

**CHAPTER 5****DYNAMIC STABILITY OF STRUCTURAL MEMBERS****5.1 DEVELOPMENT OF MASTER DYNAMIC STABILITY FORMULA FOR STRUCTURAL MEMBERS**

The master dynamic stability formula for structural members subjected to periodic loads hitherto not recognized by the other researchers is presented in this section. Prediction of the dynamic stability behavior of structural members subjected to periodic loads is necessary for assessing the integrity of the structural members, used in many fields of engineering.

The theory and application of the dynamic stability behavior of the structural members using analytical methods have been exhaustively given in the classic work of Bolotin [17]. Though rigorous analytical solutions to evaluate the dynamic stability boundaries are attractive, it is often difficult to obtain such solutions and it is easier to obtain these boundaries by employing the approximate continuum methods or the numerical method such as the versatile FE method. In either case, the governing equation of equilibrium is obtained in the form of a generalized matrix equation which is the same for all structural members, the concerned matrices involved are the system stiffness, geometric and mass matrices, the order of which varies depending on the structural member considered.

The derivation of the master dynamic stability formula and its general applicability to the structural members is the main contribution of the work in this Thesis. As the existence of the master formula is not recognized by the researchers till date, for a better appreciation of the master dynamic stability formula, some of the salient points discussed in the earlier chapters, are briefly reproduced here.

Brown *et al.* [18] observed that in their very interesting study, the first mode shapes of buckling and vibration are the same or nearly the same, which is generally satisfied for the many beam boundary conditions, the dynamic instability regions are more or less remain the same by using proper non-dimensionalization of the basic physical parameters of the problem. In this study [18], Brown *et al.* chosen these non-dimensional parameters intuitively and the similarity of the free vibration and buckling mode shapes is an observation. As such, an assertive statement cannot be made about the generality of the insensitivity of the dynamic stability regions of other structural members. For any other non-dimensionalization of these basic quantities, a series of dynamic stability curves will be obtained and are relatively difficult to analyze for proper understanding of the dynamic stability behavior of the structural members. A few earlier investigations on the dynamic stability behavior of some column boundary conditions based on brown *et al.* [18] can be seen in the works [19, 20] with the

same non-dimensionalization and obtained for the master dynamic stability regions in either the analogue or digital forms. The numerical results presented in these studies [18-20] show the near invariance of the dynamic stability boundaries. Some recent studies on dynamic stability for plates and shells can be seen with different non-dimensionalizations [92, 95] and hence the problem of analyzing a number of dynamic stability curves. The main aim of the present study is:

- a. to derive a simple, elegant and accurate formula (master dynamic stability formula) to predict the dynamic stability behavior of most of the structural members where the mode shapes of the fundamental frequency and the buckling load are same or nearly the same, subjected to an concentrated end or edge periodic axial or in- plane loads respectively
- b. to show the effectiveness of the master dynamic stability formula with reference to the beams, plate and shell problems and
- c. to establish the invariance of the dynamic stability regions, using the non-dimensional parameters derived in this chapter in the clear mathematical sense.

### 5.1.1 Formulation

When the structural members are subjected to axial or in-plane periodic loads of the form

$$N(t) = N_s + N_t \cos \theta t \quad (5.1)$$

where  $N(t)$  is the axial/edge in-plane compressive periodic load,  $N_s$  is the constant compressive load,  $N_t$  is the periodic load,  $\theta$  is the radian frequency and  $t$  is the time variable.

The dynamic stability boundaries can be obtained by Bolotin [17] following any approximate continuum method from the solution of the following matrix equation of equilibrium, as

$$[K]\{\delta\} - \left( N_s \pm \frac{N_t}{2} \right) [G]\{\delta\} - \frac{\theta^2}{4} [M]\{\delta\} = 0 \quad (5.2)$$

where  $[K]$ ,  $[G]$  and  $[M]$  are the system stiffness matrix, system geometric stiffness matrix and system mass matrix respectively and  $\{\delta\}$  is the eigenvector.

The main aim of this present study is to derive a master dynamic stability formula, which is presented in this chapter in the form this matrix equation for structural members.

From Eq. (5.2), the governing equation for the vibration problem, neglecting the load term, and after simplification is written, as

$$[M]\{\delta_1\} = \frac{1}{\omega^2} [K]\{\delta_1\} \quad (5.3)$$

Similarly, by neglecting the inertia term, the solution of the stability problem in terms of critical load can be obtained from the degenerate case of Eq. (5. 2) is written, as

$$[G]\{\delta_2\} = \frac{1}{N_{cr}}[K]\{\delta_2\} \quad (5.4)$$

Here, a reasonable assumption for the first mode which is valid for most of the structural members can be made as given in Eq. (3.114), is shown here, as

$$\{\delta_1\} = \{\delta_2\} = \{\delta\}$$

Further, it is emphasized here that a single term admissible function for the lateral displacement is used, in the case of approximate continuum methods; as a general practice for simplicity of the formulation Eq. (3.114) is exactly satisfied. Otherwise this equation, in general, may be exactly or with minor difference satisfied, depending on the boundary conditions within the tolerable limits of engineering practice.

Substituting, the Eqs. (5. 3), (5. 4) in Eq. (5.2), we get

$$[K]\{\delta\} - (N_s \pm \frac{N_t}{2}) \frac{1}{N_{cr}} [K]\{\delta\} - \frac{\theta^2}{4} \frac{1}{\omega^2} [K]\{\delta\} = 0 \quad (5.5)$$

or

$$[K]\{\delta\} - (\alpha \pm \beta/2)[K]\{\delta\} - \frac{\theta^2}{4\omega^2} [K]\{\delta\} = 0 \quad (5.6)$$

where  $\alpha = \frac{N_s}{N_{cr}}$ ,  $\beta = \frac{N_t}{N_{cr}}$  and  $\omega$  is the natural radian frequency.

