ≤ this value. Surface finish depends on feed and speed and surface finish equation from previous researchers (Dr. D.K. Patil, Dr. M.R. Patkar) is given by

\[
S = 15077 V_c^{1.52} s^{1.004} t^{0.25}
\]

\[
= \frac{15077}{\left( \frac{\pi DN}{1000} \right)^{1.52}} s^{1.004} t^{0.25}
\]

\[
= \frac{15077 \times 1000^{1.52}}{(\pi \times 50 \times x_1)^{1.52}} \times x_2^{1.004} \times 0.3^{0.25}
\]

\[
= \frac{185991}{x_1^{1.52}} \times x_2^{1.004} \leq 0.8
\]

11.3.5 Restriction on minimum tool life required

Tool life is the time elapsed between two successive grindings of a cutting tool. There are various ways of expressing tool life.

(i) Time of actual operation.
(ii) Total time of operation.
(iii) Volume of material removed.
(iv) No. of pieces machined.
(v) Equivalent cutting speed.
Taylors tool life equation

Cutting speed \( (V_c) \) is the dominant variable having considerable influence on tool life. Taylors has shown that relation between cutting speed and tool life can be written as

\[
V_cT^n = C
\]

where \( T = \) Tool life in minute
\( V_c = \) cutting speed
\( n, C = \) constants depending on work, tool pair, tool shape cutting environment etc. later it was found that change in feed and depth of cut also affect tool life.

A generalized Taylors tool life equation is given by

\[
V_c = \frac{C_v}{T^{1/m} s^{x_v} t^{y_v}}
\]

\( C_v = \) modified Taylor constant depending on tool, work pair
\( 1 \)
\( \frac{1}{m} \) - Taylor exponent \( n \)

\( x_v, y_v \) - exponent to take effect of feed and depth of cut. The exponents and constants for various tool work pair are shown in Table 11.1
### Table 11.1

**Exponents and Constants of Generalized Taylor Equation**

<table>
<thead>
<tr>
<th>Work material</th>
<th>Tool material</th>
<th>$C_v$</th>
<th>$x_v$</th>
<th>$y_v$</th>
<th>$n = (1/m)$</th>
<th>Restriction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel $\sigma_v = 75$ kg/mm$^2$</td>
<td>WC Co = 10%</td>
<td>273</td>
<td>0.15</td>
<td>0.20</td>
<td>0.20</td>
<td>$s &lt; 0.75$</td>
</tr>
<tr>
<td></td>
<td>Co = 6%</td>
<td>227</td>
<td>0.15</td>
<td>0.35</td>
<td></td>
<td>$0.3 &lt; s &lt; 0.75$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>221</td>
<td></td>
<td>0.45</td>
<td></td>
<td>$s &gt; 0.75$</td>
</tr>
<tr>
<td>Cast iron : BHN $= 190$</td>
<td>WC</td>
<td>292</td>
<td>0.30</td>
<td>0.15</td>
<td>0.18</td>
<td>$t &lt; s$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>292</td>
<td>0.15</td>
<td>0.30</td>
<td>0.18</td>
<td>$t &gt; s$</td>
</tr>
</tbody>
</table>

From table for our case material cast iron, tool material carbide tip:

$C_v = 292 \quad x_v = 0.15 \quad y_v = 0.20$

$$\frac{1}{m} = 0.20$$

Substituting the above values in tool life equation,

$$V_c = \frac{292}{T^{0.2} s^{0.2} t^{0.15}}$$

$$\pi DN = \frac{292}{1000}$$

$$292 \times 1000$$

$$\pi \times D \times N \times s^{0.2} t^{0.15} = T^{0.2}$$

$D = 50$

$t$ - depth of cut $= 0.3$ mm for best surface finish.
The left hand side should be greater than or equal to certain minimum constant tool life, required for maximum economy of cutting. If tool life is less than this value the tool fails at an earlier stage and maximum economy of cutting cannot be obtained and tool has to be replaced at a higher interval of time. Time between two tool grindings small. Hence we are keeping this constant in our model and we find the speed and feed rate which satisfies this constraint. The tool life should be at a minimum of 100 minutes for fine boring operation (from experiments).

