Chapter 5

“A subtle thought that is in error may yet give rise to fruitful inquiry that can establish truths of great value.”
- Isaac Asimov

Dynamic Performance and Stability Characteristics of Fluid Film Bearings Operating on TiO$_2$ Nanolubricants

This Chapter presents the results and discussion of studies conducted to obtain the dynamic performance and stability characteristics of a finite journal bearing operating on TiO$_2$ nanolubricants. Simultaneous influence of TiO$_2$ nanolubricant effective viscosities, at various TiO$_2$ nanoparticle concentrations, along with couple stress effects of TiO$_2$ aggregates on the dynamic and stability characteristics of journal bearings are studied. Analysis is carried out using both, linear perturbation and non-linear transient approaches of studying hydrodynamic stability characteristics. Chapter begins with the linear perturbation method and then moves on to nonlinear transient analysis.

5.1 Introduction

Hydrodynamic journal bearings are vital components of larger machineries having definite vibrational characteristics. Inherent vibration of a journal riding on the crest of a pressurized oil film is greatly influenced by disturbances or excitations in neighboring subcomponents within the machine, or completely external to the machine but are transmitted to the bearing system. The converse is also important for predictable behaviour of the machines, in such that, vibrational instabilities of journal bearing systems could also significantly affect the overall dynamic stability of the machine. It is therefore important that, journal bearings are designed to operate within the purview of stable operational speeds.
Analysis of journal bearing dynamic characteristics and stability issues has come a long way since its identification by Newkirk and Taylor [49]. Stodola [381] and Hummel [382] are credited with the analysis of oil film by likening its behavior under load to that of a spring. Subsequent modeling of dynamic characteristics of journal bearing systems based on the stiffness and damping coefficients of the oil film has led to significant breakthroughs in computing the critical speeds [383, 384, 385]. It was also realized that the variation in dynamic forces generated within the oil film with shaft position is highly nonlinear. While nonlinear transient approach of studying stability characteristics in journal bearings is well developed with many publications, such as: Majumdar and Brewe [386], Majumder and Majumdar [387], Choy et al. [388], Turaga et al. [389], and Lin et al. [390]; the determination of bearing critical speeds is generally performed using linearized stiffness and damping coefficients obtained by linearized perturbation about the steady state equilibrium [391]. A comparative study of linearized and transient approaches in determining the dynamic coefficients of journal bearings is reported by Tieu and Qiu [392]. The study claims similarity in the computed critical speeds using both the approaches. However, while linearized perturbation technique will help in computing the dynamic coefficients, the post-whirl analysis and whirl orbits could only be observed by performing nonlinear analysis. Hence the relevance of both the approaches needs to be appreciated in completely describing the dynamic behavior of journal bearing systems.

Generally, stability issues in journal bearings are categorized as oil whirl and oil whip phenomenon. From the steady state position of a journal, characterized by a stable attitude angle and eccentricity ratio, any disturbance, usually manifested as an excitation force due to sudden surge in operational parameters, will result in the development of additional momentary hydrodynamic forces within the oil film, causing the journal to experience a forward thrust. This forward motion of journal will cause injection of additional oil into the lubrication zone, leading to a further increase in dynamic forces on the journal. These dynamic forces lead the journal on a whirling path within the bearing clearance and is termed to display oil whirl. Oil whirl is the most common subsynchronous whirl instability experienced in journal bearings and is typically characterized by whirl frequency between 40% to 48% of journal speed (half-frequency whirl) [393]. Oil whirl is considered to have reached critical when the amplitude is ~ 40 to 50% of normal bearing clearance and would warrant immediate interjection to prevent seizure [394]. Oil whip or shaft whip takes place when the oil whirl frequency matches the natural frequency of the system, of which the
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bearing is a part of. Once initiated, it is interesting to note that, oil whip frequency is independent of increase in journal speed and is largely a function of mass and oil stiffness. Unchecked oil whip can result in immediate breakdown of the machine. Other than oil whirl and oil whip, additional instability could prevail due to oil starvation or severe oil degradation, and even improper choice of lubricant characterized by dry excessive friction and vibration. These instability issues are called as dry whip.

Many factors influence whirl instability in journal bearings. Load-speed combination is an important criterion, with lightly loaded rotors exhibiting whirl at high speeds. However, these incidents are rare in well-designed bearings. Increase in bearing clearance due to excessive wear is also a factor contributing to oil whirl [394]. Oil viscosity significantly influences whirl instability. Changes in oil inlet temperature is also identified as an important factor affecting whirl characteristics. Depending on the operating conditions, variation in inlet oil temperature is reported to vary the threshold speeds of journal bearings [395]. The significant effect of temperature variation on dynamic characteristics of bearing systems is due to the associated variations in oil viscosities. Another factor causing viscosity variations is the addition of lubricant additives. As described in Chapters – 3 & 4, addition of lubricant additives such as nanoparticles, cause an increase in lubricant viscosities, thereby affecting the whirl characteristics of the system. Couple stresses induced due to the presence of lubricant additives is also an important factor influencing whirl instability. The influence of couple stresses due to polymeric additives on dynamic bearing characteristics have been previously studied. Linear perturbation analysis of couple stresses due to polymeric additives by Lin [104] has reported increase in stiffness and damping coefficients leading to improved stability. A similar observation is also reported by Guha [114] and Lahmar [117]. A non-linear transient analysis of couple stress effects on dynamic characteristics of journal bearings is reported by Binu et al. [34], indicating significant increase in threshold speeds of journal bearings. The influence of nanoparticle additives on the stability characteristics of journal bearings are largely unknown. This study therefore is an attempt to understand the influence of viscosity variations induced due to the presence of TiO$_2$ nanoparticle additives at different particle concentrations and TiO$_2$ nanoparticle aggregate sizes. The influence of couple stresses due to TiO$_2$ nanoparticle aggregates on dynamic bearing characteristics is also simultaneously obtained. The journal bearing stability is studied using both linear perturbation and non-linear transient methods.
5.2 Objectives

The main objective of this study is to analyze the influence of TiO$_2$ nanoparticle lubricant additives on the dynamic characteristics of journal bearings. More specifically, the objectives are listed below.

I. Obtain a modified time-dependent Reynolds equation to simultaneously consider the influence of effective viscosities of TiO$_2$ nanolubricants at different volume fractions along with couple stress effects of TiO$_2$ nanoparticle additives.

II. Using the linearized perturbation approach compute stiffness and damping coefficients of journal bearings operating on TiO$_2$ nanolubricants at different particle concentrations and aggregate TiO$_2$ particle sizes.

III. Using the obtained dynamic characteristics perform a comparative analysis of threshold stability maps for TiO$_2$ nanolubricants at different volume fractions and aggregate particle sizes.

IV. Using non-linear transient approach, obtain journal whirl orbits for journal bearings operating on TiO$_2$ nanoparticle additives at different volume fraction and aggregate particle sizes.

V. Plot threshold stability maps for journal bearings operating on TiO$_2$ nanolubricants using non-linear transient analysis.

5.3 Experimental

The experimental measurement of shear viscosities at different volume fractions of TiO$_2$ nanoparticle dispersions in engine oil, which is explained in section 3.3.6 and discussed in section 3.4.5, forms the basis for variable viscosity analysis performed in this section. The experimental viscosities of TiO$_2$ nanoparticle dispersions formulated using the optimized procedure that is developed and explained in Chapter 3, is used as reference in identifying the nanofluid viscosity model capable of simulating shear viscosities of TiO$_2$ nanolubricant samples. The aggregate TiO$_2$ nanoparticle hydrodynamic diameter obtained through DLS particle size analysis described in section 3.3.5 and discussed in 3.4.3 is used in the analysis to model the couple stress parameter. The influence of aggregate packing fraction on effective viscosities of TiO$_2$ nanolubricants is also simulated using appropriate viscosity model.
5.4 Theoretical viscosity model

As described in section 4.4.1, the modified Krieger-Dougherty model is found to simulate shear viscosities of TiO$_2$ nanolubricant samples at different volume fractions that are in good agreement with experimental viscosities. Considering the ratio of aggregate particle size to primary particle size as 7.77, as confirmed by DLS analysis, the modified Krieger-Dougherty equation for high shear applications is expressed in equation 5.1, presented below. The expression is described in section 4.5.

$$\bar{\mu} = \frac{\mu_{nf}}{\mu_{bf}} = \left(1 - \frac{\phi}{0.605} \left(\frac{a_a}{a}\right)^{1.2}\right)^{-1.51} \tag{5.1}$$

where, $\frac{a_a}{a} = 7.77$

The above viscosity model is integrated into hydrodynamic governing equation to study the influence of effective viscosities of TiO$_2$ nanoparticle additives at different volume fractions.

5.5 Linear perturbation analysis of dynamic characteristics of journal bearing operating on TiO$_2$ nanolubricants

This section begins with a general description of linearized dynamic coefficients. The governing time-dependent Reynolds equation for journal bearings, considering the TiO$_2$ nanolubricant to be a couple stress fluid is presented. The effective viscosities of TiO$_2$ nanolubricant at varying concentrations are simulated using modified Krieger-Dougherty equation and considered in the Reynolds equation. The solution scheme for obtaining the dynamic characteristics at different TiO$_2$ nanoparticle concentrations and TiO$_2$ aggregate particle sizes is explained.

5.5.1 Theory of dynamic coefficients

In dynamic analysis of journal bearings, the journal center is not considered to be in equilibrium, but rather exhibits a whirling motion in the bearing clearance. This phenomenon is modeled by considering the oil film to behave like a spring with definite stiffness and damping coefficients. The stiffness and damping of oil film generates dynamic hydrodynamic forces that causes the journal center to whirl. The complexity in computing the dynamic coefficients arises due to the non-linear relation between dynamic hydrodynamic forces and the resulting motion of journal center. An alternate to the
computationally intensive non-linear analysis is the much simpler linearized perturbation approach. In this technique, the dynamic coefficients are obtained by numerically inducing small linearized perturbations of the journal center. The perturbed Reynolds equations are then derived and solved for complex dynamic pressures. Integrating the complex dynamic pressures will lead to the computation of dynamic stiffness and damping coefficients.

Fig. 65(a) is a schematic that models the journal bearing oil film as a spring-damper system [371]. As seen in Fig. 65(a), for a 2-D journal bearing system, two stiffness ($K_{XX}$ and $K_{YY}$) and two damping coefficients ($D_{XX}$ and $D_{YY}$) are generated due to the displacement of journal center in the same directions as dynamic forces, i.e. horizontal (X) and vertical (Y) directions. These dynamic coefficients are called as direct stiffness and direct damping coefficients. In addition, typical to many other rotor dynamic cases, cross-coupled stiffness and damping coefficients ($K_{XY}$, $K_{YX}$, $D_{XY}$ and $D_{YX}$) are also generated due to the change in hydrodynamic forces in X direction with the motion of journal center in the Y direction and vice versa. Therefore, a typical plain journal bearing will possess eight dynamic coefficients; four stiffness and four damping coefficients. A schematic of journal whirl orbit and influence of the direct and cross-coupled dynamic forces on journal whirl is shown in Fig. 65(b) & 64(c), respectively.

As shown in Fig. 65(a), during operation, initially the journal center takes up an equilibrium position under the action of static load $W$, described by eccentricity $e$ and attitude angle $\phi$. Any external excitation will disturb the prevalent steady state condition, forcing the journal center into a whirl orbit within the clearance circle. This is illustrated in Fig. 65 (b). The safe operation of bearing requires the orbits to be small and stable. A movement of journal center the X-Y plane will result in the generation of dynamic forces owing to the stiffness and damping characteristics of oil film. As seen in Fig. 65(c), the direct stiffness force $F_S$ acts on the journal, tending to push the journal center to the steady state position. Similarly, direct damping force $F_D$, which is proportional to the change in velocity of journal center, also opposes the motion and tries to retard the whirl. However, the cross-coupled stiffness and damping forces, as seen in the Fig. 65(c) act in the direction of journal orbit and hence are critical in deciding the whirl instability.