Equation (5.6), written as

$$\frac{\theta^2}{4\omega^2} = 1 - \left( \alpha \pm \frac{\beta}{2} \right) = (1 - \alpha)(1 \pm \mu) \quad (5.7)$$

Hence, the ratios of

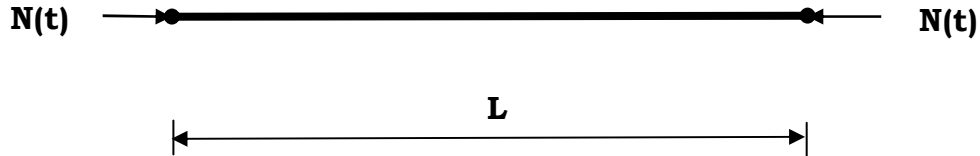
$$\frac{\theta}{\omega} = \Omega = 2\sqrt{(1 - \alpha)(1 \pm \mu)} \quad (5.8)$$

where  $\Omega$  and  $\mu$  are the non-dimensional parameters used by Brown *et al.* [18] by intuition and are systematically derived here. It is seen from the Eq.(5.8), is the same as that given in Eq.(3.30), the general dynamic stability formula derived for the beams. In Eq.(5.8), these non-dimensional parameters are independent of the characteristic values of the  $\omega$  and  $N_{cr}$  (critical load) of the structural members considered and it can be used as a master dynamic stability formula as it is the most general form and is invariant for the structural members considered. However, to obtain the absolute values of the dynamic stability boundaries of a specific structural member, the corresponding values of  $\omega$  and  $N_{cr}$  have to be used.

### 5.1.2 Numerical Results and Discussion

The master dynamic stability formula derived in this chapter to predict the dynamic stability boundaries of structural members subject to periodic loads is very general. In the non-dimensional form, the characteristic values of the structural members such as the fundamental frequency and the static buckling load parameters do not appear

explicitly and hence is valid for any structural member with complicating effects like shear deformation, rotary inertia, complex boundary constraints, effect of taper, material types used etc. However, these characteristic values appear implicitly in the definitions of  $\mu$  and  $\Omega$ .

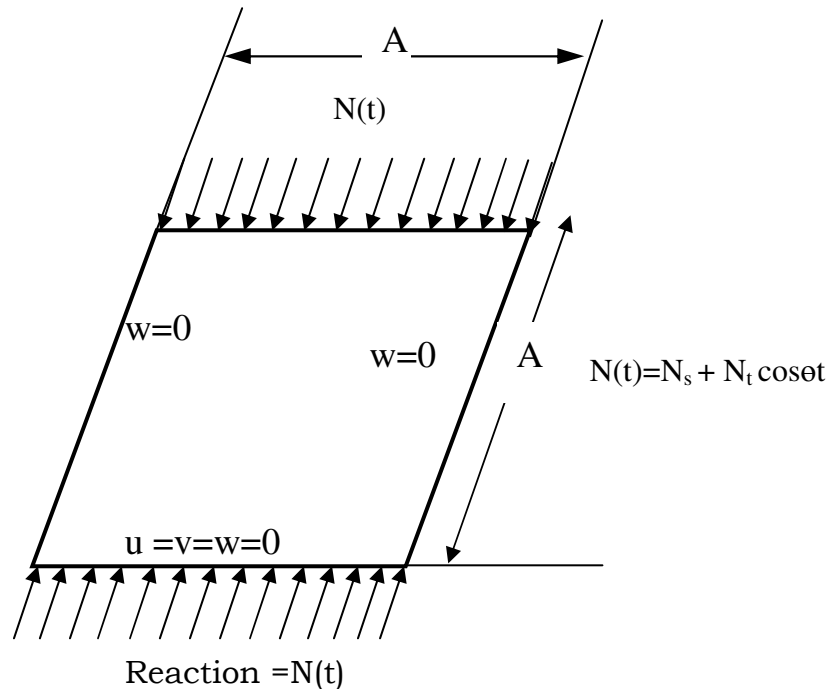


**Fig.5.1 Uniform beam subjected to end periodic compressive axial loads.**

The absolute dynamic stability boundaries for a specific structural member can be obtained by using the corresponding characteristic values available in the hand books or in literature. The main advantage of using this master dynamic stability formula is demonstrated with reference to the following structural members: a uniform beam with any boundary condition (Fig.5.1) subjected to an end axial periodic concentrated loads, a layered square plate (Fig. 5.2) studied in [92] subjected to uniform edge in-plane periodic load and a layered cylindrical shell (Fig.5.3) with delamination subjected to uniform end periodic load studied in [95].

For the beam problem, the dynamic instability boundaries  $\Omega_1$  and  $\Omega_2$ , between which it is dynamically unstable are given with varying  $\mu$  and for  $\alpha = 0.0, 0.5$  and  $0.8$  in Table 5.1. These values obtained are independent of boundary conditions. The values of Brown *et al.*[18],

