

CHAPTER V

STUDY OF $^{152, 154}\text{Sm}$ USING INTERACTING BOSON MODEL-I

5.1 INTRODUCTION

The interacting boson model-1 (IBM-1) of Arima and Iachello (1984), Iachello and Arima (1987) and Casten (1990) has been successful in describing the collective nuclear properties of even- even nuclei. In IBM-1, the nuclear structure is assumed to be a function of total boson number $N_B (=N_p+ N_n)$, where N_p and N_n are the valance proton and neutron particle or hole boson number respectively. This model is based on group theory and provides a useful theoretical explanation of various experimentally observed nuclear properties.

In even- even nuclei, the energy ratio $R_4 (= E_{4g^+} / E_{2g^+})$ is good measure of deformation and it helps in categorizing the atomic nuclei as per details given below (see Fig. 5.1):

For vibrational or SU(5) type nuclei	$R_4= 2.00$
For E(5) symmetry	$R_4= 2.20$
For γ -soft nuclei or O(6)	$R_4= 2.50$
For X(5) symmetry	$R_4= 2.90$
For rotational nuclei or SU(3)	$R_4 = 3.33$

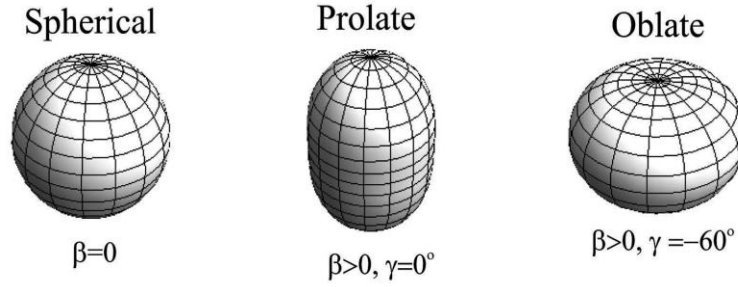


Figure 5.1: The shape of nucleus for different values of β and γ . For spherical nuclei $R_4 = 2.0$ and $\beta = 0$, transitional nuclei $R_4 \approx 2.3$ to 2.8 and for deformed nuclei [prolate ($\beta > 0$ and $\gamma = 0^\circ$) and oblate ($\beta > 0$ and $\gamma = -60^\circ$) shape] $R_4 > 2.8$.

5.2 LITERATURE REVIEW

In past years, several systematic studies of $^{146-154}\text{Sm}$ isotopes have been performed using IBM-1 by Scholten et al. (1978), Scholten (1980), Castanõs et al. (1982), Chuu et al. (1984), Yen et al. (1984), Hsieh et al. (1986), Chuu and Hsieh (1990) Han et al. (1990), Stewart et al. (1990), Kracikova et al. (1984a) and Kracikova et al. (1984b), dynamic pairing-plus-quadrupole (DPPQ) model by Kumar (1974), Kumar (1976) and Gupta (1983), boson expansion model (BEM) by Tamura et al. (1979), rotational vibrational interaction model (RVM) by Bhardwaj et al. (1983) and Bhardwaj (1983). These theories were partial successful in explaining the complex nuclear structure of $^{146-154}\text{Sm}$ isotopes.

The work of Scholten et al. (1978) and Scholten (1980) was limited to the lower bands i.e. g-, β - and γ - bands only. Castanõs et al. (1982) used an effective Hamiltonian of IBA for describing only the low lying energy spectrum of Xe, Ba, Sm, Gd and U isotopes and pointed out that the effective IBM results were in agreement with those projected from IBM calculations of Scholten (1980). Chuu et al. (1984), used an effective H_{IBA} for $N = 88$ and 90 (Ba-Yb) isotones and obtained a unified E2 transition operator to reproduce the observed $B(E2)$ values and Q_{2+} moments. These attempts of Chuu et al. (1984) were aimed to find a common set of IBM parameters for a group of nuclei (isotopes/ isotones) so that the varying nuclear structure with N, Z may be obtained by varying N_B . Gupta- Hamilton- Rammaya

