CHAPTER 6

GEOMETRICALLY NONLINEAR DYNAMIC STABILITY OF BEAMS

The importance of dynamic instability in the structural members, subjected to periodic loads is well understood by earlier works [17, 18, 19, 92 and 95]. The effect of geometric nonlinearity has to be considered when these structural members are subjected to severe operational conditions leading to large deflections in the works of [104,105,116,119,124,125,127]. Large amplitude vibrations of beam with axially immovable ends are first presented by Woinowsky – Krieger [103]. The effect of geometric nonlinearity (large amplitudes) in the dynamic stability of beam is studied by Reiss and Matkowsky [104] and Rubenfield [105].

The aim of the present work is to develop a geometrically nonlinear dynamic instability (GNDI) formula for beams with secondary effects like shear deformation and rotary inertia (Timoshenko beam theory). The same formula will be applicable for the other boundary conditions and structural members, as these are absorbed by the proper use of non-dimensional parameters of the basic quantities like the nonlinear frequency, post buckling for a specified maximum amplitude deflection and the buckling loads.

The procedure followed to solve the nonlinear dynamic instability is demonstrated with respect to the uniform shear flexible beams, which is investigated by using the FE method in [110] for the thermal loads. The efficacy of the present formula is demonstrated with the comparison of the numerical results obtained from the present formula with the finite element results of the shear flexible beams [110].

6.1 FORMULATION

If the structural members are subjected to axial periodic loads P(t), are considered in form of Eq. (3.2). The instability regions, when the beam exhibits flutter type of instability, can be obtained from the solution of any approximate continuum solution like the RR method or a numerical formulation like the FE method, by solving the matrix equation of equilibrium, derived by Bolotin [17], as given in Eq. (3.109). In this chapter, a simple master GNDI formula is derived, considering the effect of the geometric nonlinearity of von-Karman type, applicable for moderately large lateral deflections in to account. The interesting part is that the master GNDI formula derived here, using this matrix equation is similar to that proposed in Eq. (3.109), which has not been recognized till now. The FE formulation is used to solve the GNDI problem of the uniform, homogeneous Timoshenko beams [110], for the periodic thermal loads. In this work, the ends of the beam are restrained to move axially so that the thermal loads, with the axial end constraints produce equivalent mechanical loads, as discussed in Ref. [110].

Considering the geometric nonlinearity, Eq. (3.109) can be written by replacing [K]by [K_{NL}], as

$$[K_{NL}]\{\delta\}-\left(P_s \pm \frac{P_t}{2}\right)[G]\{\delta\}-\frac{\theta^2}{4}[M]\{\delta\}=0 \tag{6.1}$$

In Eq. (6.1), the matrices [K_{NL}], [G] and [M] are system nonlinear stiffness matrix, system geometric stiffness matrix and system mass matrix respectively and are applicable for any structural member after applying the boundary conditions. This equation is valid for any structural member. This fact is advantageously used to propose the master GNDI formula which is independent of the structural member and boundary conditions.

If the quantities P_s, P_t and θ in Eq. (3.2), are written in the nondimensional form as α and β using the first postbuckling load and the first nonlinear fundamental radian frequency, for a specified maximum deflection/amplitude as the reference values, the same master dynamic instability formula, derived in Eq.(3.30), for predicting the linear dynamic instability regions is obtained.

Equation (6.1) can be written, using the nondimensional quantities α and β, as

$$[K_{NL}]\{\delta\}-(\alpha \pm \frac{\beta}{2})\ P_{NL}[G]\{\delta\}-\frac{\theta^2}{4}\ [M]\{\delta\}=0 \tag{6.2}$$

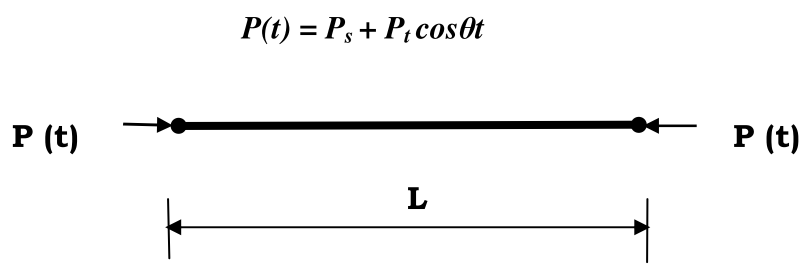

Fig.6.1 Uniform shear flexible beam subjected to end concentrated periodic axial load

where the postbuckling load P_{NL} is defined as the sum of the buckling load P_L and the incremental load ΔP that can be taken by the beam due to the geometric nonlinearity, for a specified maximum deflection a ($P_{NL}=P_L+\Delta P$). It may be noted here that ΔP = 0 in the case of the linear analysis, $\alpha = \frac{P_s}{p_{NL}}$ and $\beta = \frac{P_t}{P_{NL}}$.