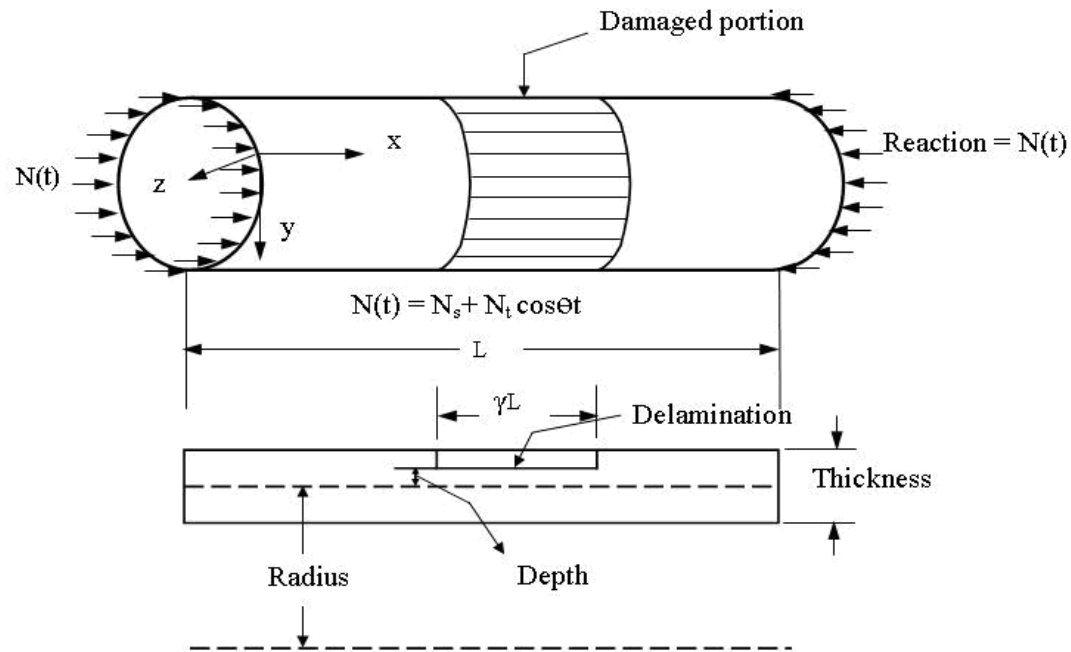
obtained from the finite element analysis evaluated for any boundary condition and reduced into the non-dimensional form  $\mu$  and  $\Omega$  for the same  $\alpha$  values from the master stability curves, named by Brown *et al.* [18], are also included in Table 5.1. The digital and analogue forms of results presented here and the numerical difference of the results between the present study and with those of [18] are shown in Table 5.1. A good agreement between the present results obtained from the simple master dynamic stability formula for the dynamic stability of the beams shows the effectiveness and universal application of the derived formula used here.



**Fig.5.2 simply supported composite skew plate under uniaxial periodic loads.**



Another problem considered is the dynamic stability of a layered square plate studied by Dey and Singha [92] using the FE method. The formulation takes care of the shear deformation and rotary inertia. The dynamic stability regions of the layered square plate is given in this study in terms of the non-dimensional parameters [92] and the present results are deduced from the master dynamic stability table for  $\alpha = 0$  as is given in Table.5.2. The excellent agreement between the two results strongly indicates the usefulness of the present simple formula.



**Fig.5.3 Composite cylindrical Shell with delamination under uniform end periodic axial loads**

The third and last problem considered is, a layered cylindrical shell with delamination throughout the circumference (Fig. 5.3). The length and depth of the delamination and the end periodic load are shown in

this figure. The non-dimensional forms used to obtain the dynamic stability curves in [95] are different in that study, the numerical results of the master dynamic stability table have to be reduced to be consistent with this non-dimensionalization and these results are shown in Table 5.3. Note that, a very good agreement between the present reduced results and with the literature [95].

These three problems considered in this study show the usefulness and the general nature of the master dynamic stability table and are valid for the condition where the eigenvectors for the characteristic values of the structural members are same or nearly the same. These three problems considered here are, entirely different structural members and the successful application of this master dynamic stability formula and the subsequent master dynamic stability table simplifies the dynamic stability analysis of the structural members by orders of magnitude. Fig.5.4 shows the master dynamic stability curves for any structural member subjected to end concentrated axial/edge in-plane periodic load, which are non-dimensional, as  $\Omega_1$  and  $\Omega_2$ , between these values the structural member will be unstable.

A master dynamic stability formula is derived in the present study. In some of the earlier studies on this topic, the non-dimensional parameter appearing in the derived formula in this study is taken by

intuition. Where as, the same non-dimensional parameters are derived systematically with rigorous mathematical sense in this section. The assumption made in deriving this formula is that the mode shapes of the lowest characteristic values such as the frequency and the buckling load parameters for most of the structural members are the same or nearly the same. This assumption is generally valid for the first mode of free vibration and buckling, which is of practical importance of many widely used structural members. However, other researchers who used the non-dimensional parameters defined in the present work observed that the first free vibration and buckling mode are more or less the same. In this study, it is shown that this condition is necessary to be used to derive the master dynamic stability formula which contains the non-dimensional parameters identified by others intuitively. The effectiveness of the master dynamic stability formula and the master dynamic stability tables is demonstrated with reference to the dynamic stability of a column, a layered square plate and a layered circular shell with delaminations, subjected to periodic loads. However, care should be exercised to use this formula indiscriminately without checking the similarity of the mode shapes. Further work is necessary to modify the present master dynamic stability formula to demonstrate its applicability if the mode shapes of the characteristic values is different. Finally, it is to be noted that the absolute values of the instability regions are dependant on the values of the frequency and the buckling load of the

structural members. Evaluation of these quantities is a standard procedure and can be easily obtained by applying continuum or numerical methods or from the Hand books or from the literature. The present formula derived can be effectively used with the secondary effects.

## **5.2 ALTERNATE MASTER DYNAMIC STABILITY FORMULAS**

With the knowledge gained based on the work presented in section 5.1, two more simple alternate master dynamic stability formulas for structural members subjected to periodic loads has been successfully developed in this section.

The motivation of the present study is to present the existence of not only one but also another two simple master dynamic stability formulas, indicating the relative ease of these formulas in the physical interpretation of the dynamic instability regions.

In all the earlier studies on the prediction of the dynamic stability regions, the natural frequency of the structural member is obtained corresponding to the stiffness of the basic structural configuration. In this chapter, the natural frequency of the structural member is evaluated corresponding to the effective stiffness by including the effect of the constant static compressive load that is treated as the initial load.

### 5.2.1 Master Dynamic Stability Formulas (MDSFs)

When the structural members are subjected to axial /edge in-plane periodic load of the form as given in Eq.(5.1), is

$$N(t) = N_s + N_t \cos \theta t$$

Using the matrix equation of equilibrium, in the form of Eq. (5.2), in the following section, three MDSFs are presented with a brief derivation for each formula.

#### 5.2.1.1 MDSF I

Using the basic structural configuration, from the Eq. (5.2), the dynamic stability regions is written, as

$$\frac{\theta}{\omega} = \Omega = 2\sqrt{(1-\alpha)(1\pm\mu)}$$

The complete explanation is not given here for the sake of brevity, as it is shown in Eq.(5.8). It is seen from the above equation that these non-dimensional parameters are independent of the characteristic values,  $\omega$  and  $N_{cr}$ , of the structural members considered and can be termed as MDSF I.