(1980) observed that the g-band spectra of the isotonic multiplets in the first quadrant of the $Z=50-82$, $N=82-126$ major closed shell vary slowly with Z and a common set of parameters should be easier to obtain. Mittal and Gupta (1990) pointed out that the approach of Chuu et al. (1984) did not reproduce the correct variation of the $E_{2\gamma}$ and $E_{0\beta}$ states with Z . Also for $N=90$ isotones, Chuu et al. (1984) results were not satisfactory for $B(E2)$ values for transition between different bands. In addition, the energy spacing in the H_{IBA} calculation of Chuu et al. (1984) were not in agreement with the observed data in β - and γ - bands (Fig. 3 of Chuu et al. (1984) for ^{152}Sm) and some states were even in reverse order in the γ -band of ^{152}Sm . The DPPQ model of Kumar (1975) had limitations for production the energies of various bands for Sm and other isotopes, because it does not have any fitting procedure of energies like other models (e.g. IBM-1). Also the energy scale is not linear in the g-band versus other bands Gupta (1983).

At present the large amount of experimental data is available from the radioactive decay, coulomb excitation and the reaction work of Lederer and Shirley (1978), Raman et al. (1987), Peker (1989), Venkova and Andrejtischeff (1981), Peker (1987), Sakai (1984) and www.nndc.bnl.gov (2015). Three quasi-bands in ^{154}Sm and four in ^{152}Sm are well established up to higher spins, which requires more detailed theoretical analysis for complete explanation of the observed collective properties. Since all the previous works were performed only for lower members of the three lower bands i.e. g-, β -, and γ -bands, it is interesting to see what the results are of a study of higher bands.

In this chapter, the IBM Hamiltonian is used for $^{152-154}\text{Sm}$ isotopes to study the nuclear properties of lower and higher bands up to high spins, which includes the energy spectrum, absolute $B(E2)$ values and $B(E2)$ branching ratios. The absolute $B(E2)$ values and $B(E2)$ branching ratios are sensitive to the wave function and provide more stringent test of a model.

5.3 THE INTERACTING BOSON MODEL AND CALCULATIONS

The two bodies effective Hamiltonian within the boson has two forms

$$\begin{aligned}
 H = & \varepsilon_s (s^+ \cdot s^-) + \varepsilon_d (d^+ \cdot d^-) + \sum_{L=0,2,4} (1/2) (2L+1)^{1/2} c_L \{ [d^+ \times d^+]^{(L)} \times [d^- \times d^-]^{(L)} \}^{(0)} \\
 & + (1/\sqrt{2}) \tilde{v}_2 \{ [d^+ \times d^+]^{(2)} \times [d^- \times d^-]^{(2)} + [d^+ \times d^+]^{(2)} \times [d^- \times d^-]^{(2)} \}^{(0)} \\
 & + (1/2) \tilde{v}_0 \{ [d^+ \times d^+]^{(0)} \times [s^- \times s^-]^{(0)} + [s^+ \times s^+]^{(0)} + [d^- \times d^-]^{(0)} \}^{(0)} \\
 & + u_2 \{ [d^+ \times s^+]^{(2)} \times [d^- \times s^-]^{(2)} \}^{(0)} + (1/2) u_0 [s^+ \times s^+]^{(0)} + [s^- \times s^-]^{(0)} \}^{(0)} \quad . \quad (5.1)
 \end{aligned}$$

Where, ε_s and ε_d are the single-boson energies and c_L , \tilde{v}_L and u_L describe the two-boson interaction. Also,

$$H' = \varepsilon'' n_d + a_0 (P^+ \cdot P) + a_1 (L \cdot L) + a_2 (Q \cdot Q) + a_3 (T_3 \cdot T_3) + a_4 (T_4 \cdot T_4) \quad (5.2)$$

where,

$$\begin{aligned}
 n_d &= (d^+ \cdot d^-), \\
 P &= (1/2) (d^- \cdot d^-) - (1/2) (s^- \cdot s^-), \\
 L &= \sqrt{10} (d^+ \times d^-)^{(1)}, \\
 Q &= [d^+ \times s^- + s^+ \times d^-]^{(2)}, \\
 T_3 &= [d^+ \times d^-]^{(2)} \quad \text{and} \\
 T_4 &= [d^+ \times d^-]^{(4)}.
 \end{aligned}$$