From Eq. (6.1) the governing equation of the degenerate case for the nonlinear vibration problem can be written, by neglecting the second term, and noting that $\frac{\theta}{2} = \omega_{NL}$, where ω_{NL} is the first radian frequency, for specified maximum amplitude a, as

$$[K_{NL}]\{\delta\} - \omega_{NL}^2 [M]\{\delta\} = 0 \tag{6.3}$$

Equation (6.3) can be written as

$$[M]\{\delta\} = \frac{1}{\omega_{NL}^2}[K_{NL}]\{\delta\} \tag{9.4}$$

Similarly, the degenerate case of problem for the postbuckling can be solved, by neglecting the third term in Eq. (6.1), as

$$[K_{NL}]\{\delta\} - P_{NL}[G]\{\delta\} = 0 \tag{6.5}$$

or

$$[G]\{\delta\} = \frac{1}{P_{NL}}[K_{NL}]\{\delta\} \tag{6.6}$$

In general, the lateral deflection curves {δ}, in Eqs.(6.3) to (6.6), though similar, are taken to be the same, as only a small tolerable difference exist in the corresponding L_2 Norms, for the large amplitude vibration and the postbuckling problems.

Substituting, Eq. (6.4) and (6.6) in Eq. (6.2), the matrices [G] and [M] can be eliminated for the nonlinear dynamic stability problem and the following equation in terms of $[K_{NL}]$ is obtained as

$$[K_{NL}]\{\delta\} - (\alpha \pm \frac{\beta}{2})\,[K_{NL}]\{\delta\} - \frac{\theta^2}{4}\frac{1}{\omega_{NL}{}^2}[K_{NL}]\{\delta\} = 0 \tag{6.7}$$

From Eq.(6.7), as $\{\delta\} \neq 0$, the nontrivial solution for the GNDI problem is

$$1 - (\alpha \pm \frac{\beta}{2}) - \frac{\theta^2}{4}\frac{1}{\omega_{NL}{}^2} = 0 \tag{6.8}$$

Equation (6.8), can be rewritten, as

$$\frac{\theta^2}{4\omega_{NL}{}^2} = 1 - \left(\alpha \pm \frac{\beta}{2}\right) = (1-\alpha)(1 \pm \mu) \tag{6.9}$$

where $\mu = \dfrac{\beta}{2(1-\alpha)}$

From Eq. (6.9), the ratio of $\dfrac{\theta}{\omega_{NL}}$ for the GNDI problem is obtained

$$\frac{\theta}{\omega_{NL}} = \Omega \;= 2\sqrt{(1-\alpha)(1\pm\mu)} \tag{6.10}$$

Equation (6.10) is the master GNDI formula in the present non-dimensional form, either the boundary conditions or the type of structural members does not appear explicitly as the corresponding matrices specific to a structural member, are eliminated. Hence, the numerical results obtained from Eq. (6.10) in terms of Ω_{NL} and μ are independent of the structural member and its boundary conditions. However, to obtain the physically meaningful values of the instability regions of the beams (or of any structural member), in the present study, the corresponding values of ω_{REF} and P_{REF}, evaluated for the given structural member and boundary conditions have to be used. The values of the first postbuckling load (P_{NL}) and the first nonlinear radian frequency (ω_{NL}) are obtained from the simple formulas developed in the recent studies [132, 133], as

$$\frac{P_{NL}}{P_L} = 1 + b\left(\frac{a}{r}\right)^2 \tag{6.11}$$

and

$$\left(\frac{\omega_{NL}}{\omega_L}\right)^2 = 1 + \frac{3}{4}b\left(\frac{a}{r}\right)^2 \tag{6.12}$$