#### 5.2.1.2 MDSF II

The significant feature of the MDSF II discussed here is that the natural frequencies of the structural members are calculated corresponding to the effective stiffness, including the constant static part of the periodic load, contrary to MDSF I.

To derive this formula, Eq. (5.2) is written as

$$[K_{eff}]\{\delta\} - \left(\pm \frac{N_t}{2}\right)[G]\{\delta\} - \frac{\theta^2}{4}[M]\{\delta\} = 0 \quad (5.9)$$

$$\text{where } [K_{eff}] = [K] - \alpha N_{cr}[G] \quad (5.10)$$

The procedure followed for MDSF I, the degenerate cases of the free vibration and the buckling problems can be obtained as

$$[M]\{\delta\} = \frac{1}{\omega^2}[K_{eff}]\{\delta\} \quad (5.11)$$

and

$$[G]\{\delta\} = \frac{1}{N_{cr}}[K]\{\delta\} \quad (5.12)$$

The discussion on the mode shapes of the vibration and the buckling presented earlier holds good here also.

Substituting Eqs. (5.11) and (5.12) in Eq. (5.9), we get

$$[K_{eff}]\{\delta\} - \left(\pm \frac{N_t}{2}\right) \frac{1}{N_{cr}}[K]\{\delta\} - \frac{\theta^2}{4} \frac{1}{\omega^2}[K_{eff}]\{\delta\} = 0 \quad (5.13)$$

Equation (5.13) can be written, considering Eq. (5.10), as

$$(1 - \alpha)[K]\{\delta\} \pm \frac{\beta}{2}[K]\{\delta\} - \frac{\theta^2}{4\omega^2}(1 - \alpha)[K]\{\delta\} = 0 \quad (5.14)$$

Equation (5.14) implies that

$$\frac{\theta^2}{4\omega^2} = 1 \pm \frac{\beta}{2(1 - \alpha)} \quad (5.15)$$

Hence, the non-dimensional parameter  $(\theta/\omega)$  is obtained from Eq. (5.15)

as

$$\frac{\theta}{\omega} = \Omega = 2\sqrt{1 \pm \frac{\beta}{2(1 - \alpha)}} \quad (5.16)$$

Using Eq. (5.16), the dynamic stability boundaries are obtained for a given  $\alpha$ , in terms of the non-dimensional parameters  $\Omega$  and  $\beta$ . It is seen from Eq. (5.16) that in these non-dimensional parameters the values  $\omega$  and  $N_{cr}$  of the structural members do not appear explicitly and in the non-dimensional form (using  $\beta$  and  $\Omega$ ) they can be used as second master dynamic stability formula (MDSF II).

### 5.2.1.3 MDSF III

The main feature of MDSF III is that the natural frequencies of the structural members are evaluated corresponding to the effective stiffness, including the static part of the periodic load and the regions of dynamic stability are evaluated in terms of the abstract non-dimensional parameter  $\mu$ , contrary to the physical non-dimensional parameter  $\beta$  as taken in MDSF II.

Substituting  $\mu = \frac{\beta}{2(1-\alpha)}$  in Eq. (5.16), we get

$$\frac{\theta}{\omega} = \Omega = 2\sqrt{1 \pm \mu} \quad (5.17)$$

Using Eq. (5.17), the dynamic stability regions/boundaries are obtained in terms of the non-dimensional physical parameters  $\Omega$  and  $\mu$ . The interesting feature of the MDSF III is that only one dynamic stability curve is obtained for all the values of  $\alpha$ . It may be noted here that the MDSF<sub>s</sub> I to III are called the master dynamic stability formulas. These

formulas are of a general forms and are invariant with respect to the structural members considered.

By using the basic stiffness, the master dynamic stability formula I (MDSFI) is obtained, with abstract non-dimensional parameter for the load terms employed by many researchers. The concept of the effective stiffness that gives a different form of the master dynamic stability formula which in turn clearly gives the dynamic stability regions without any ambiguity with physically meaningful non-dimensional constant static load, the possibility of obtaining another very simple master dynamic stability formula as a corollary of the second formula is discussed here by introducing the same non-dimensionalization of the constant load as proposed in MDSFI. Based on the relative ease of interpretation of the dynamic instability regions, obtained using the three master dynamic stability formulas, it is found that the second master dynamic stability curve is the best one, in terms of the physical interpretation. The proposed master dynamic stability formulas are valid for any structural members that exactly or approximately satisfy the condition of the mode shapes demonstrated in Table 3.2. The results of the present study are presented for isotropic slender beam, composite skew plate and composite cylindrical shell with delamination to demonstrate the existence and usefulness of the master dynamic stability regions.



#### 5.2.1.4 Numerical Results and Discussion

The two simple alternate formulas derived in the present chapter to predict the dynamic stability regions of the structural members subjected to periodic loads, including a constant static load. In the non-dimensional form, using  $\mu$ ,  $\beta$  and  $\Omega$ , the frequency and the static buckling load parameters of the structural members do not appear explicitly and hence these two formulas are valid for any structural member. The advantage of using these formulas is demonstrated with reference to the following structural members: a uniform beam with any boundary condition (Fig.5.1) subjected to an end periodic axial loads[18], a composite skew plate (Fig.5.2) studied in[92], subjected to a uniform edge periodic load and a composite cylindrical shell (Fig.5.3) with delamination [95].

Table 5.1 gives the values of the dynamic stability boundaries  $\Omega_1$  and  $\Omega_2$ , obtained from MDSF I evaluated using the buckling and the fundamental radian frequency, with varying  $\mu$ , for  $\alpha = 0.0, 0.5$  and  $0.8$ , between which the structural member is dynamically unstable. The good agreement of the dynamic stability boundaries  $\Omega_1$  and  $\Omega_2$  can be seen from the results obtained by using the proposed MDSF I with those given in [18]. These values are in excellent agreement with the three structural members considered here, using the basic stiffness (section 5.1). The curves presented in Fig. (5.4) are master dynamic stability curves (MDSF

I) for the structural members considered in this study, containing the abstract non-dimensional parameter  $\mu$  that is dependent on the values of  $\alpha$  and  $\beta$ , and the nondimensional parameter  $\Omega$  presented in Eq.(5.8).