A least square fitting technique is used to find out the optimized values of the four parameters i.e ε'' , a_0 , a_1 and a_2 ; while a_3 and a_4 are kept zero in equ. 5.2. The PHINT programme of Scholten (1979a) is used to fit the observed energy spectra of $^{152-154}\text{Sm}$ isotopes. All levels with reliable spin assignment ($I^\pi \leq 10^+$) are included up to the point that the first level with an uncertain spin assignment appears. In fitting of the energy spectra, we first determine the four parameters of H' as discussed above, that reproduce the best lower and higher bands.

The optimized values of these four boson- boson interaction parameters are listed in Table 5.1. These four parameters with E2SD ($= a_2$) and E2DD ($= \sqrt{5}\beta_2$) are the

input for the FBEM programme of Scholten (1979b). The E2 transition operator depends upon two parameters α_2 and β_2 as given below:

$$T(E2) = \alpha_2 [d^+ s^\sim + s^+ x d^\sim]^{(2)} + \beta_2 [d^+ d^\sim]^{(2)} \quad (5.3)$$

Where, α_2 is called the boson effective charge, simply the scaling parameter and affecting the B(E2) values and β_2 accounts for nuclear shape transition. The ratio E2DD/ E2SD is equal to -2.958 in the SU(3) limit and reduced to zero in the O(6) limit. The FBEM gives the B(E2) values and ratios.

Table 5.1: The Interacting Boson Model-1 parameters (all in keV) for ¹⁵²⁻¹⁵⁴Sm.

Parameter	¹⁵² Sm	¹⁵⁴ Sm
N _B	10	11
EPS	503.8	411.5
PAIR	13.1	0.1
ELL	0.5	-0.8
QQ	-26.2	-41.8
OCT	0.0	0.0
HEX	0.0	0.0
E2DD	-250.0	-250.0
E2SD	160.0	140.0
E2DD/E2SD	-1.56	-1.786

5.4 RESULT AND DISCUSSION

For ¹⁴⁶⁻¹⁵⁴Sm isotopes, experimental values of energy ratio R_4 , R_γ ($=E_{2\gamma}/E_{2g}$), R_β ($=E_{0\beta}/E_{2g}$), $R_{0,6,\beta,g}$ ($=E_{0\beta}/E_{6g}$), $R_{2,0,\beta,g}$ [$=(E_{2\beta}-E_{0\beta})/E_{2g}$], $R_{4,2,\beta,g}$ [$=(E_{4\beta}-E_{2\beta})/(E_{4g}-E_{2g})$] and $R_{4,2,\gamma,g}$ [$=(E_{4\gamma}-E_{2\gamma})/(E_{4g}-E_{2g})$] are calculated and given in Table 5.2. The experimental values of energies to calculate these ratios are taken from the website of Brookhaven National Laboratory, www.nndc.bnl.gov (2015). It is evident that ¹⁴⁶Sm