In the above equations $b\left(\frac{a}{r}\right)^2 = \frac{\lambda_T}{\lambda_L}$, where λ_T is the tensile load parameter $\lambda_T = \left(\frac{\Delta P L^2}{EI}\right)$ developed in the beam due to the large lateral deflections [103] and λ_L $\left(= \frac{P_L L^2}{EI}\right)$ is the buckling load parameter [1], a is the central deflection/amplitude of the beam, r is the radius of gyration. It is to be noted here that Eq. (6.11) is derived intuitively [132]. Equation (6.12) is derived from the master formula for predicting the vibration behavior of the initially loaded structural members, as proposed in Eq.(4.9) are the same for any structural member governed by the homogeneous Duffing's equation (temporal equation with cubic nonlinearity). However, the value of *b,* which quantifies the effect of the geometric nonlinearity is to be evaluated and is dependent on the structural member [132, 134]. For beams, in the present study, the procedure given by Woinowsky-Krieger [103] is followed to evaluate the tensile load ΔP developed due to the large deflections/amplitudes, from which by knowing λ_L the value of b is obtained.

6. 2 NUMERICAL RESULTS AND DISCUSSION

Fig. 6.1 shows the geometry of the uniform, homogeneous Timoshenko beam and also the points of application of the axial periodic load. The numerical values of the instability regions are obtained using the master

GNDI formula, derived in the present study, in terms the parameters μ and $\Omega_{NL} = \left(\frac{\theta}{\omega_{NL}}\right)$. Other geometric and material properties of the beam are not necessary in the present analysis, as the matrices [K_{NL}], [G] and [M] are eliminated, and the proposed formulation is based on the nondimensional parameters, while deriving the master GNDI formula.

Table 6.1 gives the GNDI regions for the amplitude ratio $\frac{a}{r}$=1, with reference to the nondimensional quantities μ, Ω_{NL} and α, of the Timoshenko beams. It can be verified that the master dynamic stability formula developed in this chapter is exactly the same as that proposed in Eq.(3.30), applicable for the linear analysis, by virtue of the use of the suitable nondimensional parameters in both the studies. The numerical values given in Table 6.1 brings out an important point that the dynamic instability regions are invariant with respect to the type of the structural member, boundary conditions, and the other secondary effects. This is due to the nondimensional parameters used, for the periodic loads P_t and the applied radian frequency θ with the use of the reference values corresponding to the current configuration. This simplifies the complex formulations of GNDI analysis presented in the earlier work [110], where the GNDI regions are obtained starting from Eq. (6.1), using the FE method.

The most important part of the present work is to validate the master GNDI regions of structural members. Though a good amount of work is available on this topic in the literature, most of it is theoretically oriented but do not contain any numerical results and very few papers contain the numerical results. In Ref. [110] the nonlinear dynamic instability regions of the Timoshenko beams with the slenderness ratio (L/r) equals to 25 are given for the three values of $\frac{a}{r}$. The present results obtained for the master GNDI regions have been deduced to match with the non-dimensional parameters used in Ref.[110].

It is to be noted here that the very few papers available on the GNDI, contain the numerical results for α = 0 only. For the non-zero values of α, to the best of the candidate's knowledge, based on the literature search, the numerical results are not available for non-zero values of α, and if available the nondimensional parameters used could not deduced to the same used in the present study. To validate the proposed simple formula derived to predict the nonlinear dynamic instability regions, for the values of α other than zero, the candidate considered a uniform, homogeneous and isotropic slender beam and validated the nonlinear dynamic stability results. A detailed analysis of the same is given in section 6.2.1

Table 6.2 gives the present results along with those of Ref. [110], obtained by using the FE method, for different values of the maximum deflection ratios $\frac{a}{r}$ and the non-dimensional parameters μ, for $\alpha = 0$ for the hinged Timoshenko beam for $L/r = 25$. Similar results are given in Table 6.3 for the clamped beam. The match of the present results with those of Ref. [110] is very good. It can be seen that the order, of the complexity of the formulation of Ref. [110], where the FE method is used to solve the problems starting right from the GNDI equation to obtain the GNDI regions, is orders of magnitude higher than the present formulation. As has been already mentioned, the candidate could trace a recent work reported [127], which contains a study on the nonlinear dynamic instability of thin plates. However, a similar comparison for the plates is possible with the present master GNDI formula, to the deduction of the results of recent Ref. [127], is not possible, due to non-availability of some required essential data. Based on the accuracy of the GNDI results of the present work, with respect to the Timoshenko beams and with the direct formulation given in section 6.2.1 for the slender beams, the master GNDI formula developed in the present study, can be confidently used to evaluate the GNDI regions for the other structural members also.