5.5.2 Governing equations for dynamic analysis of journal bearings

In studying the dynamic characteristics of journal bearings, the time-dependent version of the Reynolds equation needs to be considered.
The derivation of Reynolds equation for steady state hydrodynamic lubrication in journal bearing, with consideration of couple stresses, is explained in Chapter 4. A similar derivation with consideration to dynamic variation in film thickness will yield the time-dependent version of Reynolds equation for studying the dynamic characteristics of journal bearings. Further, to analyze the influence of TiO\textsubscript{2} nanoparticle additives on the dynamic characteristics, the modified Reynolds equation accounting for polar couple stresses of TiO\textsubscript{2} nanoparticle additives is used. The Reynolds equation is also modified to consider the effective viscosity of TiO\textsubscript{2} nanolubricants at different TiO\textsubscript{2} particle concentrations. Therefore, the time-dependent Reynolds equation for couple stresses used in this analysis is of the form [114, 117]:

$$\frac{\partial}{\partial x}\left(f(h,d)\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(f(h,d)\frac{\partial p}{\partial z}\right) = 6\mu U \frac{\partial h}{\partial x} + 12\mu \frac{\partial h}{\partial t}$$

(5.2)
where, \( f(h,d) = h^3 - 12hd^2 + 24d^3 \tanh\left(\frac{h}{2d}\right) \)

Further, a non-dimensional form of the governing Reynolds equation is obtained using the following non-dimensional terms \([114, 117]\).

\[
\theta = \frac{x}{R}; \quad \bar{z} = \frac{z}{L}; \quad \bar{e} = \frac{e}{C}; \quad \bar{p} = \frac{pC^2}{\mu \omega R^2}; \quad \bar{h} = \frac{h}{C}; \quad \bar{\lambda} = \frac{L}{D}; \quad \bar{t} = \omega t; \quad \bar{\mu} = \frac{\mu_n f}{\mu}; \quad \bar{d} = \frac{d}{C}
\] (5.3)

The obtained non-dimensional Reynolds equation is presented below.

\[
\frac{\partial}{\partial \theta}\left(f(\bar{h},\bar{d})\frac{\partial \bar{p}}{\partial \theta}\right) + \frac{1}{4\lambda^2} \frac{\partial}{\partial \bar{z}}\left(f(\bar{h},\bar{d})\frac{\partial \bar{p}}{\partial \bar{z}}\right) = 6\bar{\mu} \frac{\partial \bar{h}}{\partial \theta} + 12\bar{\mu} \frac{\partial \bar{h}}{\partial \bar{t}}
\] (5.4)

Where, \( f(\bar{h},\bar{d}) = \bar{h}^3 - 12\bar{d}^2\bar{h} + 24\bar{d}^3 \tanh\left(\frac{\bar{h}}{2\bar{d}}\right) \)

The instantaneous position of journal center is described using the rotational coordinates of eccentricity and attitude angle. The rotational coordinate system is illustrated in Fig. 66.

![Figure 66: Rotational coordinate system describing journal whirl](image)

Substituting the rotational coordinates will reduce the governing Reynolds equation, given in equation 5.4, to the form presented below \([104, 108, 117]\).

\[
\frac{\partial}{\partial \theta}\left(f(\bar{h},\bar{d})\frac{\partial \bar{p}}{\partial \theta}\right) + \frac{1}{4\lambda^2} \frac{\partial}{\partial \bar{z}}\left(f(\bar{h},\bar{d})\frac{\partial \bar{p}}{\partial \bar{z}}\right) = 6\bar{\mu}(1-2\dot{\phi}) \frac{\partial \bar{h}}{\partial \theta} + 12\bar{\mu} \frac{\partial \bar{h}}{\partial \bar{t}}
\] (5.5)
where,  
\[ f(h, d) = h^3 - 12d^2h + 24d^3 \tanh \left( \frac{h}{2d} \right) \]

In numerically modeling the linearized perturbations, the journal is assumed to undergo small harmonic whirling motion described by an angular velocity \( \omega_p \). Therefore, at any given instant, the position of the journal center can be expressed in terms of the rotational coordinates as \( \varepsilon = \varepsilon_0 + \varepsilon_1 e^{j\Omega t} \) and \( \phi = \phi_0 + \phi_1 e^{j\Omega t} \), where \( \varepsilon_0, \phi_0 \) are the rotational coordinates of steady state journal center position and \( \Omega \) is the whirl frequency ratio, relating the journal whirl and journal rotation frequencies, \( \Omega = \frac{\omega_p}{\omega} \). Further, during the defined harmonic whirl, first order perturbations of the journal center from the steady state position will result in changes in hydrodynamic film pressure and oil film thickness. The corresponding oil film pressure and thickness can be expressed in non-dimensional form as presented below.

\[
\begin{align*}
\bar{p} &= \bar{p}_0 + \left( \varepsilon_1 \bar{p}_e + \varepsilon_0 \phi_1 \bar{p}_\phi \right) e^{j\Omega t} \\
\bar{h} &= \bar{h}_0 + \left( \varepsilon_1 \cos \theta + \varepsilon_0 \phi_1 \sin \theta \right) e^{j\Omega t}
\end{align*}
\]

(5.6)

In the above equation 5.6, \( \bar{p}_0 \) is the steady state pressure; \( \bar{p}_e \) and \( \bar{p}_\phi \) are the perturbed pressures. Substituting the perturbed pressure and film thickness equations, expressed as equation 5.6, in non-dimensional Reynolds equation 5.5, will yield the perturbed Reynolds equations. On substituting, equation 5.5 in equation 5.6, the zero and first order terms for \( \varepsilon_1 \) and \( \varepsilon_0 \phi_1 \) are collected to obtain the following set of linear partial differential equations for steady state pressure \( \bar{p}_0 \) and perturbed pressures in \( \bar{p}_e \) and \( \bar{p}_\phi \).

Zero order equation or steady state equation:

\[
\frac{\partial}{\partial \theta} \left( f_0(h_0, d) \frac{\partial \bar{p}_0}{\partial \theta} \right) + \frac{1}{4\lambda^2} \frac{\partial}{\partial z} \left( f_0(h_0, d) \frac{\partial \bar{p}_0}{\partial z} \right) = 6\mu \frac{\partial \bar{h}_0}{\partial \theta} \quad (5.7)
\]

1st order equation for \( \varepsilon_1 e^{j\Omega t} \):

\[
\begin{align*}
\frac{\partial}{\partial \theta} \left( f_0(h_0, d) \frac{\partial \bar{p}_e}{\partial \theta} \right) &+ \frac{1}{4\lambda^2} \frac{\partial}{\partial z} \left( f_0(h_0, d) \frac{\partial \bar{p}_e}{\partial z} \right) + 3 \frac{\partial}{\partial \theta} \left( h_0^2 \frac{\partial \bar{p}_0}{\partial \theta} \cos \theta \right) + \\
3 \frac{1}{4\lambda^2} \frac{\partial}{\partial z} \left( h_0^2 \frac{\partial \bar{p}_0}{\partial z} \cos \theta \right) &= -6\mu \sin \theta + 12\mu i \Omega \cos \theta
\end{align*}
\]

(5.8)
1st order equation for $\varepsilon_0 \phi e^{j\Omega t}$:

\[
\frac{\partial}{\partial \theta} \left( \tilde{f}_0(\hat{h}_0, \hat{d}) \frac{\partial \tilde{p}_0}{\partial \theta} \right) + \frac{1}{4\lambda^2} \frac{\partial}{\partial \tilde{z}} \left( \tilde{f}_0(\hat{h}_0, \hat{d}) \frac{\partial \tilde{p}_0}{\partial \tilde{z}} \right) + 3 \frac{\partial}{\partial \theta} \left( \hat{h}_0 \frac{\partial \tilde{p}_0}{\partial \theta} \sin \theta \right) + 3 \frac{1}{4\lambda^2} \frac{\partial}{\partial \tilde{z}} \left( \hat{h}_0 \frac{\partial \tilde{p}_0}{\partial \tilde{z}} \sin \theta \right) = 6 \tilde{\mu} \cos \theta + 12 \tilde{\mu} i \Omega \sin \theta
\]

Where, \( \tilde{f}_0(\hat{h}_0, \hat{d}) = \hat{h}_0^3 - 12\hat{d}^3\hat{h}_0 + 24\hat{d}^3 \tanh \left( \frac{\hat{h}_0}{2\hat{d}} \right) \)

Equations 5.7, 5.8 and 5.9 represents the steady state and perturbed non-dimensional modified Reynolds equations for journal bearings operating on TiO$_2$ nanolubricants. The solution of equations 5.7, 5.8 and 5.9 at different TiO$_2$ nanoparticle concentrations and particle sizes will provide equivalent steady state and dynamic pressures developed within the nanolubricant oil film.

### 5.5.3 Boundary conditions

In numerically solving the steady state and dynamic Reynolds equations, expressed as equations 5.7, 5.8, and 5.9, Reynolds boundary conditions, described in section 4.4.4, are used. The boundary conditions are expressed as shown below.

\[ \tilde{p} = 0 \text{ at } \tilde{z} = 0, 1 \quad (5.10) \]
\[ \tilde{p} = 0 \text{ at } \theta = 0 \quad (5.11) \]
\[ \frac{\partial \tilde{p}}{\partial \theta} = 0 \text{ at } \theta = \theta_m \quad (5.12) \]
\[ \tilde{p} = 0 \text{ at } \theta_m \leq \theta \leq 2\pi \quad (5.13) \]

In computing the steady state pressures, the cavitated pressures are equated to zero based on Christopherson algorithm [374]. The cavitation front is described by the angular parameter \( \theta_m \). This boundary condition is expressed in equations 5.12 and 5.13.

### 5.5.4 Numerical Formulation of steady state Reynolds equation

The steady state Reynolds equation expressed as equation 5.8 is solved numerically using the finite difference approach described in Chapter 4. The central difference scheme of discretizing is employed and solved for steady state pressure using Gauss-Siedel with successive over relaxation method. The numerical formulation and solution technique is described in detail under section 4.4.6. The discretized Reynolds equation for steady state pressure is expressed in equation 5.14 presented below.
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\[(p_{0i,j}) = \frac{A \, p_{i-1,j} + B \, p_{i+1,j} + C \, p_{i,j-1} + D \, p_{i,j+1} + RHS}{E} \]  

(5.14)

where,

\[A = 4\lambda^2 P \bar{f}_0 (\bar{h}_0, \bar{d}) - 2\lambda^2 P \Delta \bar{z} \bar{f}_0 (\bar{h}_0, \bar{d})\]

\[B = 4\lambda^2 P \bar{f}_0 (\bar{h}_0, \bar{d}) + 2\lambda^2 P \Delta \bar{z} \bar{f}_0 (\bar{h}_0, \bar{d})\]

\[C = \bar{f}_0 (\bar{h}_0, \bar{d}); \quad D = \bar{f}_0 (\bar{h}_0, \bar{d}); \quad E = 8\lambda^2 P \bar{f}_0 (\bar{h}_0, \bar{d}) + 2\bar{f}_0 (\bar{h}_0, \bar{d})\]

\[RHS = 24 \bar{P} \lambda^2 P \Delta \bar{z} \Delta \theta \epsilon_0 \sin \theta\]

\[\bar{f} = \frac{\Delta \bar{z}}{\Delta \theta}; \quad \lambda = \frac{L}{2R}\]

and

\[\bar{f}_0 (\bar{h}_0, \bar{d}) = \bar{h}_0^3 - 12\bar{d}^2 \left[ \bar{h}_0 - 2\bar{d} \tanh \left( \frac{\bar{h}_0}{2\bar{d}} \right) \right] \]  

(5.15)

\[\bar{f}_0 (\bar{h}_0, \bar{d}) = -3\bar{h}_0^2 \epsilon_0 \sin \theta + 12\bar{d}^2 \epsilon_0 \sin \theta \tanh^2 \left( \frac{\bar{h}_0}{2\bar{d}} \right) \]  

(5.16)

The solution methodology for obtaining the steady state pressures is similar to the methodology presented and discussed in Chapter 4 under section 4.4.6. The computational grid for central difference scheme used in the analysis is also similar to the grid used in the computation of static performance characteristics and is illustrated in Fig. 45.

### 5.5.5 Numerical Formulation of perturbed Reynolds equation

Solution to the perturbed Reynolds equation 5.8 gives the component of the dynamic pressure along the line of centers \(\bar{P}_e\), as shown in Fig. 66. Following similar numerical formulation technique (central difference scheme) used for steady state equation 5.7, the expanded form of equation 5.8 is shown below as equation 5.17.

\[\bar{f}_0 (\bar{h}_0, \bar{d}) \frac{\partial^2 \bar{P}_e}{\partial \theta^2} + \bar{f}'_0 (\bar{h}_0, \bar{d}) \frac{\partial \bar{P}_e}{\partial \theta} + \frac{\bar{f}_0 (\bar{h}_0, \bar{d})}{4\lambda^2} \frac{\partial^2 \bar{P}_e}{\partial \bar{z}^2} + ...\]

\[...3 \left[ \bar{h}_0^2 \cos \theta \frac{\partial^2 \bar{P}_0}{\partial \theta^2} - 2\bar{h}_0 (\epsilon_0 \sin \theta) \cos \theta \frac{\partial \bar{P}_0}{\partial \theta} - \bar{h}_0^2 \sin \theta \frac{\partial^2 \bar{P}_0}{\partial \theta^2} \right] + ...\]

\[3 \bar{h}_0^2 \cos \theta \frac{\partial^2 \bar{P}_0}{\partial \bar{z}^2} + 6 \bar{h} \sin \theta - 12 \bar{h} \Omega \cos \theta = 0\]

(5.17)

Applying central difference scheme, the above equation changes to the form presented below.

