CHAPTER 1

INTRODUCTION

1.1 MOTIVATION

Complex structures used in the field of aerospace, mechanical, automobile, civil engineering, etc., are combination of simple structural members like beams, plates and shells. Study of the dynamic stability behavior of these structural members, when subjected to periodic loads, plays a vital role in confirming the structural integrity of these structural members.

The studies on static stability (Euler buckling) are sufficient only when the applied loads are independent of time. The same studies are no longer adequate to study the realistic behavior of these structural members subjected to periodic loads. Thus, to accurately determine and understand the dynamic stability behavior of structural members under periodic loading, one has to study this phenomenon of the structural members, which is an active area of research even now.

The dynamic stability analysis aims at evaluating the instability regions that arise due to the periodic loads. The Euler buckling loads and the natural frequencies for a given mode are used as reference values which are used to arrive at the non dimensional basic equations of the dynamic stability. Most often, these quantities for the first mode are of practical significance. This can be achieved by solving the differential equations, applying the classical approximate methods like the Rayleigh-Ritz (RR) method or by the versatile numerical method like the finite element (FE) method. Exact solutions to predict the dynamic stability regions of the structural members involve higher order mathematical treatment, are difficult to obtain. Though, this is not a problem for experts in this field, the practicing engineers prefer to have quick and accurate methods to obtain the solutions of the problem. Further, even the application of approximate continuum or numerical methods to obtain the solutions is also tedious because of the time consuming and cumbersome algebraic manipulations in the case of the approximate analytical method and larger computational times required even with the present day computing aids in case of the FE method, as the study basically involves multiple parameters.

Approximate analytical methods give simple and reliable formulas to predict the dynamic stability boundaries. These formulas are attractive for the structural design engineers as it is easier to obtain the dynamic stability boundaries of the structural members subjected to periodic loads. There are several analytical methods of analyzing the structural members to obtain the dynamic stability behavior, it is instructive to provide a study of the solutions obtained with the widely used energy method.

Recognizing these facts and considering the usefulness of the relatively simpler formulations, an attempt is made in this thesis to develop simple closed form solutions to investigate the dynamic stability behavior of structural members like beams, plates and shells subjected to periodic loads. *In this process, a very interesting fact is recognized, for the first time, about the existence of a master dynamic stability formula applicable to many structural members and materials with several complicating effects.*

1.2 DYNAMIC STABILITY

Dynamic stability analysis is essential for structures subjected to periodic loads. Structures loaded in this manner may appear to be stable according to static criteria while in reality they fail due to flutter instability with increasing amplitudes with time. Because of the importance of this problem, in the many fields of engineering structures and to access the structural integrity, the investigation of dynamic stability analysis is of paramount importance, in the fields of modern mechanical, aerospace, automobile, civil engineering etc. It is interesting to study the dynamic stability boundaries of the structural members under dynamic or time varying loads. The effect of damping is neglected throughout this study.

The time dependant compressive loads consist of a constant part and a periodic part. The time dependant loads can be obtained by the Fourier or harmonic analysis of half range cosine series. The dynamic stability of the structural members has to be analyzed with the highest period, as this is the most critical case for all practical purposes.

1.3 GEOMETRIC NONLINEARITY

Geometric nonlinearity exists in the structural members undergoing large deflections. To predict the response of the structural members subjected to severe operational loads, that result in large lateral deflections, the linear analysis is no longer sufficient and the geometric nonlinearity, arising in the strain-displacement relation(s), has to be considered. The von-Karman type nonlinear strain-displacement relations applicable for the moderately large deflections, are used to evaluate the axial or mid-plane stretching, which contributes to the geometric nonlinearity of the structural members. The present analysis of the geometric nonlinear behavior can result in hardening type of nonlinearity. The strain- dispalcenment (von-Karman) relation for the beam problems presented in this thesis are given by

$$\in_x = \frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2$$

where $\in_x$ is the axial strain, u is the axial displacement and w is the lateral displacement.

1.4 SCOPE OF THE PRESENT STUDY: Achievements commensurate with the scope

Beams, plates and shells are commonly used structural members in mechanical, aerospace, automobile, civil and other fields of engineering. Prediction of the dynamic stability behavior, including the geometric nonlinearity, of structural members subjected to periodic loads has been studied by many researchers. The versatile FE method and the classical Rayleigh-Ritz methods are used to study the dynamic stability analysis of beams, plates and shells. In all the studies reported earlier, buckling load parameters, frequency parameters and the dynamic stability regions are estimated by using mainly the aforementioned methods for different structural members. The emphasis of the present work is to study the dynamic stability behavior of these structural members using the energy method.