6.2.1 Validation of Master GNDI Formula for Non-Zero Values of α

To validate the derived master GNDI formula for the non-zero values of α, for the sake of better understanding, a uniform, homogeneous and isotropic slender beam of length *L*, subjected to an axial concentrated periodic load at the ends is considered. The applied periodic load $P(t)$ is given in Eq. (3.2).

The total potential energy Π is

$$\Pi = U_{NL} + U_L - W - T \tag{6.13}$$

where U_{NL} and U_L are the nonlinear and linear parts of the strain energy, *W* is the potential due to work by the periodic load *P (t)* and *T* is the kinetic energy. From the von-Karman type nonlinear strain–displacement relation, applicable to beams, the axial strain ε_x is

$$\varepsilon_x = \frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2 \tag{6.14}$$

where u is the axial displacement and w is the lateral deflection.

The expression for Π can be written as

$$\Pi = \frac{EA_b}{2}\int_0^L \varepsilon_x^{\,2} dx + \frac{EI}{2}\int_0^L w''^2 dx - \frac{P(t)}{2}\int_0^L w'^2 dx - \frac{\overline{m}}{2}\frac{\theta^2}{4}\int_0^L w^2 dx \tag{6.15}$$

The four terms on the right hand side of the Eq. (6.15) represents the corresponding energy terms given in Eq. (6.13) in the same order, where A_b is the cross- sectional area of the beam.

Considering the hinged beam, the standard one term trigonometric admissible function, for the lateral deflection w is assumed as given in Table 3.1, is

$$w = a\sin\frac{\pi x}{L}$$

The axial displacement distribution u is derived by taking note that the axial load ΔP $\left(= EA_b\varepsilon_x\right)$developed in the beam due to, large amplitude/deflections is constant. Hence, the derivative of ΔP with respect to, the axial coordinate x is zero, which gives a second order differential equation in u in terms of the known function for w. The expression for u is obtained, after integrating the differential equation twice and applying the boundary conditions u (0) = u (L) = 0 and the symmetric condition $\frac{du}{dx} = 0$ for the assumed lateral deflection curve, corresponding to the first postbuckling load or large amplitude radian frequency, as

$$u = -\frac{a^2\pi}{8L}\sin\frac{2\pi x}{L} \tag{6.16}$$

Substituting w and u (Eq. (6.16)) in Eq.(6.15), and neglecting the kinetic energy term in Eq.(6.15), the first postbuckling load P_{NL} is obtained by minimizing Π with respect to a and equating it to zero, and after simplification, as

$$P_{NL} = P_L\left[1 + \frac{1}{4}\left(\frac{a}{r}\right)^2\right] \tag{6.17}$$

where P_L is the first buckling load given by $\frac{\pi^2 EI}{L^2}$.

Similarly substituting w and u (Eq. (6.16)) in Eq.(6.15), and neglecting the potential due to work term in Eq.(6.15), the first nonlinear radian frequency due to the large amplitudes is obtained by minimizing Π with respect to a and equating it to zero, and after simplification, as

$$\omega_{NL}^{\ 2} = \omega_L^{\ 2}\left[1 + \frac{3}{16}\left(\frac{a}{r}\right)^2\right] \tag{6.18}$$

where ω_L^2 is the linear radian frequency for the first mode of vibration given by $\frac{\pi^4 EI}{\overline{m}L^4}$.

The reference values, P_{REF} = P_L = $\frac{\pi^2 EI}{L^2}$ and ω_{REF}^2 = ω_L^2 = $\frac{\pi^4 EI}{\overline{m}L^4}$ are used to define the nondimensional quantities $\alpha = \frac{P_s}{P_{REF}}$, $\beta = \frac{P_t}{P_{REF}}$ and Ω_{NL0} = $\frac{\theta}{\omega_{REF}}$ for the hinged beam.