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\[ J_0(\bar{h}_0, \bar{d}) \left[ (\bar{p}_e)_{i-1,j} - 2(\bar{p}_e)_{i,j} + (\bar{p}_e)_{i+1,j} \right] + J_0(\bar{h}_0, \bar{d}) \left[ (\bar{p}_e)_{i+1,j} - (\bar{p}_e)_{i-1,j} \right] \ldots \]
\[ + \frac{J_0(\bar{h}_0, \bar{d})}{4\lambda^2} \left[ (\bar{p}_e)_{i,j-1} - 2(\bar{p}_e)_{i,j} + (\bar{p}_e)_{i,j+1} \right] + 3\bar{h}_0^2 \cos \theta \ldots \]
\[ \left[ (\bar{p}_0)_{i-1,j} - 2(\bar{p}_0)_{i,j} + (\bar{p}_0)_{i+1,j} \right] \frac{6\bar{h}_0^2 \sin \theta}{2(\Delta \theta)} \left[ (\bar{p}_0)_{i,j-1} - 2(\bar{p}_0)_{i,j} + (\bar{p}_0)_{i,j+1} \right] \ldots \]
\[ - 3\bar{h}_0^2 \sin \theta \left[ (\bar{p}_0)_{i,j+1} - (\bar{p}_0)_{i,j} \right] + \frac{3\bar{h}_0^2 \cos \theta}{4\lambda^2} \left[ (\bar{p}_0)_{i,j} - 2(\bar{p}_0)_{i,j} + (\bar{p}_0)_{i,j+1} \right] \ldots \]
\[ + 6\bar{h}_0 \sin \theta - 12\bar{h}_i \Omega \cos \theta = 0 \]

Multiplying all the terms with \(4\lambda^2(\Delta \theta)^2\) and further expansion of the equation will yield the final discretized perturbed Reynolds equation for dynamic pressure along the line of centers \(\bar{p}_e\) of journal bearing operating on TiO\(_2\) nanolubricant with consideration to effective viscosity and TiO\(_2\) particle size. The equation for dynamic pressure along the line of centers is as expressed below.

\[ (\bar{p}_e)_{i,j} = \frac{A(\bar{p}_e)_{i-1,j} + B(\bar{p}_e)_{i+1,j} + C(\bar{p}_e)_{i,j-1} + D(\bar{p}_e)_{i,j+1} + F + RHS}{E} \quad (5.19) \]

\[ A = 4\lambda^2 \Delta \tau \bar{h}_0(\bar{h}_0, \bar{d}) - 2\lambda^2 \Delta \tau \Delta \bar{h}_0(\bar{h}_0, \bar{d}) \]
\[ B = 4\lambda^2 \Delta \tau \bar{h}_0(\bar{h}_0, \bar{d}) + 2\lambda^2 \Delta \tau \Delta \bar{h}_0(\bar{h}_0, \bar{d}) \]
\[ C = \bar{f}_0(\bar{h}_0, \bar{d}); \quad D = \bar{f}_0(\bar{h}_0, \bar{d}) \]
\[ E = 8\lambda^2 \Delta \tau \bar{h}_0(\bar{h}_0, \bar{d}) + 2\bar{f}_0(\bar{h}_0, \bar{d}) \]

where, \( F = 12\lambda^2 \Delta \tau \sin \theta \left[ (\bar{p}_0)_{i-1,j} - 2(\bar{p}_0)_{i,j} + (\bar{p}_0)_{i+1,j} \right] \ldots \]
\[ \ldots 12\lambda^2 \Delta \tau \cos \theta \left[ (\bar{p}_0)_{i+1,j} - (\bar{p}_0)_{i-1,j} \right] + 3\bar{h}_0^2 \cos \theta \ldots \]
\[ \ldots 3\bar{h}_0^2 \sin \theta \left[ (\bar{p}_0)_{i,j} - 2(\bar{p}_0)_{i,j} + (\bar{p}_0)_{i,j+1} \right] \]
\[ RHS = 24\bar{h}_0 \lambda^2 \Delta \tau \Delta \theta \sin \theta - 48i \bar{h}_0 \lambda^2 \Delta \tau \Delta \theta \Omega \cos \theta \]
\[ \bar{f} = \frac{\Delta \bar{h}}{\Delta \theta}; \quad \lambda = \frac{L}{2R} \]
\[
\tilde{f}_0(\tilde{h}_0, \tilde{d}) = \tilde{h}_0^3 - 12\tilde{d}^2 \left[ \tilde{h}_0 - 2\tilde{d} \tanh \left( \frac{\tilde{h}_0}{2\tilde{d}} \right) \right]
\]

and
\[
\tilde{f}_0(\tilde{h}_0, \tilde{d}) = -3\tilde{h}_0^2 \sin \theta + 12\tilde{d}^2 \tilde{c}_0 \sin \theta \tanh^2 \left( \frac{\tilde{h}_0}{2\tilde{d}} \right)
\] (5.21)

Similarly, discretizing and rearranging the perturbed Reynolds equation 5.9, the expression for dynamic pressure perpendicular to the line of centers \( \tilde{p}_\phi \) is obtained. The expression is presented below.

\[
\left( \tilde{p}_\phi \right)_{i,j} = \frac{A \left( \tilde{p}_e \right)_{i-1,j} + B \left( \tilde{p}_e \right)_{i+1,j} + C \left( \tilde{p}_e \right)_{i,j-1} + D \left( \tilde{p}_e \right)_{i,j+1} + F - \text{RHS}}{E}
\] (5.22)

where,

\[
A = 4\lambda^2 \tilde{r} \tilde{f}_0(\tilde{h}_0, \tilde{d}) - 2\lambda^2 \tilde{r} \Delta \tilde{x} \tilde{f}_0(\tilde{h}_0, \tilde{d})
\]

\[
B = 4\lambda^2 \tilde{r} \tilde{f}_0(\tilde{h}_0, \tilde{d}) + 2\lambda^2 \tilde{r} \Delta \tilde{x} \tilde{f}_0(\tilde{h}_0, \tilde{d})
\]

\[
C = \tilde{f}_0(\tilde{h}_0, \tilde{d}); \quad D = \tilde{f}_0(\tilde{h}_0, \tilde{d}); \quad E = 8\lambda^2 \tilde{r} \tilde{f}_0(\tilde{h}_0, \tilde{d}) + 2\tilde{f}_0(\tilde{h}_0, \tilde{d})
\]

\[
F = 12\lambda^2 \tilde{r} \tilde{h}_0 \sin \theta \left[ \left( \tilde{p}_0 \right)_{i-1,j} - 2\left( \tilde{p}_0 \right)_{i,j} + \left( \tilde{p}_0 \right)_{i,j+1} \right]
\]

\[
\ldots 6\lambda^2 \tilde{r} \Delta \tilde{x} \tilde{h}_0^2 \cos \theta \left[ \left( \tilde{p}_0 \right)_{i+1,j} - \left( \tilde{p}_0 \right)_{i-1,j} \right] + 3\tilde{h}_0^2 \sin \theta \ldots
\]

\[
\ldots \left[ \left( \tilde{p}_0 \right)_{i,j-1} - 2\left( \tilde{p}_0 \right)_{i,j} + \left( \tilde{p}_0 \right)_{i,j+1} \right]
\]

\[
\text{RHS} = 24 \tilde{\mu} \lambda^2 \tilde{r} \Delta \tilde{x} \Delta \theta \cos \theta + 48i \tilde{\mu} \lambda^2 \tilde{r} \Delta \tilde{x} \Delta \theta \Omega \sin \theta
\]

\[
\bar{r} = \frac{\Delta \tilde{x}}{\Delta \theta}, \lambda = \frac{L}{2\bar{r}}
\]

and
\[
\tilde{f}_0(\tilde{h}_0, \tilde{d}) = \tilde{h}_0^3 - 12\tilde{d}^2 \left[ \tilde{h}_0 - 2\tilde{d} \tanh \left( \frac{\tilde{h}_0}{2\tilde{d}} \right) \right]
\] (5.25)

\[
\tilde{f}_0(\tilde{h}_0, \tilde{d}) = -3\tilde{h}_0^2 \tilde{c}_0 \sin \theta + 12\tilde{d}^2 \tilde{c}_0 \sin \theta \tanh^2 \left( \frac{\tilde{h}_0}{2\tilde{d}} \right)
\]

A computational code is developed using MATLAB to solve equation 5.19 and 5.22 to obtain the dynamic pressures using Gauss-Seidel method.

### 5.5.6 Dynamic coefficients of journal bearings operating on TiO$_2$ nanolubricants

The computed dynamic pressure components along the line of centers \( \tilde{p}_e \) and perpendicular to the line of centers \( \tilde{p}_\phi \) are used in determining the stiffness and damping coefficients of TiO$_2$ nanolubricant oil film under the perturbed conditions of journal center. Integration of dynamic force components along and perpendicular to the line of centers in
rotational coordinates will provide the dynamic coefficients. A general description of
dynamic coefficients provided by Lahmar [117] is expressed below.
\[
\begin{bmatrix}
Z_{ex} & Z_{ep} \\
Z_{ex} & Z_{ep}
\end{bmatrix}^T = -\int_0^{2\pi} \int_0^{2\pi} \left\{ \begin{array}{c}
\overline{p}_x \\
\overline{p}_y
\end{array} \right\} \times (\cos \theta \cdot \sin \theta) \, d\varpi \, d\phi
\]  
(5.26)

where, \( Z_{i,j} = K_{i,j} + i\Omega D_{i,j} \) are the complex dynamic coefficients. The real components,
\( K_{i,j} \) are the stiffness coefficients and imaginary components \( D_{i,j} \) are the damping
coefficients. The nodal coordinates \( i, j \) represents the rotational coordinates \( \varepsilon, \phi \).

Expanding equation 5.27, basic expressions for dynamic coefficients are obtained as shown
below.
\[
\begin{bmatrix}
Z_{ex} & Z_{ep} \\
Z_{ex} & Z_{ep}
\end{bmatrix}^T = -\int_0^{2\pi} \int_0^{2\pi} \left\{ \begin{array}{c}
\overline{p}_x \cos \theta \\
\overline{p}_y \cos \theta \\
\overline{p}_x \sin \theta \\
\overline{p}_y \sin \theta
\end{array} \right\} d\varpi \, d\phi
\]  
(5.27)

\[
\therefore Z_{ex} = \int_0^{2\pi} \overline{p}_x \cos \theta \, d\varpi \, d\theta; \quad Z_{ep} = -\int_0^{2\pi} \overline{p}_y \cos \theta \, d\varpi \, d\theta
\]  
(5.28)

\[
Z_{ex} = -\int_0^{2\pi} \overline{p}_x \sin \theta \, d\varpi \, d\theta; \quad Z_{ep} = -\int_0^{2\pi} \overline{p}_y \sin \theta \, d\varpi \, d\theta
\]  
(5.29)

Considering the complex form of perturbed pressures, the non-dimensional dynamic
coefficients are expressed as given below.
\[
\therefore Z_{ex} = \overline{K}_{ex} + i\Omega \overline{D}_{ex}; \quad Z_{ep} = \overline{K}_{ep} + i\Omega \overline{D}_{ep}
\]
\[
Z_{ex} = \overline{K}_{ex} + i\Omega \overline{D}_{ex}; \quad Z_{ep} = \overline{K}_{ep} + i\Omega \overline{D}_{ep};
\]  
(5.29)

The dynamic coefficients can then be transformed into linear coordinate system,
characteristic of steady state load \( (\theta, X, Y) \), using the general expressions given below [117].