Table 5.4 gives the values of the dynamic stability boundaries  $\Omega_1$  and  $\Omega_2$ , for the three structural members, considering the effective stiffness with varying  $\beta$ , for  $\alpha = 0.0, 0.2$  and  $0.4$ . The dynamic stability boundaries of  $\Omega_1$  and  $\Omega_2$  can be seen from the results obtained by using the proposed MDSF II, which match well with those given for the composite shell problem in Jinhua and Yaming [95]. In Fig.5.5, the corresponding master dynamic stability curves (MDSF II) are presented for the three structural members considered in this study. It is observed from this figure, that the dynamic stability curves start with the value of 2.0 when  $\beta = 0.0$  for all values of  $\alpha$  considered and the dynamic stability regions are increasing with increasing constant static load factor ( $\alpha$ ). The curves presented in this figure unambiguously show the physical trend that the compressive static load has an effect on the increasing, the dynamic instability regions.

It may be noted here that the master dynamic stability curves (MDSF I) contain the abstract nondimensional parameter  $\mu$  that is dependant on the values of  $\alpha$  and  $\beta$ . These curves do not give directly the physical trends observed from the dynamic stability curves proposed

using MDSF II. These curves, given in Figs.5.4 and 5.5, for different values of  $\alpha$ , give an impression that the dynamic instability regions decrease with increasing  $\alpha$ , which is difficult to appreciate.

Table 5.5 gives the values of the dynamic stability boundaries  $\Omega_1$  and  $\Omega_2$ , with varying  $\mu$ , obtained by using MDSF III for a composite skew plate, obtained considering the effective stiffness, subjected to periodic load for  $\alpha = 0.0$ . Table 5.6 gives the values of the dynamic stability boundaries  $\Omega_1$  and  $\Omega_2$ , for the three structural members, considering the effective stiffness with varying  $\mu$ . The instability boundaries of  $\Omega_1$  and  $\Omega_2$  can be seen from the results obtained by using the proposed MDSF III, which match well with those given in Table 5.5 for the composite skew plate problem. The curve presented in Fig. 5.6 is the master dynamic stability curve obtained from MDSF III for the three structural members considered in this study, and is different from those given in Fig.5.4 and 5.5, as it contains the abstract nondimensional parameter  $\mu$  (as shown in Fig.4.4) and the dynamic load factor  $\beta$ , and the nondimensional parameter  $\Omega$ . It is also observed from this Fig.5.6, that the dynamic stability curve start with the value of 2.0 when  $\mu = 0$ , and it is a single curve unlike the master dynamic stability curves obtained from the MDSF I and II.

However, these three master dynamic stability formulas give the results when the nondimensional values are converted to the absolute

values of the instability boundaries of a specific structural member by taking the corresponding values of  $\omega$  and  $N_{cr}$ .

### **5.3 CONCEPT OF MASTER DYNAMIC STABILITY POINT**

Based on the work presented in earlier sections, the existence of the dynamic stability point for structural members subjected to periodic loads has been successfully identified in this section.

The aim of the present study is to demonstrate the use of the dynamic stability point applicable to any structural member. In all the earlier studies on the prediction of the regions of dynamic instability, the natural frequency of the structural members is obtained corresponding to

- i) the stiffness of the basic structural configuration, by incorporating the constant static part of the periodic load in the geometric stiffness matrix.
- ii) the effective stiffness by including the effect of the constant static part of the periodic load that is treated as the initial load.

In this section, the natural frequency of the structural member is evaluated corresponding to the effective stiffness by considering the combination of the constant static and periodic loads that is treated as the initial load.

The concept of the effective stiffness by including the effect of constant static load and periodic part of the load that is treated as the initial load, in this section, the natural frequency correspond to effective stiffness, that gives a different form of the master dynamic stability formula, which in turn clearly gives all the information presented in the earlier discussed three simple dynamic stability formulas to predict the dynamic stability regions by a point.

Finally, the motivation of the present work is to propose a novel concept of the dynamic stability point from which the dynamic stability regions, of any structural member with different boundary conditions, can be easily evaluated. This concept further simplifies the evaluation of the dynamic instability regions of any structural member with different boundary conditions and secondary effects mentioned earlier. The procedure to evaluate the dynamic stability regions, using the concept of the master dynamic stability point, from the general dynamic stability equation, using a different effective stiffness by adding the static and dynamic parts of the periodic load to the stiffness, is clearly presented in the following section.

### 5.3.1 Formulation

The standard dynamic stability equation for a given structural member, subjected to axial /edge in-plane periodic loads, in the matrix equation of equilibrium is rewritten as

$$[K_{eff}]\{\delta\} - \frac{\theta^2}{4}[M]\{\delta\} = 0 \quad (5.18)$$

$$\text{where } [K_{eff}] = [K] - \left(N_s \pm \frac{N_t}{2}\right)[G] \quad (5.19)$$

Following the procedure followed in the previous sections 5.1 and 5.2, the governing matrix equation for the free vibration problem of the structural member considered, with the same matrix equation given in Eq. (5.18), can be written as

$$[M]\{\delta_1\} = \frac{1}{\omega^2}[K_{eff}]\{\delta_1\} \quad (5.20)$$

Substituting, Eq. (5.20) in Eq. (5.18), we get

$$[K_{eff}]\{\delta\} - \frac{\theta^2}{4} \frac{1}{\omega^2}[K_{eff}]\{\delta\} = 0 \quad (5.21)$$

The discussion on the mode shapes of dynamic stability and vibration presented earlier holds good here also.