The nondimensional GNDI equation is obtained, by substituting lateral deflection w (Eq. (3.22)) and Eq. (6.16) in Eq. (6.15), and taking the linear buckling load P_L and the linear radian frequency ω_L as the reference values P_{REF} and ω_{REF} given above to obtain the nondimensional quantities α, β and Ω_{NL0} in the form

$$\frac{\theta}{\omega_{REF}} = \Omega_{NL0} = 2\sqrt{(1-\alpha)(1 \pm \mu) + \frac{3}{16}\left(\frac{a}{r}\right)^2} \tag{6.19}$$

For the clamped beam the corresponding expressions for w (as given in Table 3.1 or Ref. [88]), u (derived through the same procedure given for the hinged beam), P_{NL}, ω_{NL} and $\frac{\theta}{\omega_{REF}}$ are

$$w = \frac{a}{2}\left[1 - \cos\frac{2\pi x}{L}\right] \tag{6.20}$$

$$u = \frac{a^2\pi}{16L}\sin\frac{4\pi x}{L} \tag{6.21}$$

$$P_{NL} = P_L\left[1 + \frac{1}{16}\left(\frac{a}{r}\right)^2\right] \tag{6.22}$$

$$\omega_{NL}^2 = \omega_L^2\left[1 + \frac{3}{64}\left(\frac{a}{r}\right)^2\right] \tag{6.23}$$

For the clamped beam, the reference values used in defining the nondimensional quantities α, β and Ω_{NL0} are $P_{REF} = P_L = \frac{4\pi^2 EI}{L^2}$ and $\omega_{REF}^2 = \omega_L^2 = \frac{16\pi^4 EI}{3mL^4}$ to define the nondimensional quantities α, β and Ω_{NL0}.

$$\frac{\theta}{\omega_{REF}} = \Omega_{NL0} = 2\sqrt{(1-\alpha)(1 \pm \mu) + \frac{3}{64}\left(\frac{a}{r}\right)^2} \tag{6.24}$$

Figs.6.2 and 6.3 show the GNDI regions of the hinged and clamped beams respectively for the amplitude ratio of $\frac{a}{r} = 1$. Here, the reference values are taken as P_L and ω_L to evaluate the nondimensional quantities.

It can be seen from these figures that the GNDI regions differ for the hinged and clamped beams for these reference values.

Alternately, if the postbuckling load P_{NL} and the nonlinear radian frequency ω_{NL} are taken as the reference values, to define the nondimensional quantities α, β and Ω_{NL}, the nondimensional GNDI formula becomes

$$\frac{\theta}{\omega_{NL}} = \Omega_{NL} = 2\sqrt{(1-\alpha)(1 \pm \mu)} \qquad (6.25)$$

for both the hinged and clamped beams.

The GNDI regions obtained from Eqs. (6.19) and (6.24) are shown in Figs.6.2 and 6.3 respectively. From these figures, it is evident that the GNDI regions given in Figs. 6.2 and 6.3 using the reference values P_L and ω_L differs from the same obtained using the reference values P_{NL} and ω_{NL} as shown in Fig.6.4. The regions of GNDI obtained from Eq. (6.25), shown in Fig.6.4, are the same for the different values of α, as given in Table 6.1 represent the master GNDI regions. This shows that these regions are the same for both the linear (Table 3.4) and the GNDI behavior for the Euler- Bernoulli and Timoshenko beams.

This clearly shows the importance of the reference values used and Eq. (6.25) is independent of boundary conditions and the effect of the shear deformation and rotary inertia. Further Eq.(6.10) proposed in the formulation section and Eq.(6.25) derived directly, to obtain the GNDI regions are exactly the same suggesting that this equation is applicable not only for beams with or without the secondary effects but also to any other structural member provided the reference values used corresponding to the specific structural member.

The following salient points from this work may be noted:

a. The one term admissible function chosen, in the case of the hinged beam, is exact for both the postbuckling and the nonlinear vibration problems, for the first lateral deflection curve corresponding to the first postbuckling and large amplitude vibration problems, for a specified maximum lateral deflection/amplitude. As the spatial distribution gets canceled, if the admissible function is substituted in the governing nonlinear differential equations.

b. Similarly in the case of the clamped beam, the one term admissible function chosen, for the first deflection curve is exact for the postbuckling problem and slightly approximate for the large amplitude vibration problem. This is due to the fact that the spatial distribution gets canceled for the postbuckling problem and does not get canceled for the large amplitude vibration problem, for a specified maximum lateral deflection, if the admissible function is substituted in the corresponding governing nonlinear differential equations.

c. The GNDI formula derived with the reference values corresponding to the buckling load and the linear radian frequency are dependent on the boundary conditions.

d. If the reference values corresponding to the postbuckling load and the nonlinear radian frequency are used, the resulting GNDI formula is independent of the structural member and the boundary conditions. This formula is called as the master GNDI formula and this fact is not recognized till now by many researchers on this topic.

e. In most of the studies on the nonlinear dynamic instability, the results are available only for $\alpha = 0$. In this study, it has been shown that the master GNDI formula is valid for all the values of α.