\[
\left( \frac{292 \times 1000}{\pi \times 50 \times x_1 \times x_2^{0.2} \times 0.3^{0.15}} \right)^{5} \geq T
\]

\[
\left( \frac{2227}{x_1 \times x_2^{0.2}} \right)^{5} \geq T
\]

11.3.6 Restriction on power

The power observed by the cutting process cannot exceed power capacity of main drive. From Researcher J. Wang,

\[
0.367 \ V_c^{0.91} \times a^{0.78} \times t^{0.75} \leq p
\]
(\(V_c\) - cutting speed, \(s\) - feed, \(t\) - depth of cut, \(p\) - power of spindle of fine boring machine = 2.2 kw)

\[
0.367x \left(\frac{\pi \times D \times N}{1000}\right)^{0.91} x \ s^{0.78} x \ 0.3^{0.75} \leq 2.2
\]

\[
\left(\frac{0.367 \pi \times 50 \times x_1}{1000}\right)^{0.91} x \ x_2^{0.78} x \ 0.3^{0.75} \leq 2.2
\]

\(x_1^{0.91} x_2^{0.78} \leq 79.6\)

Hence the optimization problem is

Maximise M.R.R. = 47.12 \(x_1 \ x_2\)

Subject to

\(g_1(x) = 500 - x_1 \leq 0\)

\(g_2(x) = x_1 - 800 \leq 0\)

\(g_3(x) = x_2 - 0.07 \leq 0\)

\(g_4(x) = \frac{70}{x_1 \ x_2} \leq 2.33\)

\(g_5(x) = \frac{185991}{x_1^{1.52}} x_2^{1.004} \leq 0.8\)

\(g_6(x) = x_1 x_2^{0.2} \leq 886.58\)

\(g_7(x) = x_1^{0.91} x_2^{0.78} \leq 79.6\)

11.4 SOLUTION TO THE ABOVE OPTIMIZATION PROBLEM

Since there are only two design variables the problem can be solved graphically as shown below:
First the constraint surfaces are to be plotted in two dimensional design space where the two axes represent two design variables $x_1$ and $x_2$.

To plot first constraint surface we have

$$g_1(x) = 500 - x_1 \leq 0$$
$$x_1 = 500$$

To plot second constraint we have

$$g_2(x) = x_1 - 800 \leq 0$$

The curve $x_1 = 500, x_1 = 800$ represent constraint surfaces $g_1(x) = 0$, $g_2(x) = 0$.

Plot $x_1 = 500$ and plot $x_1 = 800$ which is plotted as in Figure 11.2

$P_1Q_1$ and $P_2Q_2$ represent $g_1(x) = 0$ and $g_2(x) = 0$.

The feasible region is shown as shaded.

To plot 3rd constraint, we have $g_3(x) = x_2 - 0.07 \leq 0$.

Plot $x_2 = 0.07$, which is plotted as shown in Figure 11.2.

Straight line $P_3Q_3$ represent $g_3(x) = 0$.

Feasible region shown as shaded.

To plot 4th constraint we have

$$g_4(x) = \frac{70}{x_1 x_2} \leq 2.33$$
$$x_1 x_2 \geq \frac{70}{2.33} = 30.04$$
The curve \( x_1, x_2 = 30.04 \) represents the constraint surface \( g_4(x) = 0 \). This curve can be plotted by plotting several points on the curve. The points on the curve can be found by giving a series of values to \( x_1 \) and finding the corresponding values \( x_2 \) that satisfy the relation \( x_1 \cdot x_2 = 30.04 \).

**TABLE 11.2**

**TABLE TO DRAW CURVE FOR CUTTING TIME CONSTRAINT**

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
<th>1000</th>
<th>1100</th>
<th>1200</th>
<th>1300</th>
<th>1400</th>
<th>1500</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_2 )</td>
<td>0.30</td>
<td>0.15</td>
<td>0.10</td>
<td>0.075</td>
<td>0.05</td>
<td>0.042</td>
<td>0.0375</td>
<td>0.0333</td>
<td>0.03</td>
<td>0.027</td>
<td>0.025</td>
<td>0.023</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These points are plotted and a curve \( P_4Q_4 \) passing through all these points is drawn shown in Figure 11.2. The feasible region is shown as shaded.