Since  $\{\delta\} \neq 0$ , Equation (5.21) becomes

$$\theta = 2\omega \quad (5.22)$$

Hence, the non-dimensional parameter containing  $\theta$  and  $\omega$  can be written, from Eq. (5.22) as

$$\Omega = \frac{\theta}{\omega} = 2 \quad (5.23)$$

From Eq. (5.23), it can be seen that the dynamic instability regions collapse to a point, in terms of the nondimensional parameter  $\Omega$ . In Eq.

(5.23),  $\omega$  is obtained with the initial load of  $N_s \pm \frac{N_i}{2}$ . In the present study,

the initial load parameters are  $\lambda_i = \frac{N_i L^2}{EI}$  for beams and  $\frac{N_i L^2}{\pi^2 D}$  for the

square plate, and the buckling load parameters are  $\lambda_b = \frac{NL^2}{EI}$  and  $\frac{NL^2}{\pi^2 D}$ .

The radian frequency parameter of the initially loaded structural member

is  $\lambda_f = \frac{\bar{m} \omega^2 L^4}{EI}$  and  $\frac{\bar{m} \omega^2 L^4}{D}$  and the radian frequency parameter without

initial load is  $\lambda_{f_0} = \frac{\bar{m} \omega_o^2 L^4}{EI}$  and  $\frac{\bar{m} \omega_o^2 L^4}{D}$ , for the beams and plates

respectively.

where  $N_i$  and  $\lambda_i$  are the initial load and initially loaded buckling

parameter respectively,  $L$  is the length of the beam,  $D$  is the flexural

rigidity of the plate,  $\omega_o$  and  $\lambda_{f_0}$  are the natural frequency and frequency

parameter with out initial load of the structural members respectively.

Form the Eq. (4.10), the initially loaded frequency parameter is

$$\lambda_f = \lambda_{f_0} \left[ 1 - \frac{\lambda_i}{\lambda_b} \right] \quad (5.24)$$

Substituting, the load parameters in the above equation is written as,

$$\lambda_f = \lambda_{f_0} \left[ 1 - \frac{\frac{N_i L^2}{EI}}{\frac{N_{cr} L^2}{EI}} \right] \quad (5.25)$$

Substituting, the frequency parameters in the above equation is written, as

$$\frac{\bar{m}\omega^2 L^4}{EI} = \frac{\bar{m}\omega_0^2 L^4}{EI} \left[ 1 - \left( \alpha \pm \frac{\beta}{2} \right) \right] \quad (5.26)$$

Please note that  $\beta$  is used in the second alternate master dynamic stability formula as given in Eq. (5.16).  $N_{cr}$  is the buckling load, which is used to non-dimensionalize the components of the periodic load

$$\text{as } \alpha = \frac{N_s}{N_{cr}}, \quad \beta = \frac{N_t}{N_{cr}} \quad \text{and} \quad \mu = \frac{\beta}{2(1-\alpha)}$$

Equation (5.26), after simplification gives the relation between  $\omega^2$  and  $\omega_0^2$  as

$$\omega^2 = (1-\alpha)(1\pm\mu)\omega_0^2 \quad (5.27)$$

Equation (5.27) can be used to evaluate the radian frequency of the initially loaded structural member in terms of the radian frequency without the initial load. The value of  $\omega$ , evaluated from Eq. (5.27) can be used in Eq. (5.23) to obtain the master dynamic stability formula of any structural member as

$$\Omega = \frac{\theta}{\omega_0} = 2\sqrt{(1-\alpha)(1\pm\mu)} \quad (5.28)$$

### 5.3.2 Numerical Results and Discussion

The effectiveness of the novel concept of the dynamic stability point, proposed in this chapter, is demonstrated with the results of the dynamic stability problems related to uniform beams and square plates.



For the beam problem the results of section 5.1, which gives the same general dynamic stability results with the Ref.[18] is considered. The dynamic stability results using the concept of the master dynamic stability point are exactly the same, when compared with those given in section 5.1 and Ref.[18], for all the boundary conditions, as the master dynamic stability formulas are the same. Hence the numerical results are not presented here for the sake of brevity. The most important proof of validity of master dynamic stability point is shown through the results of the dynamic stability of a square plate under edge uniaxial load, which is studied very recently [93].

Table 5.7 gives the numerical results for the dynamic stability regions of the square plate obtained using the master dynamic stability point. The present results are in good agreement, with those obtained following a entirely different method [93], for  $\alpha = 0$  for several values of the nondimensional parameter  $\beta$  and the  $\Omega$ . Similar results for  $\alpha = 0.6$  given for the square plate, presented in Table 5.8, compare very well with those presented in [93]. In Table 5.1, the results obtained from the master dynamic stability point are given for  $\alpha = 0, 0.5$  and  $0.8$  and for several values of  $\mu$  and  $\Omega$ . This Table can be used to predict the dynamic stability of any structural member. For better comprehension, these numerical results for the square plates and also those for the beams are presented in the analogue form in Fig. 5.4, for the same values of  $\alpha, \mu$

and  $\Omega$ . It can be seen that the dynamic stability regions for the beams and square plate are exactly the same for the same nondimensional parameters. This establishes the usefulness of the concept of the master dynamic stability point.

#### **5.4 CONCLUDING REMARKS**

The three master dynamic stability formulas (MDSF<sub>s</sub>) developed in this chapter, are applicable to the structural members with complicating secondary effects like shear deformation, rotary inertia, etc. However, care should be exercised to apply this formula only when the similarity of the buckling and the vibration mode shapes of the structural member are established. The successful application of the three master dynamic stability formulas to get the master dynamic stability regions simplifies the dynamic stability analysis by orders of magnitude, which is not recognized by the researchers till now. The master dynamic stability point is proposed in this chapter and the successful application of the dynamic stability point identified here, to get the master dynamic stability curves, simplifies the dynamic stability analysis by orders of magnitude.