($R_4 = 1.85$) and ^{148}Sm ($R_4 = 2.14$) nuclei are the spherical in nature i.e. SU(5) type because their R_4 is close to 2. The ^{150}Sm nucleus ($R_4 = 2.31$) is a transitional one i.e. lying on transition from SU(5) to SU(3) symmetry. The ^{152}Sm is a best example of X(5) symmetry because its experimental value of R_4 is 3.01 compared to X(5) symmetry value 2.9, R_γ is 8.9 compare to X(5) value 8.16, R_β is 5.62 compared to X(5) value 5.65 and the values of other energy ratios i.e., $R_{0,6,\beta,g}$, $R_{2,0,\beta,g}$, $R_{4,2,\beta,g}$ and $R_{4,2,\gamma,g}$ are near to the X(5) values. The ^{154}Sm is rotor type i.e. close to SU(3) symmetry. The $^{152-154}\text{Sm}$ isotopes are lying on transition from SU(5) to SU(3) and ^{152}Sm is close to X(5) symmetry (see Casten's symmetry triangle Fig. 5.2). For ^{152}Sm , the present IBM calculation gives the energy ratio $E_{0\beta}/E_{6g}$ equal to 0.9509 compared to observed value of 0.9685 and X(5) value 1.0405. Hence present IBM calculation is supporting the X(5) nature of ^{152}Sm .

The variation of experimental values of ratios R_4 and $R_{0,6,\beta,g}$ versus A for $^{146-154}\text{Sm}$ is shown in Fig. 5.3. The corresponding values of these ratios in X(5) limit are shown for useful comparison. It is clear from the Fig. 5.3 that the ratio R_4 increases from 1.85 to 3.25 as A increases from 146 to 154 and ^{152}Sm is very close to X(5) limiting value. However, the ratio $R_{0,6,\beta,g}$; decreases initially when A increases from 146 to 150; increases while A increases from 150 to 154 and for ^{152}Sm is very close to X(5) limiting value (see Fig. 5.3).

Table 5.2: The experimental values of energy ratio R_4 ($=E_{4g}/E_{2g}$), R_γ ($=E_{2\gamma}/E_{2g}$), R_β ($=E_{0\beta}/E_{2g}$), $R_{0,6,\beta,g}$ ($=E_{0\beta}/E_{6g}$), $R_{2,0,\beta,g}$ ($=E_{2\beta}-E_{0\beta}/E_{2g}$), $R_{4,2,\beta,g}$ ($=E_{4\beta}-E_{2\beta}/E_{4g}-E_{2g}$) and $R_{4,2,\gamma,g}$ ($=E_{4\gamma}-E_{2\gamma}/E_{4g}-E_{2g}$) are given for $^{146-154}\text{Sm}$ isotopes. The experimental values are taken from www.nndc.bnl.gov (2015). The IBM calculated ratios for $^{152-154}\text{Sm}$ are shown for comparison in last rows as Present Work.

A	R_4	R_γ	R_β	$R_{0,6,\beta,g}$	$R_{2,0,\beta,g}$	$R_{4,2,\beta,g}$	$R_{4,2,\gamma,g}$
146	1.8486	2.2059	1.9433	0.8014	0.2124	-	1.2471
148	2.1446	2.6427	2.5919	0.7483	0.4325	0.3669	1.0425
150	2.3157	3.5748	2.2172	0.5789	0.9155	0.91723	1.02137
152	3.0102	8.9146	5.6215	0.9685	1.0325	0.8679	1.1675
154	3.2532	17.5701	13.414	2.02039	0.9576	1.04591	1.19207
X(5)	2.904	8.16	5.649	1.0405	1.801	1.701	1.071
152 Present Work	2.8121	7.8281	5.0563	0.9509	1.1536	1.3969	1.7088
154 Present Work	3.3138	19.1755	14.531	2.1062	1.4202	1.2798	1.2374