The fifth constraint \( g_5(x) = 0 \) can be expressed as

\[
g_5(x) = \frac{185991}{x_1^{1.52}} x_2^{1.004} = 0.8
\]

Points lying on the constraints surface \( g_5(x) = 0 \) can be obtained as follows:

**TABLE 11.3**

**TABLE TO DRAW CURVE FOR SURFACE FINISH CONSTRAINT**

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
<th>1000</th>
<th>1100</th>
<th>1200</th>
<th>1300</th>
<th>1400</th>
<th>1500</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_2 )</td>
<td>0.049</td>
<td>0.14</td>
<td>0.06</td>
<td>0.04</td>
<td>0.36</td>
<td>0.073</td>
<td>0.1</td>
<td>0.135</td>
<td>0.159</td>
<td>0.18</td>
<td>0.209</td>
<td>0.25</td>
<td>0.283</td>
<td>0.292</td>
<td></td>
</tr>
</tbody>
</table>

These points are plotted as \( P_5Q_5 \) and feasible region identified (shaded) as shown in Figure 11.2.
The 6th constraint \( g_6(x) = 0 \) can be expressed as \( x_1 x_2^{0.2} = 886.58 \) and points lying on the constraint surface \( g_6(x) = 0 \) can be plotted as follows:

**TABLE 11.4**

**TABLE TO DRAW CURVE FOR TOOL LIFE CONSTRAINT**

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
<th>1000</th>
<th>1100</th>
<th>1200</th>
<th>1300</th>
<th>1400</th>
<th>1500</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_2 )</td>
<td>17.5</td>
<td>7.0</td>
<td>3.2</td>
<td>1.6</td>
<td>0.92</td>
<td>0.54</td>
<td>0.34</td>
<td>0.22</td>
<td>0.14</td>
<td>0.10</td>
<td>0.07</td>
</tr>
</tbody>
</table>

These points are plotted on the curve \( P_6Q_6 \) and feasible region identified (shaded) as shown in Figure 11.2.

The 7th constraint \( g_7(x) = 0 \) can be expressed as

\[ x_1^{0.91} x_2^{0.78} = 79.6 \]

and points lying on the constraint surface \( g_7(x) = 0 \) can be plotted as follows:

**TABLE 11.5**

**TABLE TO DRAW CURVE FOR POWER CONSTRAINT**

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
<th>1000</th>
<th>1100</th>
<th>1200</th>
<th>1300</th>
<th>1400</th>
<th>1500</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_2 )</td>
<td>1.26</td>
<td>0.68</td>
<td>0.35</td>
<td>0.25</td>
<td>0.19</td>
<td>0.15</td>
<td>0.13</td>
<td>0.11</td>
<td>0.09</td>
<td>0.08</td>
<td>0.07</td>
<td>0.069</td>
<td>0.063</td>
<td>0.058</td>
<td>0.054</td>
</tr>
</tbody>
</table>

These points are plotted on the curve \( P_7Q_7 \) and feasible region identified (shaded) as shown in Figure 11.2.

In Figure 11.2 ABCD is the feasible region with points A,B,C & D. Point A and B are bound acceptable points. Since point A and B are on the extreme line of the material constraint these points can not be optimum. Let us try with point C and D.
At point C  \( x_1 = 588 \)  \( x_2 = 0.07 \)

The objective function and material removal rate

\[
\text{M.R.R.} = 47.12 \times x_1 \times x_2 \\
= 47.12 \times 588 \times 0.07 \\
= 1939 \text{ mm}^3/\text{min}
\]

At point D, \( x_1 = 517.5 \)  \( x_2 = 0.0575 \)

\[
\text{M.R.R.} = 47.12 \times 517.5 \times 0.0575 \\
= 1402 \text{ mm}^3/\text{min}
\]

Since objective function is maximum at point C

Point C  =  588, 0.07 is the optimum point

It was also experimentally found out that for cast iron material as work piece and carbide tip as tool material the speed 588 rpm and feed .07 mm/rev gave best results. (best surface finish, best tool life, optimum power utilization). Hence the optimum speed and feed rate for fine boring operation are 588 rpm and .07 mm/rev.

The same model can be repeated for different materials by changing only the material constraint.
GRAPHICAL OPTIMIZATION OF SINGLE PASS FINE BORING OPERATION

FIG. 11.2