**Stiffness coefficients coordinate transformation:**
\[
\begin{bmatrix}
\overline{K}_{XX} & \overline{K}_{XY} \\
\overline{K}_{XY} & \overline{K}_{YY}
\end{bmatrix} = \begin{bmatrix}
\cos \phi_0 & -\sin \phi_0 \\
\sin \phi_0 & \cos \phi_0
\end{bmatrix} \times \begin{bmatrix}
\overline{K}_{ex} & \overline{K}_{ep} \\
\overline{K}_{ep} & \overline{K}_{ep}
\end{bmatrix} \times \begin{bmatrix}
\cos \phi_0 & \sin \phi_0 \\
-\sin \phi_0 & \cos \phi_0
\end{bmatrix}
\]  
(5.30)

\[
\Rightarrow \begin{bmatrix}
\overline{K}_{XX} & \overline{K}_{XY} \\
\overline{K}_{XY} & \overline{K}_{YY}
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} \times \begin{bmatrix}
\overline{K}_{ex} & \overline{K}_{ep} \\
\overline{K}_{ep} & \overline{K}_{ep}
\end{bmatrix}
\]

\[
\Rightarrow \begin{bmatrix}
\overline{K}_{XX} & \overline{K}_{XY} \\
\overline{K}_{XY} & \overline{K}_{YY}
\end{bmatrix} = \begin{bmatrix}
\overline{K}_{ex} & \overline{K}_{ep} \\
\overline{K}_{ep} & \overline{K}_{ep}
\end{bmatrix}
\]  
(5.31)

**Damping coefficients coordinate transformation:**

Similar to the derivation of equation 5.31, coordinate transformation of damping
coefficients can be expressed as shown below.

\[
\begin{bmatrix}
D_{XX} & D_{XY} \\
D_{YX} & D_{YY}
\end{bmatrix}
= \begin{bmatrix}
D_{xx} & D_{x\phi} \\
D_{y\phi} & D_{\phi\phi}
\end{bmatrix}
\]  \hspace{1cm} (5.32)

Therefore, comparing equation 5.28 with the coordinate transformation equations 5.31 and 5.32, the non-dimensional stiffness and damping coefficients can be computed as follows:

\[
\overline{K}_{XX} = \overline{K}_{xx} = \text{Real}\left(Z_{xx}\right) = -\text{Re}\int_0^{2\pi} \int_0^1 \overline{p}_e \cos \theta d\bar{z} d\theta \\
\overline{K}_{YX} = \overline{K}_{y\phi} = \text{Real}\left(Z_{y\phi}\right) = -\text{Re}\int_0^{2\pi} \int_0^1 \overline{p}_\phi \cos \theta d\bar{z} d\theta \\
\overline{K}_{XY} = \overline{K}_{x\phi} = \text{Real}\left(Z_{x\phi}\right) = -\text{Re}\int_0^{2\pi} \int_0^1 \overline{p}_\phi \sin \theta d\bar{z} d\theta \\
\overline{K}_{YY} = \overline{K}_{\phi\phi} = \text{Real}\left(Z_{\phi\phi}\right) = -\text{Re}\int_0^{2\pi} \int_0^1 \overline{p}_\phi \sin \theta d\bar{z} d\theta
\]  \hspace{1cm} (5.33)

\[
\overline{D}_{XX} = \overline{D}_{xx} = \text{imag}\left(Z_{xx}\right) = -\text{Im}\int_0^{2\pi} \int_0^1 \overline{p}_e \cos \theta d\bar{z} d\theta / \Omega \\
\overline{D}_{YX} = \overline{D}_{y\phi} = \text{imag}\left(Z_{y\phi}\right) = -\text{Im}\int_0^{2\pi} \int_0^1 \overline{p}_\phi \cos \theta d\bar{z} d\theta / \Omega \\
\overline{D}_{XY} = \overline{D}_{x\phi} = \text{imag}\left(Z_{x\phi}\right) = -\text{Im}\int_0^{2\pi} \int_0^1 \overline{p}_\phi \sin \theta d\bar{z} d\theta / \Omega \\
\overline{D}_{YY} = \overline{D}_{\phi\phi} = \text{imag}\left(Z_{\phi\phi}\right) = -\text{Im}\int_0^{2\pi} \int_0^1 \overline{p}_\phi \sin \theta d\bar{z} d\theta / \Omega
\]  \hspace{1cm} (5.34)

The above eight coefficients constitute the dynamic coefficients of journal bearing system.

**5.5.7 Stability Analysis**

As pointed out in section 5.5.1 and with reference to Fig. 65, whirl instability of journal bearing is dependent on the stiffness and damping characteristics of TiO$_2$ nanolubricant film hydrodynamically developed in the journal bearing system. The
threshold of stable operation of a journal bearing is described by two parameters, which are: i) critical mass parameter $\bar{M}_{cr}$ and ii) critical whirl ratio $\Omega_{cr}$. Established theory on journal bearing stability provides equations to compute the above mentioned threshold parameters using the computed dynamic coefficients [385]. At the margin of stable operation, the non-dimensional critical mass parameter and critical whirl ratio are expressed as [385, 117]:

$$
\bar{M}_{cr} = \frac{M_{cr} \omega^2 C}{W_0} = \frac{A_{eq}}{\Omega^2_{cr}} \tag{5.41}
$$

$$
\Omega^2_{cr} = \left( \frac{\omega_p}{\omega} \right)^2 \frac{(\bar{K}_{XX} - A_{eq}) (\bar{K}_{YY} - A_{eq}) - \bar{K}_{XY} \bar{K}_{YX}}{D_{XX} D_{YY} - D_{XY} D_{YX}} \tag{5.42}
$$

where, $A_{eq} = \frac{\bar{K}_{XX} D_{YY} + \bar{K}_{YY} D_{XX} - \bar{K}_{XX} \bar{D}_{XX} - \bar{K}_{YY} \bar{D}_{YY} - \bar{K}_{XY} \bar{D}_{YX}}{D_{XX} + D_{YY}} \tag{5.43}$

The computed critical mass parameter is then used to evaluate the non-dimensional threshold journal speed as shown below.

$$
\bar{\omega}_{cr} = \omega_{cr} \sqrt{\frac{M_{cr} C}{W_0}} = \sqrt{\bar{M}_{cr}} \tag{5.44}
$$

From the above equation, it could therefore be stated that, a journal bearing system is asymptotically stable when the non-dimensional journal mass $\bar{M}$ is less than $\bar{M}_{cr}$. Likewise, a system is asymptotically stable when the non-dimensional operating speed of the rotor is less than $\bar{\omega}_{cr}$. A negative value of $\bar{M}_{cr}$ means that the journal will always be stable for all values of journal mass $\bar{M}$. Similarly, a negative value of $\Omega^2_{cr}$ implies the absence of whirl.

Therefore, studying the stability analysis of journal bearings operating on TiO$_2$ nanolubricants involves understanding the changes in linearized stiffness and damping coefficients of journal bearings due to the improved effective viscosity of nanolubricants at different additive volume fractions. The results of such as study performed as part of this research is presented in paper P7$^5$ (accepted for publication) and enclosed in Appendix – A [35]. The study will also consider the influences of couple stresses due to the presence of TiO$_2$ nanoparticle aggregates in the lubricant film.

5.5.8 Solution Methodology

The solution methodology used is as presented below.

i. Obtain input parameters:
   a) *Bearing geometry and operating parameters:* Bearing length to diameter ratio \( \lambda = \frac{L}{2R} \), eccentricity ratio \( \varepsilon \), and radial clearance \( C \).
   b) *Nanolubricant properties:* Viscosity of the base oil \( \mu_{bf} \), nanolubricant viscosity model, ratio of TiO\(_2\) nanoparticle aggregate to primary particle size \( \frac{a_u}{d} \), TiO\(_2\) nanoparticle additive volume fraction \( \phi \), maximum particle packing fraction \( \phi_m \) (0.5 for low shear and 0.605 for high shear applications)
   c) *Whirl ratio:* An initial assumed whirl ratio \( \Omega \) is sought as input (E.g. \( \Omega = 0.5 \)).
   d) *Couple stress factor:* TiO\(_2\) nanoparticle aggregate particle size \( d \).
   e) Mesh set-up and iteration constants: grid size \( (N \times M) \), terminating residual values for iterations, number of iterations for convergence, SOR (successive over relaxation) factor for Gauss-Siedel.

ii. Initialization:
   
   Grid intervals:
   
   \[
   \Delta \theta = \frac{2\pi}{N - 1}, \Delta \zeta = \frac{1}{M - 1}, \quad \text{and} \quad r = \frac{\Delta \zeta}{\Delta \theta}
   \]

   Initialize arrays to zero: Arrays are initialized for the following parameters: non-dimensionaľ film thickness \( \overline{h} \); non-dimensionaľ steady state pressure \( \overline{p}_0 \); non-dimensionaľ dynamic pressures \( \overline{p}_e \) and \( \overline{p}_\phi \); couple stress factors \( \overline{f}(\overline{h}, \overline{d}) \) and \( \overline{f'}(\overline{h}, \overline{d}) \); variable used in identifying the cavitation front; steady state load and dynamic forces.

   Iteration constants: zero is assigned to iteration count, a significant higher value is assigned to the variable that stores the iteration error, in comparison to, the maximum terminating residual value.

   Viscosity model: Depending on nanolubricant viscosity model provided as input, a suitable case-switch variable is initialized to select the specified model from the list of nanofluid viscosity models considered for analysis.

iii. Compute constants:
**Effective viscosity of nanolubricant**: based on the selection of nanolubricant viscosity model, the effective viscosity of nanolubricant is computed as a function of volume fraction and aggregate nanoparticle size $\bar{\mu} = \frac{\mu_{nf}}{\mu_{bf}}$.

iv. Compute film thickness:

*Non-dimensional film thickness* $\bar{h}$: non-dimensional film thickness is computed across the grid using the standard relation $\bar{h} = 1 + \varepsilon \cos \theta$.

v. Compute couple stress factors:

*Couple stress characteristic factor and its derivative* $\bar{f}(\bar{h}, \bar{d})$ and $\bar{f}'(\bar{h}, \bar{d})$: the non-dimensional film thickness values across the grid is used to compute the couple stress characteristic factor and its derivative, which are dependent on the nanoparticle aggregate particle size.

\[
\bar{f}_0(\bar{h}_0, \bar{d}) = \bar{h}_0^3 - 12\bar{d}^2 \left[ \bar{h}_0 - 2\bar{d} \tanh \left( \frac{\bar{h}_0}{2\bar{d}} \right) \right] \\
\bar{f}'(\bar{h}_0, \bar{d}) = -3\bar{h}_0^2\varepsilon \sin \theta + 12\bar{d}^2\varepsilon \sin \theta \tanh \left( \frac{\bar{h}_0}{2\bar{d}} \right)
\]

vi. Compute steady state pressure distribution:

*Steady state pressure* $(\bar{p}_0)_{i,j}$: Iterative solution of modified non-dimensional Reynolds equation 5.14 is carried out using central difference scheme with SOR – Gauss-Siedel. The convergence is checked at each iteration:

\[
\max \left( \frac{\bar{p}_{i,j}\text{Current} - \bar{p}_{i,j}\text{Old}}{\bar{p}_{i,j}\text{Current}} \right) \leq 10^{-5}.
\]

Iteration is continued till the conditions of termination with regard to residual error or number of iterations is met. Grid continuity criteria is set during the computation to ensure the overlapping of end nodes to facilitate continuity.

vii. Identify cavitation front:

After computation of steady state pressures on all nodes, the commencement of negative pressures is identified across the grid length. All negative pressures are substituted with zero. This vanishing of negative pressures is performed in the computation of steady state load only.

viii. Computation of steady state load:
Numerical integration of steady state pressures across the solution domain using Simpson’s 1/3 rule provides the steady state load.

ix. **Compute dynamic pressure distributions:**

The dynamic pressures developed along the line of center and perpendicular to the line of center is obtained by solving the perturbed Reynolds equations 5.19 and 5.22. The solution scheme is similar to that employed in obtaining the steady state pressures. The iteration criteria and terminal values are similar to the steady state computations described in point vi) listed above.

x. **Computation of dynamic loads:**

Numerical integration of dynamic pressures across the solution domain using Simpson’s 1/3 rule provides the dynamic loads. The dynamic loads are computed along and perpendicular to the line of centers.

xi. **Computation of dynamic coefficients:**

As described in section 5.5.6, the stiffness and damping coefficients are computed for journal bearings operating at different TiO$_2$ nanoparticle concentrations and aggregate particle sizes.

xii. **Stability analysis:**

The dynamic coefficients are then employed in studying the influence of TiO$_2$ nanoparticle additive concentrations and aggregate particle sizes on the threshold speeds and critical mass parameters of journal bearings. The underlying theory is previously explained in section 5.5.7.

### 5.5.9 Validation of computational code

The computational code that is developed on the basis of solution scheme explained in section 5.5.8 is validated by comparing the obtained journal bearing dynamic coefficients with the published results of Guha [114]. For Newtonian fluids, corresponding to the case of zero couple stress parameter ($\overline{d} = 0$) and zero additive volume fraction ($\phi = 0$), the stiffness and damping coefficients are obtained for increasing eccentricity ratio and presented in Figs. 67 and 68. On comparison, the variation in stiffness and damping coefficients are found to be in qualitative agreement with the results published by Guha [114]. Fig. 67(b) provides the stiffness coefficients from Guha [114] for comparison and Fig. 68(b) provides the damping coefficients from Guha [114]. For validation, the dynamic coefficients for $\overline{d} = 0$ and $\phi = 0$ represented as Newtonian in Guha [114] is compared with
coefficients obtained from the code as presented in Fig. 67 and 68. The validated code is then used in the study to analyze the influence of TiO$_2$ nanoparticle additive concentration and aggregate particle sizes on the journal bearing dynamic characteristics.

![Diagram](image)

*Figure 67: Variation in stiffness coefficients with eccentricity ratio for Newtonian fluids $(d = 0, \phi = 0)$*

### 5.5.10 Results and discussions

The modified Reynolds equation – 5.5, presented in section 5.5.2 of this chapter, incorporates parameters which enable us to study the influence of TiO$_2$ nanoparticle additive size, as well as, TiO$_2$ nanoparticle additive concentration on the dynamic journal bearing characteristics. The relative viscosity term $\bar{\mu}$, computed using the modified Krieger-Dougherty equation – 5.1, integrates the effects of TiO$_2$ nanoparticle additive concentration on the dynamic characteristics of journal bearings. The influence of TiO$_2$ nanoparticle additive size on the dynamic characteristics of journal bearings is studied using the couple stress parameter $\tilde{d} = \frac{d}{C}$, which is in accordance with the Stokes couple stress theory.