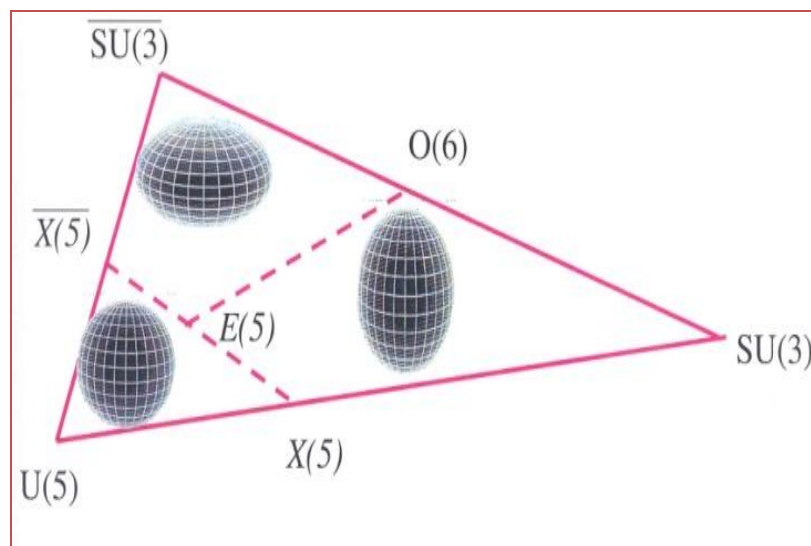


Figure 5.2 Casten's symmetry triangle.

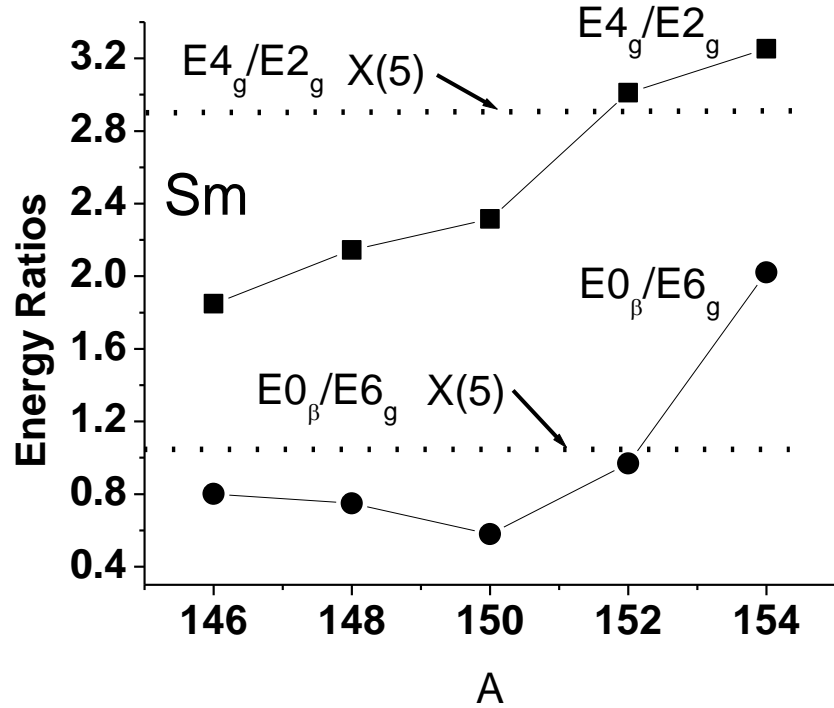


Fig. 5.3: The variation of experimental values of ratio R_4 and $R_{0,6,\beta,g}$ versus A for $^{146-154}\text{Sm}$. The data points of R_4 are shown by solid squares (■) and $R_{0,6,\beta,g}$ by solid circles(●). The corresponding values of these ratios in X(5) limit are shown by dotted lines(--) for useful comparison. The experimental values are taken from www.nndc.bnl.gov (2015).

The variation of experimental values of ratios R_γ and R_β versus A for $^{146-154}\text{Sm}$ is shown in Fig. 5.4. The corresponding values of these ratios in X(5) limit are shown for useful comparison. It is clear from the Fig. 5.4 that the ratios R_γ and R_β both; increases as A increases from 146 to 148; decreases slowly as A increases from 148 to 150 and increases sharply as A increases from 150 to 154 indicating shape phase transition from SU(5) to SU(3). Both the experimental ratios for ^{152}Sm is very close to X(5) limiting values indication the X(5) character. In the present IBM calculation, the R_γ and R_β ratios for ^{152}Sm are close to X(5) values (see Table 5.2) and our calculation is supporting X(5) nature of ^{152}Sm .