**Table 5.1 Variation of  $\Omega_1$  and  $\Omega_2$  for beam with end periodic loads**

$\mu$	$\alpha = 0.0$		$\alpha = 0.5$		$\alpha = 0.8$	
	$\Omega_1$	$\Omega_2$	$\Omega_1$	$\Omega_2$	$\Omega_1$	$\Omega_2$
0	2.0000 (2.00) <sup>1</sup>	2.0000 (2.00)	1.4142 (1.40)	1.4142 (1.40)	0.8944 (0.91)	0.8944 (0.91)
0.1	1.8973 (1.89)	2.0976 (2.09)	1.3416 (1.35)	1.4832 (1.48)	0.8485 (0.85)	0.9380 (0.95)
0.2	1.7888 (1.79)	2.1908 (2.18)	1.2649 (1.27)	1.5491 (1.56)	0.8000 (0.79)	0.9797 (0.98)
0.3	1.6733 (1.68)	2.2803 (2.28)	1.1832 (1.19)	1.6124 (1.62)	0.7483 (0.74)	1.0198 (1.04)
0.4	1.5491 (1.57)	2.3664 (2.37)	1.0954 (1.12)	1.6733 (1.69)	0.6928 (0.69)	1.0583 (1.08)
0.5	1.4142 (1.44)	2.4494 (2.47)	1.0000 (1.02)	1.7320 (1.78)	0.6324 (0.65)	1.0954 (1.11)

<sup>1</sup>Values given in the parentheses are read from the graph [18]

**Table 5.2 Variation of  $\Omega_1$  and  $\Omega_2$  for a layered square plate under uniform uniaxial edge periodic loads for  $\alpha = 0$**

$\beta$	Present Study		Dey & Singha [92] <sup>2</sup>	
	$\Omega_1$	$\Omega_2$	$\Omega_1$	$\Omega_2$
0.0	2.0000	2.0000	2.00	2.00
0.25	1.8708	2.1213	1.87	2.13
0.5	1.7320	2.2360	1.74	2.24
0.75	1.5811	2.3452	1.57	2.34

<sup>2</sup>Values read from the graph [92]

**Table 5.3 Variation of  $\Omega_1$  and  $\Omega_2$  of a layered cylindrical shell subjected to end uniform periodic loads with circumferential delamination for  $\alpha = 0.2$**

$\beta$	Present Study <sup>a</sup>		Ref. [95] <sup>3</sup>	
	$\Omega_1$	$\Omega_2$	$\Omega_1$	$\Omega_2$
0.0	1.7888	1.7888	1.80	1.80
0.1	1.7320	1.8439	1.73	1.84
0.2	1.6733	1.8973	1.68	1.91
0.3	1.6124	1.9493	1.61	1.95
0.4	1.5491	2.0000	1.55	2.00
0.5	1.4832	2.0433	1.47	2.05

<sup>3</sup>Values read from the graph [95]

**Table 5.4 Variation of  $\Omega_1$  and  $\Omega_2$ , of three structural members, considering effective stiffness with end periodic load**

$\beta$	$\alpha = 0.0$		$\alpha = 0.2$		$\alpha = 0.4$	
	$\Omega_1$	$\Omega_2$	$\Omega_1$	$\Omega_2$	$\Omega_1$	$\Omega_2$
0.0	2.0000 (2.0000)*	2.0000 (2.0000)	2.0000 (2.0000)	2.0000 (2.0000)	2.0000 (2.0000)	2.0000 (2.0000)
0.1	1.9493 (1.9523)	2.0493 (2.0476)	1.9364 (1.9206)	2.0615 (2.0793)	1.9148 (1.8888)	2.0816 (2.0952)
0.2	1.8973 (1.9047)	2.0976 (2.0952)	1.8708 (1.8571)	2.1213 (2.1269)	1.8257 (1.8095)	2.1602 (2.1904)
0.3	1.8439 (1.8571)	2.1447 (2.1428)	1.8027 (1.7777)	2.1794 (2.1904)	1.7320 (1.7142)	2.2360 (2.2698)
0.4	1.7888 (1.7936)	2.1908 (2.1904)	1.7320 (1.7142)	2.2360 (2.2380)	1.6329 (1.5873)	2.3094 (2.3333)
0.5	1.7320 (1.7301)	2.2360 (2.2222)	1.6583 (1.6825)	2.2912 (2.2857)	1.5275 (1.4604)	2.3804 (2.4126)

\*Values given in the parentheses are read, from the composite shell problem from the graph [95], for comparison.

**Table 5.5 Variation of  $\Omega_1$  and  $\Omega_2$  for composite skew plate, considering effective stiffness, with end periodic Load for  $\alpha = 0.0$**

$\mu$	$\alpha = 0.0$	
	$\Omega_1$	$\Omega_2$
0.0	2.0000	2.0000
0.125	1.8708	2.1213
0.25	1.7320	2.2360
0.375	1.5811	2.3454

**Table 5.6 Variation of  $\Omega_1$  and  $\Omega_2$  for the three structural members, considering the effective stiffness, with end periodic load.**

$\mu$	$\Omega_1$	$\Omega_2$
0.0	2.0000	2.0000
0.1	1.8973	2.0976
0.2	1.7888	2.1908
0.3	1.6733	2.2803
0.4	1.5491	2.3664
0.5	1.4142	2.4494

**Table 5.7 Variation of  $\Omega_1$  and  $\Omega_2$  for square plate subjected to uniaxial Periodic load for  $\alpha = 0$**