Upon validating the computational code, dynamic coefficients are computed for different values of couple stress parameters and nanoparticle concentrations. Table 11 provides the values of various operating parameters used in the analysis. The validated computational code is then used to obtain the dynamic characteristics of journal bearings.
operating on TiO$_2$ nanolubricants at different TiO$_2$ nanoparticle concentrations and aggregate particle sizes.

Figure 67(b): Stiffness coefficients from Guha [114]

Figure 68: Variation in damping coefficients with eccentricity ratio for Newtonian fluids

\[ \bar{d} = 0, \phi = 0 \]
Figs. 69 - 76 provides the variation of stiffness and damping coefficients at TiO$_2$ nanoparticle volume fractions of 0.001, 0.005, 0.01, and 0.02; for couple stress parameters of $\tau = 0.01, 0.03108, \text{ and } 0.5$. It is observed from Figs. 69 - 76 that, for a given TiO$_2$ nanoparticle additive concentration, increasing the nanoparticle aggregate size (simulated by varying couple stress parameter) results in an increase in stiffness and damping coefficients.

Study also reveals an increase in stiffness and damping coefficients for an increase in TiO$_2$ nanoparticle concentrations of fixed aggregate TiO$_2$ nanoparticle size. Figs. 77 and 78 provides the variation in dynamic coefficients of TiO$_2$ nanoparticle dispersions of aggregate particle size 777 nm, which results in a couple stress parameter of 0.03108, with TiO$_2$ nanoparticle additive concentrations of $\phi = 0.001, 0.005, 0.01, 0.02$.

The comparative increment in stiffness and damping coefficients due to the influence of both, TiO$_2$ nanoparticle concentration and TiO$_2$ nanoparticle aggregate size, is presented in Tables 12 and 13.
Table 11: Operating parameters

<table>
<thead>
<tr>
<th>Bearing Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio: 1.0</td>
</tr>
<tr>
<td>Bearing Angle: $\theta = 360^\circ$ Full Bearing</td>
</tr>
<tr>
<td>Bearing Type: Plain Compliant Bearing</td>
</tr>
<tr>
<td>Bearing Clearance: 25 microns</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lubricant Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type: Couple stress fluid</td>
</tr>
<tr>
<td>Additives: TiO$_2$ nanoparticles</td>
</tr>
<tr>
<td>Couple stress parameter: $\bar{d} = \frac{d}{c} = 0.03108$ corresponding to aggregate nanoparticle size of 777 nm and bearing clearance of 25 microns. Analysis is also performed for $\bar{d} = 0.01$ and $\bar{d} = 0.05$, for comparison.</td>
</tr>
<tr>
<td>TiO$_2$ nanoparticle concentrations: $\phi = 0.001, 0.005, 0.01, 0.02$</td>
</tr>
</tbody>
</table>

Table 12 offers a comparison of stiffness coefficients of journal bearings operating with TiO$_2$ nanolubricants with those of plain oil at a TiO$_2$ nanoparticle volume fraction $\phi = 0.01$ and couple stress parameter $\bar{d} = 0.03108$. Table 13 offers a similar comparison for damping coefficients.

As observed in Table 12, the stiffness coefficients are considerably higher for nanolubricants in comparison to plain oil. This increase in stiffness coefficients being more prominent at higher eccentricities. A similar observation is also obtained for damping coefficients. The results also present the comparative influence of couple stress parameter and additive volume fraction on the dynamic coefficients.

Table 12: Comparison of Stiffness Coefficients

<table>
<thead>
<tr>
<th>Eccentricity Ratio $\varepsilon$</th>
<th>Plain oil</th>
<th>Nanolubricant at $\phi = 0.01$ and $\bar{d} = 0.03108$</th>
<th>Percentage increase (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>KRR</td>
<td>KPP</td>
<td>KPR</td>
</tr>
<tr>
<td>0.2</td>
<td>0.28</td>
<td>0.65</td>
<td>-3.25</td>
</tr>
<tr>
<td>0.5</td>
<td>2.06</td>
<td>2.38</td>
<td>-5.16</td>
</tr>
<tr>
<td>0.8</td>
<td>23.81</td>
<td>9.12</td>
<td>-17.3</td>
</tr>
<tr>
<td>0.9</td>
<td>176.7</td>
<td>18.8</td>
<td>-50.8</td>
</tr>
</tbody>
</table>
Figure 69: Variation in stiffness coefficients at a volume fraction $\phi=0.001$ for varying couple stress parameters $\tilde{\alpha} = 0.01, 0.03108,$ and $0.5$

Figure 70: Variation in stiffness coefficients at a volume fraction $\phi=0.005$ for varying couple stress parameters $\tilde{\alpha} = 0.01, 0.03108,$ and $0.5$
Figure 71: Variation in stiffness coefficients at a volume fraction $\phi=0.01$ for varying couple stress parameters $\overline{d} = 0.01, 0.03108, \text{ and } 0.5$

Figure 72: Variation in stiffness coefficients at a volume fraction $\phi=0.02$ for varying couple stress parameters $\overline{d} = 0.01, 0.03108, \text{ and } 0.5$
Figure 73: Variation in damping coefficients at a volume fraction $\phi=0.001$ for varying couple stress parameters $\bar{d} = 0.01, 0.03108,$ and $0.5$

Figure 74: Variation in damping coefficients at a volume fraction $\phi=0.005$ for varying couple stress parameters $\bar{d} = 0.01, 0.03108,$ and $0.5$
Figure 75: Variation in damping coefficients at a volume fraction $\phi=0.01$ for varying couple stress parameters $\overline{d} = 0.01, 0.03108, \text{ and } 0.5$

Figure 76: Variation in damping coefficients at a volume fraction $\phi=0.02$ for varying couple stress parameters $\overline{d} = 0.01, 0.03108, \text{ and } 0.5$
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Figure 77: Variation in stiffness coefficients at a couple stress parameter of $\overline{\alpha} = 0.03108$ for varying TiO$_2$ nanoparticle concentrations of $\phi = 0.001, 0.005, 0.01$ and 0.02.

Figure 78: Variation in damping coefficients at a couple stress parameter of $\overline{\alpha} = 0.03108$ for varying TiO$_2$ nanoparticle concentrations of $\phi = 0.001, 0.005, 0.01$ and 0.02.
Considering the cross-stiffness coefficient $K_{RP}$ at an eccentricity ratio of 0.8, it is observed that adding TiO$_2$ nanoparticle additives of aggregate size 250 nm (corresponding to $\bar{d} = 0.01$), at a volume fraction of $\phi = 0.01$ will increase the cross stiffness by 42% in comparison to plain oil. This percentage increase moves up to 51% and 63% when increasing the aggregate particle size to 777 nm (corresponding to $\bar{d} = 0.03108$) and 1250 nm (corresponding to $\bar{d} = 0.05$) respectively.

However, when TiO$_2$ nanoparticle size is kept constant at 777 nm and TiO$_2$ additives are added at a volume fraction of 0.001, the cross stiffness $K_{RP}$ is found to increase by 10% in comparison to plain oil.

The increase in stiffness and dynamic coefficients could be related to the following reasons:

a. The addition of nanoparticle additives causes an increase in the dynamic viscosity of the lubricant which results in increased stiffness and damping capacity of the fluid film.

b. The couple stresses of the nanoparticle additives cause an increase in the dynamic pressures developed in the fluid film. The dynamic coefficients being a direct function of the pressures, as observed in equations 5.33 to 5.40, will therefore exhibit an increase in its magnitude as reflected in Table 12.

This increase in cross stiffness is more pronounced when TiO$_2$ nanoparticles are added at higher volume fractions. Addition of 0.01 and 0.02 volume fractions increases the cross stiffness parameter $K_{RP}$ by 51% and 140% respectively. However, it has to be mentioned that higher particle sizes and volume fractions will change the flow behavior and physical interactions of the particles leading to quicker sedimentation. Optimum values of TiO$_2$ nanoparticle sizes and volume fractions for acceptable dispersion stability has to be determined. Experimentation to determine optimum values are necessary. It also needs to be mentioned that, considering the changes in viscosity due to temperature variation will
also provide a more accurate picture. The viscosity variations dealt with in this study does not account for the temperature variation prevalent in bearing area for high load and high speed applications. Hence an thermohydrodynamic analysis of dynamic characteristics will offer more accurate insights into the dynamic behaviour of journal bearings operating on nanolubricants.

Influence of couple stress parameter and TiO$_2$ nanoparticle concentration on whirl instability of journal bearings is studied and presented below. The stability parameters comprising of critical mass parameter, critical threshold speed (angular), and whirl ratio are computed and their variations with TiO$_2$ aggregate particle size and concentration of TiO$_2$ nanoparticle additives are studied.

Fig. 79 shows the variation in critical mass parameter $\sigma_{cr}$ with eccentricity ratio for different couple stress parameters, corresponding to TiO$_2$ aggregate nanoparticle sizes, at a constant volume fraction of $\phi = 0.01$.

![Graph showing variation in critical mass parameter](image)

*Figure 79: Variation in Critical Mass Parameter with varying couple stress parameters at a constant volume fraction of $\phi = 0.01$*

It is observed from Fig. 79 that, the addition of nanoparticles of different sizes lead to an increase in stability of journal bearing systems at higher eccentricities (0.7 to 0.9). At lower eccentricity ratio, the influence of nanoparticle size (couple stress parameter) on
stability is not of much significance. At higher eccentricities, significant increase in stable operating region is observed in the plot.

Influence of TiO$_2$ nanoparticle concentration at a constant particle size of 777 nm on the critical mass parameter is obtained and the results are shown in Fig. 80. As seen in Fig. 80, stability of journal bearing systems is found to improve with addition of TiO$_2$ nanoparticles at increasing concentrations. The increase in stability is more prominent at eccentricity ratio higher than 0.5.

Improvement in stability of journal bearing system, as observed in Figs. 78 and 79, is further analysed by plotting the threshold stability maps for varying couple stress parameters, as well as, varying nanoparticle concentrations. The threshold speeds for varying couple stress parameters at a TiO$_2$ nanoparticle volume fraction of 0.01 is presented in Fig. 81.

The threshold stability map for varying TiO$_2$ nanoparticle concentrations at a constant particle size of 777 nm is shown in Fig. 82.

![Figure 80: Variation in Critical Mass Parameter with varying TiO$_2$ additive volume fractions at a constant couple stress parameter of $d^* = 0.03108$](image)

The improvement in stability of journal bearing system due to the addition of TiO$_2$ nanoparticles at higher eccentricities is evident in threshold stability maps provided in Figs. 81 and 82. This fact is also observed in Fig. 83, which shows the variation of critical whirl ratio for various couple stress parameters at a constant TiO$_2$ volume fraction of 0.01.
Chapter 5: Dynamic Performance and Stability Characteristics

Figure 81: Threshold stability map for varying couple stress parameters at TiO$_2$ volume fraction of $\phi = 0.01$

Figure 82: Threshold stability map for varying TiO$_2$ volume fractions at a constant couple stress parameter of $\overline{d} = 0.03108$

The reason for pronounced influence of TiO$_2$ nanoparticle additives on stability characteristics of journal bearings at higher eccentricities could be attributed to comparable nanoparticle size and film thickness at higher eccentricities. Reduced film thickness associated with higher eccentricities could permit increased physical interactions between particles and between particles and surfaces, resulting in pronounced damping coefficients.
However, its true influence will also be greatly influenced by dispersion stability and temperature reduction of oil film viscosity.

Figure 83: Critical whirl ratio for varying couple stress parameters at constant volume fraction of $\phi = 0.01$

Fig. 84 shows the variation of critical whirl ratio for varying TiO$_2$ volume fractions of constant aggregate particle size of 777 nm.

Figure 84: Critical whirl ratio for varying TiO$_2$ volume fractions at constant couple stress parameter of $d = 0.03108$
Results of the analysis as illustrated in Figs. 69 to 78 indicate that the impact of TiO$_2$ nanoparticle additives, with increasing volume fraction and particle size, on the dynamic coefficients are significant only at higher eccentricities. The reason for pronounced influence of TiO$_2$ nanoparticle additives on stability characteristics of journal bearings at higher eccentricities could be attributed to comparable nanoparticle size and film thickness at higher eccentricities. Reduced film thickness associated with higher eccentricities could permit increased physical interactions between particles and between particles and surfaces, resulting in pronounced damping coefficients. However, the analysis also points to a significant increase in dynamic coefficients for TiO$_2$ nanolubricants at volume fraction of 0.01 and particle size of 777 nm in comparison to plain oil. Increasing the volume fraction to 0.02 and the particle size to 1250 is observed to have very little impact of dynamic coefficients at lower eccentricities (below 0.5). This is also revealed in the critical mass parameter and threshold speed results presented in Figs. 79-80 and 81-82 respectively. Therefore, in conclusion it can be stated that while the presence of TiO$_2$ nanoparticles influences stability in comparison to plain oil, increasing the concentration and particle sizes to 0.02 and 1250 respectively, does not influence the stability at lower eccentricities. It is also stated that, the limiting values of 0.02 volume fraction and 1250 particle size are chosen considering the severe reduction in flow characteristics of the lubricant above these limiting values.