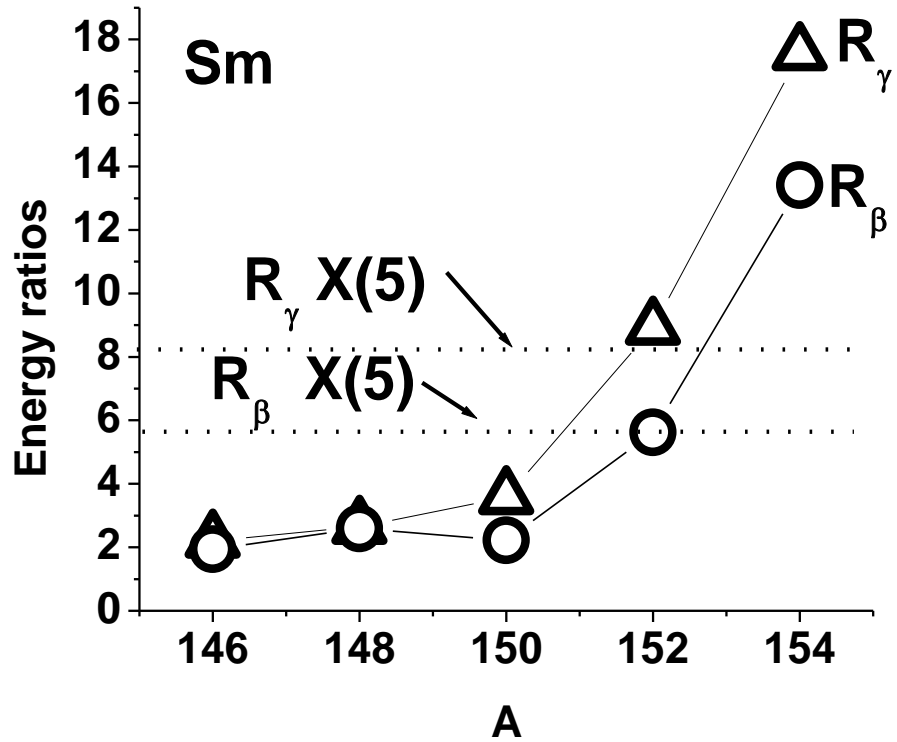


Fig. 5.4: Same as Fig. 5.3 for ratio R_γ and R_β versus A . The data points of R_γ are shown by hollow triangles (Δ) and R_β by hollow circles (\circ). The corresponding values of these ratios in X(5) limit are shown by dotted lines (--) for useful comparison. The experimental values are taken from www.nndc.bnl.gov (2015).

The experimental values of $B(E2;4g \rightarrow 2g)/B(E2;2g \rightarrow 0g)$, $B(E2;2_\gamma \rightarrow 0_g/2_g)$ and $B(E2;2_\beta \rightarrow 0_g/2_g)$ branching ratio are given in Table 5.3 for $^{146-152}\text{Sm}$ isotopes. The corresponding values of N_p , N_n , $N_B (=N_p+N_n)$ and N_pN_n are also given. The values for X(5) symmetry of IBM, vibrational model (VM) and rotor model (RM) are also given for useful comparison. The experimental ratios are taken from www.nndc.bnl.gov (2015). It is noted that for ^{152}Sm , the observed value of $B(E2;4g \rightarrow 2g)/B(E2;2g \rightarrow 0g)$ ratio is close to X(5) limiting values and our calculated value is 1.499 indicating X(5) nature.

Table 5.3: The experimental values of energy ratio $B(E2;4g \rightarrow 2g)/B(E2;2g \rightarrow 0g)$, $B(E2; 2_\gamma \rightarrow 0_g/2_g)$ and $B(E2; 2_\beta \rightarrow 0_g/2_g)$. The corresponding values of N_p , N_n , $N_B (=N_p+N_n)$ and $N_p N_n$ are also listed for $^{146-152}\text{Sm}$ isotopes. The values for X(5) symmetry of IBM, vibrational model (VM) and rotor model (RM) are also given.