6.3 CONCLUDING REMARKS

The master GNDI formula for predicting the nonlinear dynamic instability behavior of shear flexible (Timoshenko) beams subjected to periodic loads is derived by using proper nondimensional parameters and is applicable for the structural members, where the effect of geometric nonlinearity is of von-Karman type, used extensively for considering moderately large lateral deflections, is identically the same as the linear dynamic instability formula derived in earlier chapters.

Table 6.1 Variation of Ω_{NL1} and Ω_{NL2} for Timoshenko beams subjected to end concentrated periodic axial load (evaluated from present master formula)

μ	α = 0.0		α = 0.5		α = 0.8	
	Ω_{NL1}	Ω_{NL2}	Ω_{NL1}	Ω_{NL2}	Ω_{NL1}	Ω_{NL2}
0	2.0000	2.0000	1.4142	1.4142	0.8944	0.8944
0.1	1.8973	2.0976	1.3416	1.4832	0.8485	0.9380
0.2	1.7888	2.1908	1.2649	1.5491	0.8000	0.9797
0.3	1.6733	2.2803	1.1832	1.6124	0.7483	1.0198
0.4	1.5491	2.3664	1.0954	1.6733	0.6928	1.0583
0.5	1.4142	2.4494	1.0000	1.7320	0.6324	1.0954

Table 6.2 Variation of Ω_{NL01} and Ω_{NL02} for hinged Timoshenko (shear flexible) beam subjected to end concentrated periodic axial load for α =0

μ	a/r = 0.4		a/r = 1.0		a/r = 1.4	
	Ω_{NL01}	Ω_{NL02}	Ω_{NL01}	Ω_{NL02}	Ω_{NL01}	Ω_{NL02}
0	2.0312 (2.0305)[a]	2.0312 (2.0305)	2.1869 (2.1837)	2.1869 (2.1837)	2.3540 (2.3466)	2.3540 (2.3466)
0.1	1.9268 (1.9295)	2.1303 (2.1267)	2.0746 (2.0901)	2.2936 (2.2734)	2.2331 (2.2598)	2.4688 (2.4303)
0.2	1.8167 (1.8229)	2.2249 (2.2187)	1.9560 (1.9921)	2.3956 (2.3598)	2.1054 (2.1695)	2.5785 (2.5113)
0.3	1.6994 (1.7096)	2.3158 (2.3071)	1.8297 (1.8891)	2.4934 (2.4431)	1.9694 (2.0752)	2.6839 (2.5897)

[a] Values given in the parentheses are Suresh *et al.*[110]

Table 6.3 Variation of Ω_{NL01} and Ω_{NL02} for clamped Timoshenko (shear flexible) beam subjected to end concentrated periodic axial load for $\alpha = 0$

μ	a/r = 0.4		a/r = 1.0		a/r = 1.4	
	Ω_{NL01}	Ω_{NL02}	Ω_{NL01}	Ω_{NL02}	Ω_{NL01}	Ω_{NL02}
0	2.0150 (2.0081)[a]	2.0150 (2.0081)	2.0449 (2.0505)[a]	2.0449 (2.0505)	2.0898 (2.0978)	2.0898 (2.0978)
0.1	1.9115 (1.9059)	2.1133 (2.1054)	1.9399 (1.9505)	2.1447 (2.1458)	1.9824 (2.0002)	2.1917 (2.1911)
0.2	1.8022 (1.7979)	2.2072 (2.1983)	1.8290 (1.8451)	2.2401 (2.2371)	1.8691 (1.8976)	2.2891 (2.2805)
0.3	1.6858 (1.6830)	2.2974 (2.2875)	1.7109 (1.7333)	2.3315 (2.3247)	1.7567 (1.7891)	2.3826 (2.3666)

[a] Values given in the parentheses are from Suresh *et al.*[110]