As observed in Fig. 84, there is negligible change in critical whirl ratio with increasing nanoparticle concentrations. However, it is seen in Fig. 82 that, the threshold speed of journal bearing system is increasing with TiO$_2$ volume fractions. Therefore, even though the whirl ratio does not reveal significant variation with volume fraction, there is a significant increase in stable operating region at higher eccentricities due to the addition of TiO$_2$ nanoparticle additives. Quantitative comparisons of threshold speed $\bar{\omega}_{cr}$ for journal bearing operating on TiO$_2$ nanolubricant in comparison to plain engine oil for varying particle concentration and aggregate nanoparticle size in presented in Tables 14 and 15 respectively.

Threshold stability maps and whirl ratio variation shown in Figs. 83 and 84 reveals that, the influence of TiO$_2$ nanoparticle size, controlled in the analysis by varying the couple stress factor, is more significant that TiO$_2$ nanoparticle concentration. However, as seen in all stability characteristic variations, the presence of TiO$_2$ nanoparticle additives clearly demonstrates an increase in stable operating region of journal bearing systems.
It is observed from Tables – 13 and 14 that, the presence of TiO\textsubscript{2} nanoparticles at a concentration of $\phi = 0.01$ and couple stress parameter $\overline{d} = 0.03108$ is found to increase the critical threshold journal speed by nearly ~20%.

**Table 14: Percentage increase in threshold speed due to addition of TiO\textsubscript{2} nanoparticle additives at constant couple stress parameter $\overline{d} = 0.03108$ at varying particle volume fractions (Linear perturbation analysis)**

<table>
<thead>
<tr>
<th>Eccentricity Ratio $\varepsilon$</th>
<th>Plain engine oil</th>
<th>Critical Threshold Speed $\overline{\omega_{cr}}$ for Nanolubricants at $\overline{d} = 0.03108$</th>
<th>Percentage increase (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{d} = 0, \phi = 0$</td>
<td>1.47</td>
<td>1.51, 1.47, 1.51, 1.51, 1.47, 1.76, 9.56, 19.93</td>
<td>51.46</td>
</tr>
<tr>
<td>$\overline{d} = 0.001 \phi$</td>
<td>2.44</td>
<td>2.52, 2.44, 2.52, 2.44, 3.12, 9.93, 20.35</td>
<td>51.98</td>
</tr>
<tr>
<td>$\overline{d} = 0.005 \phi$</td>
<td>3.04</td>
<td>3.14, 3.04, 3.14, 3.04, 3.38, 10.21, 20.65</td>
<td>52.36</td>
</tr>
<tr>
<td>$\overline{d} = 0.01 \phi$</td>
<td>3.75</td>
<td>3.89, 3.75, 3.89, 3.75, 3.76, 10.62, 21.09</td>
<td>52.92</td>
</tr>
<tr>
<td>$\overline{d} = 0.02 \phi$</td>
<td>4.67</td>
<td>4.88, 4.67, 4.88, 4.67, 4.48, 11.39, 21.94</td>
<td>53.99</td>
</tr>
</tbody>
</table>

**Table 15: Percentage increase in threshold speed due to addition of TiO\textsubscript{2} nanoparticle additives at constant TiO\textsubscript{2} volume fraction $\phi = 0.01$ with varying aggregate particle size characterized by couple stress parameter**

<table>
<thead>
<tr>
<th>Eccentricity Ratio $\varepsilon$</th>
<th>Plain engine oil</th>
<th>Critical Threshold Speed $\overline{\omega_{cr}}$ for Nanolubricants at $\phi = 0.01$</th>
<th>Percentage increase (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{d} = 0, \phi = 0$</td>
<td>1.47</td>
<td>1.74, 1.76, 1.79, 18.63, 19.93, 22.15</td>
<td></td>
</tr>
<tr>
<td>$\overline{d} = 0.01 \phi$</td>
<td>2.44</td>
<td>2.90, 2.94, 3.01, 18.68, 20.35, 23.18</td>
<td></td>
</tr>
<tr>
<td>$\overline{d} = 0.05 \phi$</td>
<td>3.04</td>
<td>3.60, 3.66, 3.76, 18.71, 20.65, 23.93</td>
<td></td>
</tr>
<tr>
<td>$\overline{d} = 0.01 \phi$</td>
<td>3.75</td>
<td>4.46, 4.55, 4.69, 18.76, 21.09, 25.04</td>
<td></td>
</tr>
<tr>
<td>$\overline{d} = 0.02 \phi$</td>
<td>4.67</td>
<td>5.56, 5.70, 5.95, 18.85, 21.94, 27.27</td>
<td></td>
</tr>
</tbody>
</table>

**Highlights of dynamic characteristics study of journal bearing systems operating on TiO\textsubscript{2} nanolubricants using linear perturbation approach**

In this study, dynamic characteristics of journal bearing systems operating on TiO\textsubscript{2} nanoparticle dispersions in engine oil is obtained using linear perturbation approach.

- A modified Reynolds equation is developed, which integrates the influence of TiO\textsubscript{2} nanoparticle additive concentration and nanoparticle aggregate sizes on the dynamic characteristics of journal bearing systems.
5.6 Nonlinear transient analysis of dynamic characteristics of journal bearings operating on TiO$_2$ nanolubricants

In the previous section 5.5, the dynamic characteristics and whirl instabilities of journal bearing operating on TiO$_2$ nanolubricants is studied using linearized perturbation approach in which the dynamic coefficients were used to simulate the threshold journal mass and speeds delineating stable and unstable operations. A more accurate method of studying the stability characteristics of journal bearings is the non-linear transient analysis in which the trajectories of journal center are obtained at various eccentricity ratios as a function of TiO$_2$ concentration and aggregate particle size.

Obtaining the journal center trajectories separating stable and unstable operation of the system is an efficient method of computing the threshold journal mass and critical journal speeds. Comparative studies on whirl instability characteristics of journal bearings using both nonlinear transient analysis and linear perturbation approach was carried out by Gadangi et al. [396] and Turaga et al. [389, 397]. The limits of operational stability with respect to whirl of rigid rotors supported on journal bearings were obtained using linearized
perturbation method and nonlinear approach. Results reported a significant difference in critical journal mass and critical whirl ratios obtained by both the methods. The studies also claim that the results of nonlinear method are more accurate in comparison to linear perturbation and pseudo-transient methods of whirl instability analysis.

In this study, the journal center trajectories are obtained for journal bearing operating on TiO$_2$ nanolubricants. Influence of TiO$_2$ nanoparticle additive concentration and TiO$_2$ aggregate particle size on dynamic oil film forces is studied. The whirl instability threshold is then illustrated by plotting the threshold stability maps for various combinations of TiO$_2$ additive concentration and aggregate particle sizes.

5.6.1 Governing Reynolds equation

Schematic of a fluid film bearing used in the analysis is illustrated in Fig. 85(a). The generated fluid film forces acting on the bearing system is also represented in Fig. 85(b). Influence of TiO$_2$ additive aggregate particle sizes is modeled in the analysis by considering the nanolubricant as a couple stress fluids. Similar to static analysis presented in Chapter 4, the influence of TiO$_2$ nanoparticle concentration is integrated into the analysis using the relative viscosities of TiO$_2$ nanolubricants modeled using modified Krieger-Dougherty viscosity model.

Assuming the absence of body forces and body couples, along with the usual assumptions of hydrodynamic lubrication as mentioned in Pinkus [29], the governing Reynolds equation is expressed below.

$$ \frac{\partial}{\partial x} \left( f(h, d) \frac{\partial \rho}{\partial x} \right) + \frac{\partial}{\partial z} \left( f(h, d) \frac{\partial \rho}{\partial z} \right) = 6\mu U \frac{\partial h}{\partial x} + 12\mu \frac{\partial h}{\partial t} \quad (5.45) $$

where, $$ f(h, d) = h^3 - 12hd^2 + 24d^3 \tanh \left( \frac{h}{2d} \right) $$

Further, a non-dimensional form of the governing Reynolds equation is obtained using the following non-dimensional terms [114, 117].

$$ \theta = \frac{x}{R}; \bar{z} = \frac{z}{L}; \varepsilon = \frac{e}{C}; \bar{\rho} = \frac{pC^2}{\mu_0R^2}; \bar{h} = \frac{h}{C}; \bar{\lambda} = \frac{L}{D}; \bar{\bar{\lambda}} = \omega t; \bar{\mu} = \frac{\mu_0}{\mu_{bf}}; \bar{d} = \frac{d}{C} \quad (5.46) $$

The obtained non-dimensional Reynolds equation is presented below.

$$ \frac{\partial}{\partial \theta} \left( f(h, d) \frac{\partial \bar{\rho}}{\partial \theta} \right) + \frac{1}{4\lambda^2} \frac{\partial}{\partial \bar{z}} \left( f(h, d) \frac{\partial \bar{\rho}}{\partial \bar{z}} \right) = 6\bar{\mu} \frac{\partial \bar{h}}{\partial \theta} + 12\bar{\mu} \frac{\partial \bar{h}}{\partial \bar{\bar{\lambda}}} \quad (5.47) $$
Equations 5.47 represents the time dependent non-dimensional modified Reynolds equations for journal bearings operating on TiO$_2$ nanolubricants. The solution of equations 5.47 at different TiO$_2$ nanoparticle concentrations and particle sizes will provide equivalent steady state pressures developed within the nanolubricant oil film. In numerically solving the steady state Reynolds equations, expressed as equations 5.47, the Reynolds boundary
conditions, described in section 4.4.4, are used.

The boundary conditions are as expressed below.

\[ p = 0 \text{ at } \bar{z} = 0, 1 \tag{5.48} \]

\[ p = 0 \text{ at } \theta = 0 \tag{5.49} \]

\[ \frac{\partial p}{\partial \theta} = 0 \text{ at } \theta = \theta_m \tag{5.50} \]

\[ p = 0 \text{ at } \theta_m \leq \theta \leq 2\pi \tag{5.51} \]

In computing the steady state pressures, the cavitated pressures are equated to zero based on Christopherson algorithm [374]. The cavitation front is described by the angular parameter \( \theta_m \). The steady state Reynolds equation is solved numerically using the finite difference approach described in Chapter 4. The central difference scheme of discretizing is employed and solved for steady state pressure using Gauss-Siedel with successive over relaxation method. The numerical formulation and solution technique is described in detail under section 4.4.6. The discretized Reynolds equation for steady state pressure is expressed in equation 5.52 presented below.

\[
(p_{0})_{i,j} = \frac{\left( A \bar{p}_{i-1,j} + B \bar{p}_{i+1,j} + C \bar{p}_{i,j-1} + D \bar{p}_{i,j+1} + RHS \right)}{E} \tag{5.52}
\]

where,

\[ A = 4\lambda^2 \bar{\tau}^2 \bar{f}_0 \left( \bar{h}_0, \bar{d} \right) - 2\lambda^2 \bar{\tau} \Delta \bar{z} \bar{f}_0' \left( \bar{h}_0, \bar{d} \right) \]

\[ B = 4\lambda^2 \bar{\tau}^2 \bar{f}_0 \left( \bar{h}_0, \bar{d} \right) + 2\lambda^2 \bar{\tau} \Delta \bar{z} \bar{f}_0' \left( \bar{h}_0, \bar{d} \right) \]

\[ C = \bar{f}_0 \left( \bar{h}_0, \bar{d} \right); \quad D = \bar{f}_0 \left( \bar{h}_0, \bar{d} \right); \quad E = 8\lambda^2 \bar{\tau}^2 \bar{f}_0 \left( \bar{h}_0, \bar{d} \right) + 2\bar{f}_0 \left( \bar{h}_0, \bar{d} \right) \]

\[ RHS = 24 \bar{\tau} \lambda^2 \bar{\tau} \Delta \bar{z} \Delta \theta \varepsilon_0 \sin \theta \]

\[ \bar{\tau} = \frac{\Delta \bar{z}}{\Delta \theta}; \lambda = \frac{L}{2R} \]

\[ \bar{f}_0 \left( \bar{h}_0, \bar{d} \right) = \bar{h}_0^3 - 12\bar{d}^2 \left[ \bar{h}_0 - 2\bar{d} \tanh \left( \frac{\bar{h}_0}{2\bar{d}} \right) \right] \tag{5.54} \]

\[ \bar{f}_0' \left( \bar{h}_0, \bar{d} \right) = -3\bar{h}_0^2 \varepsilon_0 \sin \theta + 12\bar{d}^2 \varepsilon_0 \sin \theta \tan^2 \left( \frac{\bar{h}_0}{2\bar{d}} \right) \]

The solution methodology for obtaining the steady state pressures is similar to the methodology presented and discussed in Chapter 4 under section 4.4.6. The computational grid for central difference scheme used in the analysis is also similar to the grid used in the computation of static performance characteristics and is illustrated in Fig. 45.