The contributions of the work presented here are summarized as:

- A general formula to predict the dynamic stability behavior of the beams subjected to axial periodic loads.
- To study the effect of one parameter (Winkler) and two parameter (Pasternak) elastic foundations on the dynamic stability regions of beams.
- To bring out the effect of static tensile load coupled with the periodic load on the dynamic stability behavior of beams.

- A general formula to predict the dynamic stability behavior of the non uniform (with symmetric linear taper) beam subjected to axial periodic loads.
- Prediction of the dynamic stability boundaries of square plate subjected to edge in- plane periodic load in one direction and a compressive static load in the perpendicular direction.

During these studies an interesting observation, till now not identified by the researchers, about the existence of a master dynamic stability formula, applicable to any structural member is made. These structural members may be of different materials with several complicating effects. The usefulness of this unique formula right from the beams to shells is verified. The effectiveness of the master dynamic stability formula is shown through the following studies:

- The use of the master dynamic stability formula for any structural member subjected to periodic loads and its utility is demonstrated for the first time.
- The subsequent identification of two more master dynamic stability formulas for any structural member subjected to periodic loads.
- Finally, based on the earlier studies a revolutionary concept the existence of a very simple master dynamic stability point for any structural member subjected to periodic loads is explored.
- The master formula to predict the geometrically nonlinear dynamic instability of shear flexible beams subjected to periodic loads.

The importance of the present work is considered to be unique because of the recognition of the several simple master dynamic stability formulas that can be confidently used for linear and nonlinear dynamic stability analysis of several structural members.

During these studies, another interesting feature is also observed, the existence of a master formula to evaluate the fundamental frequency of any initially loaded structural member. This formula is useful to determine the reference frequency required in the dynamic stability analysis used to define the non-dimensional frequency of the applied periodic load for any structural member.

1.5 ORGANIZATION OF THESIS

The present work mainly deals with the theoretical analysis of the dynamic stability behavior of structural members like beams, plates and shells. This thesis contains six chapters and the contents of each chapter are given below:

- Chapter 1 contains the motivation, importance of the dynamic stability behavior of the structural members subjected to periodic axial/edge in-plane loads and the scope of the thesis.
- In chapter 2, a concise literature study is reported pertaining to dynamic stability of structural members on the proposed work.

- Chapter 3 presents the prediction of the dynamic stability behavior of beams subjected to periodic loads by choosing simple single term admissible functions for commonly used boundary conditions of the beams and plates. The effects of elastic foundations on dynamic stability regions of the beam with elastic foundations are proposed. The dynamic stability behavior, in terms of regions of stability, of the beam subjected to axial periodic load and static tensile load has also been studied, in contrast to all the earlier studies where compressive static load only is considered. The dynamic stability boundaries of square plate subjected to biaxial load, which is periodic in one direction and constant compressive load in the perpendicular direction has been studied.
- In Chapter 4, a simple universal (master) formula for evaluation of the natural frequencies of initially loaded structural members has been developed and its usefulness in the dynamic stability analysis is discussed.
- Chapter 5 represents a master dynamic stability formula for structural members, right from beams to shells with complicating effects, subjected to periodic loads to predict the dynamic stability regions is proposed and also focuses on the development of the existence of two more alternate master dynamic stability formulas for the structural members subjected to periodic loads using different non-dimensional parameters. The standard and the

effective stiffness are used to obtain the non-dimensional parameters. Further, an interesting novel concept of the master dynamic stability point for the structural members subjected to periodic loads with effective stiffness, combining the static and periodic part of loads in stiffness is identified.

- Chapter 6 presents, the master geometrically nonlinear dynamic instability formula for predicting the nonlinear dynamic instability of shear flexible beams subjected to periodic loads is proposed.
- Finally, overall conclusions drawn from the present thesis reported in chapters 3-6 along with suggestions for the further scope of the work that can be carried as an extension have been presented.

In all these studies on the dynamic stability, it is shown that how simple is the concept of master dynamic stability formulas when compared with the formulations involving higher order mathematical treatment.