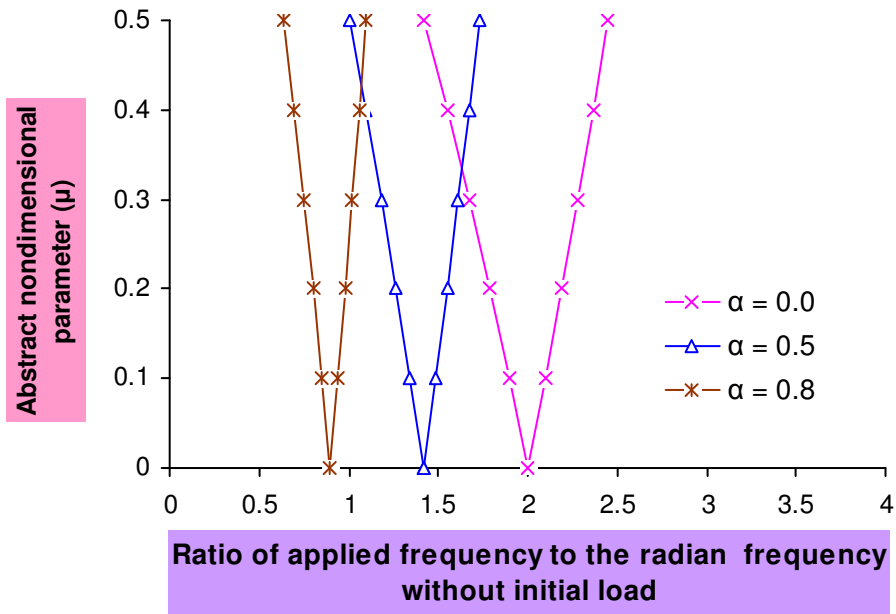
$\beta$	Present study		Ramachandra & Sarat Kumar[93] <sup>1</sup>	
	$\Omega_1$	$\Omega_2$	$\Omega_1$	$\Omega_2$
0.0	39.4801	39.4801	39.09	39.09
0.1	38.4790	40.4531	38.63	40.00
0.2	37.4527	41.4066	37.27	40.91
0.3	36.4657	42.3363	36.59	42.04
0.4	35.3109	43.2463	35.22	42.73
0.5	34.1890	44.1386	34.54	43.64
0.6	33.0309	45.0131	32.72	44.54
0.7	31.8287	45.8698	31.81	45.45
0.8	30.5792	46.7127	30.68	46.36

<sup>1</sup>Values in the parenthesis are read from the graph [93]

**Table 5.8 Variation of  $\Omega_1$  and  $\Omega_2$  for square plate subjected to uniaxial Periodic load for  $\alpha = 0.6$**

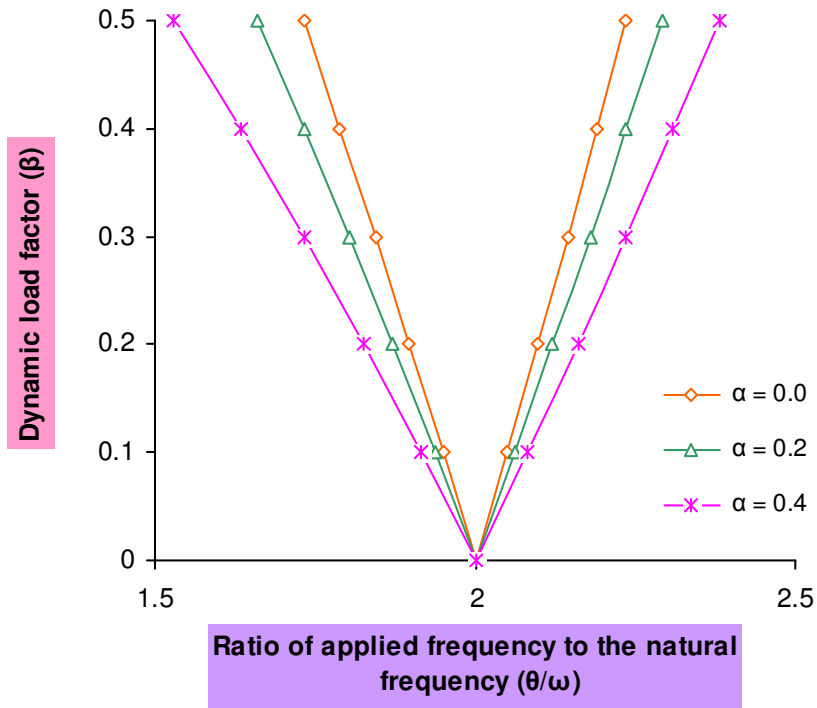
$\beta$	Present Formula		Ramachandra & Sarat Kumar[93] <sup>1</sup>	
	$\Omega_1$	$\Omega_2$	$\Omega_1$	$\Omega_2$
0.0	24.9693	24.9693	25.03	25.03
0.05	24.1755	25.7370	24.18	25.64
0.1	23.3563	26.4831	23.45	26.36
0.15	22.5055	27.2096	22.48	27.09
0.2	21.6231	27.9163	21.51	27.93
0.25	20.7033	28.6052	20.79	28.54
0.3	19.7393	29.2783	19.81	29.27

<sup>1</sup>Values in the parenthesis are read from the graph [93]

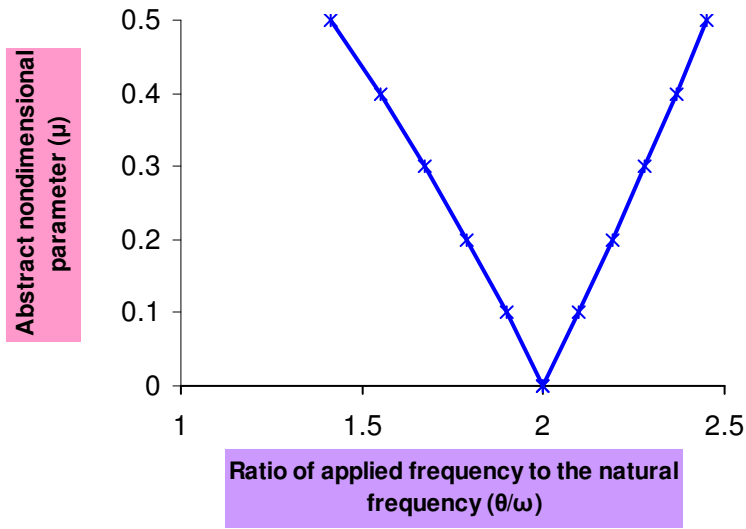


**Fig. 5.4 Master dynamic stability curves for structural members subjected to end concentrated periodic axial/in-plane loads.**





**Fig.5. 5 Master dynamic stability curves II, for the three structural members, with effective stiffness, subjected to periodic load.**



**Fig.5. 6 Master dynamic stability curve III, for the three structural members, with effective stiffness, subjected to Periodic load.**