The dimensionless pressures obtained by the solution of Reynolds equation is then used in computing the generated hydrodynamic forces shown in Fig. 85(b). The
hydrodynamic forces $F_{\theta}$ and $F_r$ are then used in the nonlinear transient analysis as explained below.

### 5.6.2 Nonlinear transient stability analysis

Nonlinear analysis of journal bearing dynamic characteristics commences with the equations of motion generated with the application of Newton’s second law to the hydrodynamic force system illustrated in Fig. 85(b). The resulting equations of motion are presented below.

\[
MC \left[ \ddot{\phi} - \epsilon \left( \dot{\phi} \right)^2 \right] = F_r + W \cos \phi \tag{5.55}
\]

\[
MC \left[ \epsilon \ddot{\phi} + 2 \epsilon \dot{\phi} \right] = F_{\theta} - W \sin \phi \tag{5.56}
\]

where, $F_r$ and $F_{\theta}$ are the generated bearing forces which is numerically computed as a solution of the steady state Reynolds equation. $M$ and $C$ are the mass of the journal and radial bearing clearance respectively.

The dimensionless equations of motion are obtained as presented below.

\[
\ddot{\phi} = \epsilon \left( \dot{\phi} \right)^2 + \frac{F_r}{W_0M} + \frac{\cos \phi}{\bar{M}} \tag{5.57}
\]

\[
\dot{\phi} = - \frac{2 \epsilon \dot{\phi}}{\epsilon} + \frac{F_{\theta}}{W_0M \epsilon} \cdot \frac{\sin \phi}{\bar{M} \epsilon} \tag{5.58}
\]

Where, $\ddot{\phi}, \dot{\phi}$ represent the velocities of journal center, $\dddot{\phi}, \ddot{\phi}$ are the acceleration derivatives, $\bar{M} = \frac{MC\omega^2}{W_0}$ is the dimensionless mass parameter and $\bar{W_0} = \frac{W_0C^2}{\mu \omega R^3 L}$ is the dimensionless static load. The second order equations of motion pertaining to $\epsilon, \phi$ represented as equations 5.57 and 5.58 are solved using the fourth order Runge Kutta method for fixed values of mass parameter $\bar{M}$, and steady state load components $\bar{F}_{\theta}$ and $\bar{F}_r$ to obtain the next set of journal center coordinates. Repeated computations of the above procedure will result in the complete trajectory of the journal center. The assumed mass parameter $\bar{M}$ values are modified till the limit cycle corresponding to critical mass parameter $\bar{M}_{cr}$ is obtained which points to the threshold of stable operation.
Upon arriving at the critical mass parameter $\bar{M}_{cr}$, the analysis moves on to the computation of critical threshold journal speed using the dimensionless equation presented below.

\[ \bar{\sigma}_{cr} = \omega_{cr} \sqrt{\frac{M_{cr} C}{W_0}} = \sqrt{\bar{M}_{cr}} \]  

(5.59)

Obtaining the limit cycles of journal center also provides for the computation of critical whirl speed $\bar{\omega}_{pcr}$ of the journal center. This is done by finding out the time steps required for the completion of one whirl orbit of the journal center. In addition to critical mass parameter, critical journal speed, and critical whirl speed, an additional parameter used in stability analysis is the critical whirl ratio $\Omega_{cr}$. The critical whirl ratio is obtained as the ratio of critical whirl speed to critical journal speed.

\[ \Omega_{cr} = \frac{\omega_{pcr}}{\omega_{cr}} \]  

(5.60)

5.6.3 Solution Methodology

The solution methodology used is as presented below.

i. Obtain input parameters:

a) **Bearing geometry and operating parameters**: Bearing length to diameter ratio $\lambda = \frac{L}{2R}$, eccentricity ratio $\varepsilon$, radial clearance $C$, assumed mass parameter $\bar{M}$, assumed time steps for whirl orbit $t$, journal rotational speed in RPM.

b) **Nanolubricant properties**: Viscosity of the base oil $\mu_{bf}$, nanolubricant viscosity model, ratio of TiO$_2$ nanoparticle aggregate to primary particle size $\frac{a_a}{a}$, TiO$_2$ nanoparticle additive volume fraction $\phi$, maximum particle packing fraction $\phi_m$ (0.5 for low shear and 0.605 for high shear applications)

c) **Couple stress factor**: TiO$_2$ nanoparticle aggregate particle size $d$.

d) **Mesh set-up and iteration constants**: grid size $(N \times M)$, terminating residual values for iterations, number of iterations for convergence, SOR (successive over relaxation) factor for Gauss-Siedel.

ii. Initialization:

Grid intervals: $\Delta \theta = \frac{2\pi}{N-1}$, $\Delta \zeta = \frac{1}{M-1}$, and $\bar{r} = \frac{\Delta \zeta}{\Delta \theta}$
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Initial variables: journal mass, angular velocity of the journal, initial time variable \( t_0 = 0 \), time interval is initialized as say \( \frac{5\pi}{180} \) radians, the coordinates \( \varepsilon, \phi \), and \( \dot{\varepsilon}, \dot{\phi} \) are initialized to zero.

Initialize arrays to zero: Arrays are initialized for the following parameters: non-dimensional film thickness \( \overline{h} \); non-dimensional steady state pressure \( p_0 \); couple stress factors \( \overline{f}(\overline{h}, \overline{d}) \) and \( \overline{f}'(\overline{h}, \overline{d}) \); variable used in identifying the cavitation front; components of steady state load, and resultant steady state load; arrays for storage of journal center coordinates \( \varepsilon, \phi \), and \( \dot{\varepsilon}, \dot{\phi} \) values.

Iteration constants: zero is assigned to iteration count, a significant higher value is assigned to the variable that stores the iteration error, in comparison to, the maximum terminating residual value.

Viscosity model: Depending on nanolubricant viscosity model provided as input, a suitable case-switch variable is initialized to select the specified model from the list of nanofluid viscosity models considered for analysis.

iii. Compute constants:

Effective viscosity of nanolubricant: based on the selection of nanolubricant viscosity model, the effective viscosity of nanolubricant is computed as a function of volume fraction and aggregate nanoparticle size \( \mu = \frac{\mu_{nf}}{\mu_{hf}} \).

iv. Compute film thickness:

Non-dimensional film thickness \( \overline{h} \): non-dimensional film thickness is computed across the grid using the standard relation \( \overline{h} = 1 + \varepsilon \cos \theta \).

v. Compute couple stress factors:

Couple stress characteristic factor and its derivative \( \overline{f}(\overline{h}, \overline{d}) \) and \( \overline{f}'(\overline{h}, \overline{d}) \): the non-dimensional film thickness values across the grid is used to compute the couple stress characteristic factor and its derivative, which are dependent on the nanoparticle aggregate particle size.

\[
\overline{f}_0(\overline{h}_0, \overline{d}) = \overline{h}_0^3 - 12\overline{d}^2 \left[ \overline{h}_0 - 2\overline{d} \tanh \left( \frac{\overline{h}_0}{2\overline{d}} \right) \right]
\]

\[
\overline{f}'(\overline{h}_0, \overline{d}) = -3\overline{h}_0^2 \varepsilon \sin \theta + 12\overline{d}^2 \varepsilon \sin \theta \tanh \left( \frac{\overline{h}_0}{2\overline{d}} \right)
\]
vi. *Compute steady state pressure distribution*

vii. *Identify cavitation front*

viii. *Computation of steady state load components, resultant load, and attitude angle*

ix. *Solution of equations of motion*

   A fourth order Runge Kutta function is then called upon to solve the equations of motion. The initial computations of eccentricity ratio, attitude angle, assumed mass parameter, time steps, time interval value, assumes values of $\dot{\varepsilon}$, $\dot{\phi}$, and the computed load components are passed on as inputs to the RK function.

   The next set of journal center coordinates is then obtained from the solution of equations of motion.

x. *Stability analysis:*

   On completion of defined time steps, the polar whirl orbit plot is obtained from the code. The nature of whirl is studied. The procedure is then repeated by varying the assumed mass parameter till the limit cycle is attained. Further, the threshold speed and whirl ratio is computed at various eccentricity ratio values.

   The influence of TiO$_2$ nanolubricants on whirl instability is then analysed by performing the limit circle analysis at different TiO$_2$ additive concentration and aggregate particle sizes.

5.6.4 **Validation of computational code**

The developed computational code is validated by comparing the whirl orbits generated from the code with the whirl orbits published by Majumdar and Brewe [386] for similar sets of initial conditions. The initial conditions used in the validation is presented in Table 16.

*Table 16: Initial conditions used by Majumdar and Brewe [386]*

<table>
<thead>
<tr>
<th>Initial conditions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length to diameter ratio, L/D</td>
<td>1.0</td>
</tr>
<tr>
<td>Eccentricity ratio, $\varepsilon_0$</td>
<td>0.8</td>
</tr>
<tr>
<td>Non dimensional mass parameter, $\bar{M}$</td>
<td>5</td>
</tr>
<tr>
<td>Initial value of whirl ratio</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Fig. 86 provides a comparison of whirl orbits obtained from the code with the published results of Majumdar and Brewe [386] at the initial conditions mentioned in Table 16. A comparison of pressure distribution is also presented in Fig. 87. As observed in Figs. 86 and 87, the results obtained from the code are in good agreement with published results, thereby validating the code.

Figure 86: a) Whirl orbit obtained from code b) Whirl orbit from Majumdar and Brewe [386]

The computational code is also employed in studying the whirl instability characteristics of journal bearing operating on conventional lubricants with polymer additives. The lubricant is modeled as a couple stress fluid with couple stress factor \( T = 0 \) (Newtonian), 0.2, 0.4, and 0.6. The results of the study are published in Binu et al. [34] and attached in the Appendix as paper P36.

5.6.5 Results and discussions

Journal center trajectories for plain engine oil without nanoparticle additives at an eccentricity ratio of 0.6 is shown in Fig. 88. Fig. 88 shows the whirl orbits for stable, unstable and limit cycle cases of system operation. The trajectory is obtained for an eccentricity ratio \( \varepsilon = 0.6 \). The mass parameter values are modified in a trial and error method to obtain the three cases. As observed in Fig. 88, at the considered eccentricity ratio of 0.6, a mass parameter of 5 demonstrates stable whirl whereas, a mass parameter of 8.1 causes

---

unstable whirl. Therefore, the critical mass parameter is expected to be situated between 5 and 8.1.

Figure 87: a) Pressure distribution obtained from code b) Pressure distribution from Majumdar and Brewe [386]

On a trial and error approach, the limit cycle is observed at a mass parameter value of 7.1. For the identified mass parameter, the threshold journal speed, critical journal speed, and critical whirl speed is computed. The whirl ratio $\Omega$ is also computed. Fig. 89 presents the limit cycles and corresponding critical mass parameters for a plain engine oil. The computation is carried out for $\overline{d} = 0$ and $\phi = 0$ (Newtonian fluid). The limit cycles are observed for eccentricity ratio $\varepsilon = 0.01$, $0.1$, $0.2$, $0.4$, $0.6$, and $0.7$, representing light to heavily loaded bearing operations.