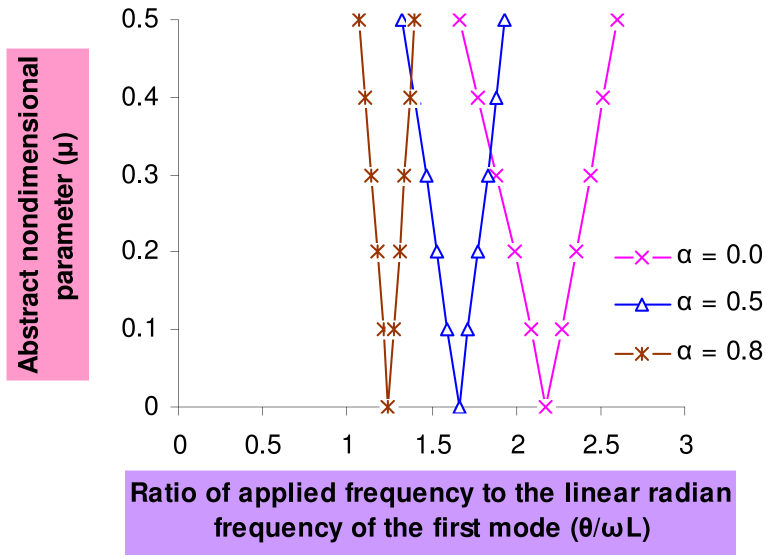

Fig. 6.2 GNDI regions for hinged beam with reference values P_L and ω_L.

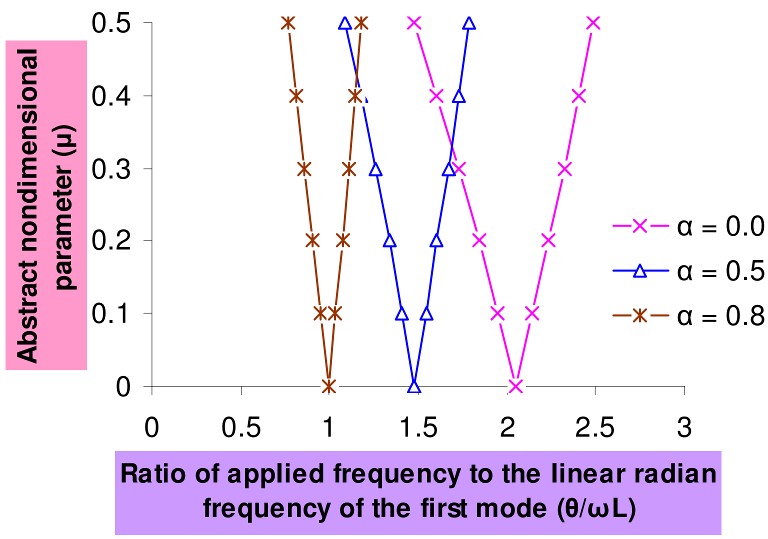

Fig. 6.3 GNDI regions for clamped beam with reference values P_L and ω_L.

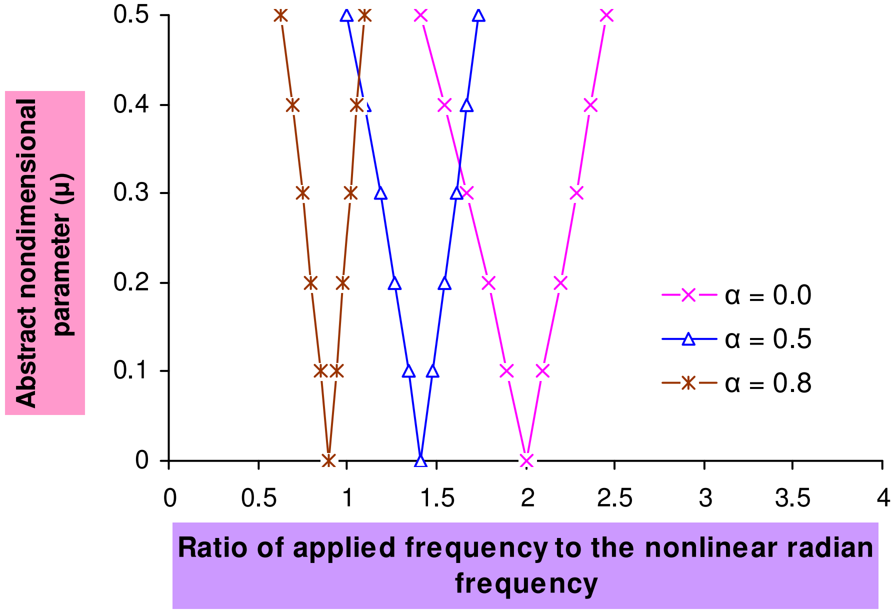

Fig.6.4 Master GNDI curves with reference values P_{NL} and ω_{NL}.