A	N_p	N_n	N_B	$N_p N_n$	$B(E2;4g \rightarrow 2g)/B(E2;2g \rightarrow 0g)$	$B(E2; 2_\gamma \rightarrow 0_g/2_g)$	$B(E2; 2_\beta \rightarrow 0_g/2_g)$
146	6	1	7	6	1.7941^a $\geq 1.27(26)^b$ $\geq 1.30^g$	$0.0012(4)^b$ $>0.01^g$	$0.066(13)^b$ 0.02^g
148	6	2	8	12	1.6598^c $1.65(21)^a$	0.11^c 0.067^h	0.07^c 0.086^h
150	6	3	9	18	1.856^d $1.9(3)^a$	0.26^h $0.33(8)^d$	$0.012(2)^d$ 0.012^h
152	6	4	10	24	1.5574^e $1.445(22)^a$	$0.38(1)^e$ $0.40(1)^i$	$0.17(1)^e$ $0.169(7)^i$
154	6	5	11	30	1.60465^f $1.39(3)^a$	$0.60(11)^f$	0.44^f
X(5)					1.6	0.666	0.429
VM					2.0	0	0
RM					1.42	0.7	0.7

^awww.nndc.bnl.gov

^bKracikova et al. (1984a)

^cPeker (1990)

^dMateosian (1986)

^ePeker (1989)

^fPeker (1987)

^gPeker (1984)

^hLederer and Shirley (1978)

ⁱStewart et al. (1990)

5.4.1 The B(E2) Branching Ratios in the SU(5) and SU(3) Limit

In the SU(5) limit, the one d-boson excitation $n_d = 1$ is 2^+_1 state, the $n_d = 2$ d-boson excitation is a triplet of 0^+_2 , 2^+_2 and 4^+_1 states and $n_d = 3$ boson excitation is a quintuplet of 0^+_3 , 2^+_3 , 3^+_1 , 4^+_2 and 6^+_1 . The $\Delta n_d = 0, \pm 1$ transitions are allowed and $\Delta n_d = \pm 2, \pm 3$, etc. transitions are prohibited.

In the SU(3) limit, these states are regrouped into different bands. The absolute B(E2) values for ($\gamma \rightarrow g$) and ($\beta \rightarrow g$) transitions depend on the intrinsic matrix elements and geometrical factors Bohr and Mottelson (1975). The B(E2) branching ratio for two transitions from a particular level in a given band to the two states of other band i.e. ($I_i \rightarrow I_f/I_f'$) depends on the Alaga value Bohr and Mottelson (1975). In the SU(3) limit these rules are slightly modified because the ($\gamma \rightarrow g$) and ($\beta \rightarrow g$) transitions are prohibited, but in the slightly broken symmetry the ($\gamma \rightarrow g$) transition should be faster than ($\beta \rightarrow g$) transition. The observed B(E2) ratios are obtained from the γ -ray spectrum data, using the relation Alaga et al. (1955),

$$B(E2; I_i \rightarrow I_f / I_f') = [I_\gamma / I_{\gamma'}] \{E_{\gamma'} / E_\gamma\}^5, \quad (5.4)$$

where E_γ and $E_{\gamma'}$ are the γ -ray energies for ($I_i \rightarrow I_f$) and ($I_i \rightarrow I_f'$) transitions; I_γ and $I_{\gamma'}$ are the intensities, respectively.

5.4.2 The ^{152}Sm isotope

5.4.2.1 Energy spectrum

In ^{152}Sm the members of g-band and β_1 -band are available up to 14^+ , for β_2 up to 2^+ and γ_1 up to 5^+ (see Sakai (1984). The experimental energy values of Sakai (1984) and Peker (1989) are compared with the present calculation and DPPQ Gupta (1983) in Table 5.4. In the present calculation the band-head of the g-, β - and γ -bands are very close to the experiment and the spacing of different members in the different bands is also like in the experiment Sakai (1984) and Peker (1989). For $K^\pi = 0^+_{3-}$ band the calculated 0^+ state lies at 1.496 MeV compared to the 1.0829 MeV in experiment. The variation of E_I with spin I^+ for different bands is presented in Fig. 5.5. The slopes of E_I versus I^+ of different bands in experiment are similar to the theoretical slopes.