The operating parameters for the analysis is presented in Table 17. The limit cycles obtained for a journal bearing operating on TiO$_2$ nanolubricants at different eccentricity ratios for an aggregate particle size of 777 nm corresponding to couple stress parameter $\overline{d} = 0.03108$, at TiO$_2$ nanoparticle concentrations $\phi = 0.001$, $0.005$, $0.01$, and $0.02$ are presented in Figs. 90 to 93. Fig. 94 presents the variation of critical mass parameter as a function of eccentricity ratio for varying TiO$_2$ nanoparticle concentration at a couple stress parameter of $\overline{d} = 0.03108$. The threshold stability maps illustrating the variation in critical threshold speed with eccentricity ratio for varying TiO$_2$ nanoparticle additive concentration is shown in Fig. 95.
Figure 88: A sample limit cycle analysis delineating the stable, unstable, and limit cycle journal whirl for plain lubricant.
Figure 89: Limit cycles for plain engine oil
### Table 17: Operating parameters for nonlinear transient analysis

<table>
<thead>
<tr>
<th>Bearing Details</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ratio:</strong> 1.0</td>
</tr>
<tr>
<td><strong>Bearing Angle:</strong> $\theta = 360^\circ$ Full Bearing</td>
</tr>
<tr>
<td><strong>Bearing Type:</strong> Plain Compliant Bearing</td>
</tr>
<tr>
<td><strong>Bearing Clearance:</strong> 25 microns</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lubricant Details</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type:</strong> Couple stress fluid</td>
</tr>
<tr>
<td><strong>Additives:</strong> TiO$_2$ nanoparticles</td>
</tr>
<tr>
<td><strong>Couple stress parameter:</strong> $\overline{d} = \frac{d}{C} = 0.03108$ corresponding to aggregate nanoparticle size of 777 nm and bearing clearance of 25 microns. Analysis is also performed for $\overline{d} = 0.01$ and $\overline{d} = 0.05$, for comparison.</td>
</tr>
<tr>
<td><strong>TiO$_2$ nanoparticle concentrations:</strong> $\phi = 0.001, 0.005, 0.01, 0.02$</td>
</tr>
<tr>
<td><strong>Viscosity of base fluid:</strong> 0.1036 Pa-s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Computational Details</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of time steps employed:</strong> 800</td>
</tr>
<tr>
<td><strong>Mass of the journal:</strong> 10.8 Kg (In line with the shaft employed in the test rig)</td>
</tr>
<tr>
<td><strong>Time interval:</strong> $\frac{5\pi}{180}$ rad</td>
</tr>
</tbody>
</table>
Figure 90: Limit cycles at different eccentricity ratios for TiO$_2$ volume fraction, $\phi = 0.001$
Figure 91: Limit cycles at different eccentricity ratios for TiO$_2$ volume fraction, $\phi = 0.005$
Figure 92: Limit cycles at different eccentricity ratios for TiO$_2$ volume fraction, $\phi = 0.01$
Figure 93: Limit cycles at different eccentricity ratios for TiO$_2$ volume fraction, $\phi = 0.02$
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Figure 94: Critical mass parameter for varying TiO$_2$ concentrations at a constant couple stress parameter of $\bar{d} = 0.03108$

Figure 95: Critical threshold speed for varying TiO$_2$ concentrations at a constant couple stress parameter of $\bar{d} = 0.03108$

It is observed in Fig. 94 that, critical mass parameter $M_{cr}$ for nanolubricants at the considered low concentrations are nearly equal to that for plain engine oil. Slight increase is observed at higher eccentricities. However, improvement in stable operating conditions of journal bearing using TiO$_2$ nanolubricants of aggregate particle size 777 nm at the
considered low volume fractions is observed in the threshold stability maps plotted in Fig. 95. It is seen in Fig. 95 that, increasing concentrations of TiO$_2$ additives of constant particle size characterised by couple stress parameter $\overline{a} = 0.03108$, increases the threshold speeds resulting in enhanced stable operating region for the bearing. Pronounced increment is observed at higher eccentricity ratios.

The quantitative increase in threshold speeds of TiO$_2$ nanolubricants in comparison to plain engine oil is presented in Table – 18.

Table 18: Percentage increase in threshold speed due to addition of TiO$_2$ nanoparticle additives at constant aggregate particle size 777 nm at varying particle volume fractions

<table>
<thead>
<tr>
<th>Eccentricity Ratio $\varepsilon$</th>
<th>Plain engine oil</th>
<th>Critical Threshold Speed $\overline{\omega}_{cr}$ for Nanolubricants at $\overline{a} = 0.03108$</th>
<th>Percentage increase (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\overline{a} = 0$, $\phi = 0$</td>
<td>$\phi = 0.001$</td>
<td>$\phi = 0.005$</td>
</tr>
<tr>
<td>0.2</td>
<td>21.438</td>
<td>22.158</td>
<td>23.691</td>
</tr>
<tr>
<td>0.4</td>
<td>31.675</td>
<td>32.627</td>
<td>34.784</td>
</tr>
<tr>
<td>0.5</td>
<td>39.298</td>
<td>40.575</td>
<td>43.257</td>
</tr>
<tr>
<td>0.6</td>
<td>50.807</td>
<td>52.991</td>
<td>56.493</td>
</tr>
<tr>
<td>0.7</td>
<td>76.917</td>
<td>83.715</td>
<td>89.248</td>
</tr>
</tbody>
</table>

From Table 18 it is observed that, presence of TiO$_2$ nanoparticle additives characterized by couple stress parameter of $\overline{a} = 0.03108$ at a concentration of $\phi = 0.01$ will increase the threshold speed by ~20% in comparison to plain engine oil.

The influence of variation in TiO$_2$ nanoparticle aggregate particle size characterized by couple stress parameter $\overline{a}$ on whirl instability is then analyzed using the limit cycle approach. The TiO$_2$ nanoparticle concentration is kept constant at $\phi = 0.01$ and the couple stress parameter $\overline{a}$ is varied to a value less than ($\overline{a} = 0.01$) and greater than ($\overline{a} = 0.05$) the DLS measured value of $\overline{a} = 0.03108$. The variation in critical mass parameter with eccentricity ratio is presented in Fig. 96. The threshold stability maps for varying couple stress parameters are presented in Fig. 97. The limit cycles along with the identified critical mass parameters are presented in Figs. 98 to 100.
Figure 96: Critical mass parameter for varying couple stress parameters at a constant TiO$_2$ additive concentration of $\phi = 0.01$

Figure 97: Threshold speeds for varying couple stress parameters at a constant TiO$_2$ additive concentration of $\phi = 0.01$
Figure 98: Limit cycles at $\phi = 0.01$ for couple stress parameter $\tau = 0.01$
Figure 99: Limit cycles at $\phi = 0.01$ for couple stress parameter $\tilde{d} = 0.03108$
Figure 100: Limit cycles at $\phi = 0.01$ for couple stress parameter $\overline{T} = 0.05$
Quantitative improvement in threshold speeds of journal bearing operating on TiO$_2$ nanolubricant at a particle volume fraction of $\phi = 0.01$ for varying couple stress parameters in comparison to plain engine oil is illustrated in Table 19.

**Table 19: Percentage increase in threshold speed due to addition of TiO$_2$ nanoparticle additives at constant TiO$_2$ volume fraction $\phi = 0.01$ with varying aggregate particle size characterized by couple stress parameter**

<table>
<thead>
<tr>
<th>Eccentricity Ratio $\varepsilon$</th>
<th>Plain engine oil</th>
<th>Critical Threshold Speed $\bar{\Omega}_{cr}$ for Nanolubricants at $\phi = 0.01$</th>
<th>Percentage increase (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{d} = 0$, $\phi = 0$</td>
<td>$\bar{d} = 0.01$</td>
<td>$\bar{d} = 0.03108$</td>
</tr>
<tr>
<td>0.2</td>
<td>21.438</td>
<td>25.566</td>
<td>25.934</td>
</tr>
<tr>
<td>0.4</td>
<td>31.675</td>
<td>37.495</td>
<td>38.079</td>
</tr>
<tr>
<td>0.5</td>
<td>39.298</td>
<td>46.571</td>
<td>47.354</td>
</tr>
<tr>
<td>0.6</td>
<td>50.807</td>
<td>60.493</td>
<td>61.844</td>
</tr>
<tr>
<td>0.7</td>
<td>76.917</td>
<td>91.765</td>
<td>97.701</td>
</tr>
</tbody>
</table>

It is observed in Fig. 95 that, similar to variation in volume fraction, changes in aggregate particle size characterized by couple stress parameter, causes negligible change in critical mass parameter in comparison to plain engine oil. Threshold speed shows improvement due to the variation in couple stress parameter as seen in Fig. 97. At a volume fraction of 0.01, couple stress parameter of 0.03108 causes a near 20% increase in threshold speed.

The variation in critical whirl ratio $\Omega_{cr}$ with TiO$_2$ aggregate nanoparticle size and TiO$_2$ nanoparticle additive volume fraction is illustrated in Fig. 101 and 102 respectively. As observed in Figs. 101 and 102, the whirl ratio, even though it suggests an apparent improvement in stable operating region, is not significantly different from the whirl ratios obtained for plain engine oil.

The analysis therefore, reveal that at low concentrations of TiO$_2$ nanoparticle additives, critical journal speed offers the clearest improvement in threshold journal operating conditions, in comparison to critical mass parameter and whirl ratio.
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Figure 101: Variation in critical whirl ratio with couple stress parameter $\vec{d}$ at constant $\text{TiO}_2$ volume fraction of $\phi = 0.01$

Figure 102: Variation in critical whirl ratio with $\text{TiO}_2$ volume fraction at constant couple stress parameter of $\vec{d} = 0.03108$

Highlights on nonlinear transient analysis
Key observations from the nonlinear transient analysis of journal bearing operating on $\text{TiO}_2$ nanolubricants described in this section 5.6 is mentioned below.
5.7 Conclusions

In this study, dynamic and whirl instability characteristics of journal bearings operating on TiO₂ nanolubricants are obtained using both, linearized perturbation and nonlinear transient approach. Results of the study reveal an improvement in dynamic and whirl instability performance of journal bearings using TiO₂ nanolubricants in comparison to plain engine oil.

Analysis of dynamic coefficients using linearized perturbation technique reveals a significant increase in stiffness and damping coefficients of journal bearings operating on TiO₂ nanolubricants in comparison to plain engine oil. The cross stiffness KRP, which is generally considered as the primary impact coefficient on dynamic bearing performance, is observed to increase by 51% with the use of TiO₂ nanolubricants at a particle volume fraction of $\phi = 0.01$ and aggregate particle size of 777 nm amounting to couple stress parameter of $\phi = 0.03108$.

Limit cycles were obtained at eccentricity ratios ranging from 0.01 to 0.07, for both conditions of, varying TiO₂ nanoparticle additive concentration and TiO₂ aggregate nanoparticle size.

Critical mass parameter is used as a control variable in identifying limit cycles based on a trial and error approach.

Analysis reveal an apparent improvement in stability characteristics of journal bearings operating on TiO₂ nanolubricants.

Critical threshold speed $\bar{\omega}_{cr}$ is found to be more sensitive to variations in TiO₂ nanolubricant additives at various eccentricities in comparison to critical mass parameter $\bar{M}_{cr}$ and critical whirl ratio $\Omega_{cr}$.

A 20% increase in threshold speed $\bar{\omega}_{cr}$ is observed with the use of TiO₂ nanoparticle additives at a volume fraction of $\phi = 0.01$ and couple stress parameter of $\phi = 0.03108$. 

- Limit cycles were obtained at eccentricity ratios ranging from 0.01 to 0.07, for both conditions of, varying TiO₂ nanoparticle additive concentration and TiO₂ aggregate nanoparticle size.
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- Analysis reveal an apparent improvement in stability characteristics of journal bearings operating on TiO₂ nanolubricants.
- Critical threshold speed $\bar{\omega}_{cr}$ is found to be more sensitive to variations in TiO₂ nanolubricant additives at various eccentricities in comparison to critical mass parameter $\bar{M}_{cr}$ and critical whirl ratio $\Omega_{cr}$.
- A 20% increase in threshold speed $\bar{\omega}_{cr}$ is observed with the use of TiO₂ nanoparticle additives at a volume fraction of $\phi = 0.01$ and couple stress parameter of $\phi = 0.03108$. 

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Stability analysis using nonlinear transient technique confirms the beneficial effect of TiO$_2$ nanolubricants on the dynamic and stability characteristics of journal bearings. Limit cycle whirl orbits are obtained for journal bearings operating on TiO$_2$ nanolubricants at varying particle concentration and aggregate particle size. Nonlinear transient analysis also shows a $\sim 20\%$ increase in threshold speed with the use of TiO$_2$ nanolubricants at a particle volume fraction of $\phi = 0.01$ and aggregate particle size of 777 nm.

Study does not present any significant difference in stability characteristics observed in both linearized perturbation and nonlinear transient approach. The less significant observable difference being the comparatively lesser sensitivity of critical mass parameter $M_{cr}$ and critical whirl ratio $\Omega_{cr}$ with TiO$_2$ volume fraction and aggregate particle size in nonlinear transient analysis, in comparison to linearized perturbation approach. Enhanced threshold speed of journal is the most significant influence of TiO$_2$ nanolubricants on the dynamic stability characteristics of journal bearings.

Even though linearized perturbation approach offers all functional details pertaining to dynamic and whirl instability characteristics of journal bearings, the journal behavior post the onset of whirl could only be studied using nonlinear transient analysis. Therefore, combined analysis using both, linearized perturbation and nonlinear techniques in essential in complete study of dynamic bearing performance.

5.8 Future Scope of Work

As a continuation of the performed dynamic characteristics of journal bearings operating on TiO$_2$ nanolubricants, the following future endeavors could be considered.

I. Experimental analysis of dynamic and stability characteristics would provide clearer insights. Literature review also points to the necessity of more experimental validation of dynamic and stability characteristics of journal bearings operating on multiphase lubricants.

II. Considering the significant changes in thermal characteristics of nanolubricants in comparison to plain lubricants, thermohydrodynamic analysis will provide more clarity on the dynamic and stability characteristics.

III. Considering the proven benefits of Magnetorheological Fluids as an efficient damper, the possibility of employing magnetic nanoparticles as lubricant additives for better control of damping characteristics could be studied.