Overall Conclusions

The work carried out in the present thesis is aimed to develop simple analytical formulas to study the dynamic stability behavior of the structural members, used in many fields of engineering, subjected to periodic loads. These formulas developed in the present thesis, when compared to rigorous mathematical formulations, are simple and attractive to the practicing engineers and researchers in this field, to get quick, accurate and reliable solutions for predicting the dynamic stability behavior of the structural members.

Based on the results obtained in the present work, the main conclusions of the methods presented in the thesis are highlighted below:

- A general analytical dynamic stability formula has been developed to predict the dynamic stability behavior of uniform beams subjected to axial periodic loads. It is observed that these formulas are independent of the beam boundary condition. The effect of one (Winkler) and two parameter (Pasternak) elastic foundations on the dynamic stability regions of beams, below and above the first transition foundation value, clearly brings out the effectiveness of the formulas.

- It is shown that the general dynamic stability formula is applicable not only for the uniform beams but also for the non uniform (linearly tapered) beams.

The general analytical formulas for predicting the dynamic stability behavior of i) the beam subjected to an axial periodic and static compressive and tensile loads and ii) the dynamic stability of square plate subjected to periodic in-plane load in one direction and a compressive static load in the perpendicular direction clearly show the effectiveness of these formulas for both the one and two dimensional problems.

- A simple master formula has been developed for the evaluation of natural frequencies of any initially loaded structural member.

A critical observation of these general formulas developed for the specific structural members like the beams with or without elastic foundation and the plates mentioned above, if written with the proper non-dimensional form, gives the same dynamic stability formulas. This is an indication to postulate the existence of a master dynamic stability formula applicable for any structural member with several complicating effects. Further studies, based on this postulation, on the dynamic stability studies are enumerated below:

1. A master dynamic stability formula applicable for any structural member, with complicating effects, subjected to the periodic loads has been developed.

2. Two more different master dynamic stability formulas, using the concept of effective stiffness of the structural members are developed as,

   a) Using the effective stiffness of the structural members, the regions of the dynamic stability curves are obtained with the dynamic load factor, which are easier to understand in physical terms.

   b) The regions of dynamic stability curves starting from 2 on the Ω-axis developed with the non-dimensional parameter defined as in item 1, for the dynamic part of the periodic load, gives only one single dynamic stability curve, irrespective of the non-dimensional static part of the periodic load.

3. Finally a very simple dynamic stability point has been proposed.

It is seen from the dynamic stability regions that the number of master dynamic stability curves exhibiting the instability regions decrease successively for item 1 to 2(b) and reduces to a point.

- A master formula has been developed to predict the dynamic instability including the effect of geometric nonlinearity, of shear flexible beams subjected to end axial periodic loads. This formula is the same as the master dynamic stability formula obtained without considering the geometric nonlinearity, when proper reference frequency and buckling load are used.

The results obtained from the present investigation are in good agreement with those obtained by applying the finite element and other solutions.

Based on the present study it is emphasized here that the dynamic stability behavior of the structural members can be quickly and accurately predicted, without resorting to the higher order mathematical formulations and this reduces the effort of the practicing engineers in this field by orders of magnitude.

Further Scope of Work

Further scope for improvement in the simple formulations and master dynamic stability formulas developed during the course of this work need to be extended to study:

- The effect of damping on the dynamic stability regions of structural members. The effect of both the internal and external damping on the dynamic stability behavior can be studied.

- To study the response of the dynamic stability behavior of the structural members for the materials that exhibit nonlinearly elastic and inelastic behavior. This study is necessary when the critical loads of the structural member are beyond the yield point of the material.
- The dynamic stability behavior of structural members can be studied, using a general time function, involved in the dynamic part of the load, instead of the single periodic part.
- The regions of dynamic stability can be studied, when multiple periodic loads, with a phase difference is applied on the structural members.
- The validity of the present numerical studies needs to be augmented with the experimental studies.
- Finally, the feasibility of the development of simple formulas to study the dynamic stability behavior of structural systems that are combinations of the structural members has to be established.