Table 5.4: The values of energy (in MeV) for ^{152}Sm . The theoretical result from present IBM calculation and DPPQ Gupta (1983) are also shown.

State	K^π	Expt. ^a	Present	DPPQ ^b
2_g	0_1^+	0.1218	0.1315	0.121
4_g	0_1^+	0.366648	0.3698	0.361
6_g	0_1^+	0.70694	0.6992	
8_g	0_1^+	1.12537	1.1097	
0_β	0_2^+	0.6847	0.6649	1.000
2_β	0_2^+	0.81047	0.8166	1.211
4_β	0_2^+	1.02296	1.1495	
6_β	0_2^+	1.31051	1.5402	
8_β	0_2^+	1.66648	1.9983	
2_γ	2_1^+	1.08589	1.0294	1.556
3_γ	2_1^+	1.23388	1.1005	
4_γ	2_1^+	1.37175	1.4366	
5_γ	2_1^+	(1.5595)	1.4807	
6_γ	2_1^+	1.7283 ^c	1.9086	
7_γ	2_1^+	1.9458 ^c	1.9312	
8_γ	2_1^+	2.1397 ^c	2.4472	
$0_{\beta 2}$	0_3^+	1.08286	1.4960	
$2_{\beta 2}$	0_3^+	(1.2928)	1.5890	

^a Sakai (1984), ^b Gupta (1984), ^c Peker (1989)

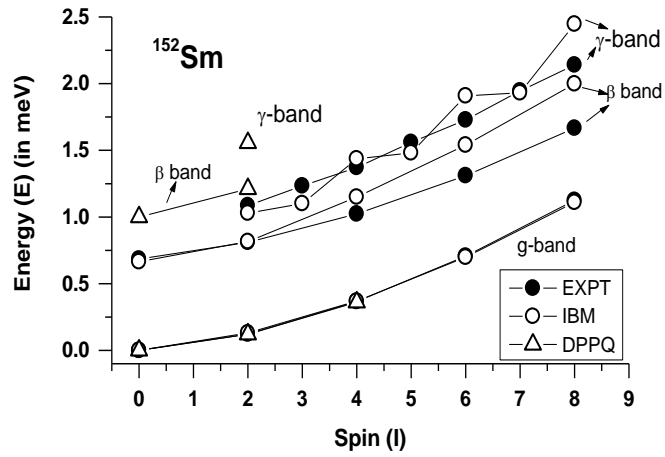


Fig 5.5: The variation of E_I with spin I^+ for different bands in ^{152}Sm . The experimental data Sakai (1984) and Peker (1989), points are shown by solid circles (\bullet), present calculation IBM by hollow circles (\circ) and DPPQ Gupta (1984) by hollow triangles (Δ).

5.4.2.2 B(E2) values

For ^{152}Sm , the observed and calculated B(E2) values are listed in Table 5.5 for (g→g), (β→g) and (γ→g) transitions.

Table 5.5: The B (E2; I_i→I_f) values (in e²b² unit) in ¹⁵²Sm.

I _i →I _f	Expt.	Present	BEM4	BEM6	DPPQ	RVM1	RVM2	IBM
Ref.	A		c	c	d	e	e	F
2 _g →0 _g	0.6806	0.6806	0.673	0.673	0.64	0.669	0.669	0.75
4 _g →2 _g	1.06(4)	1.02	0.99	0.98	0.96	1.057	1.057	1.0
6 _g →4 _g	1.18 ^b	1.14	1.12	1.09	--	--	--	0.97
8 _g →6 _g	1.36 ^b	1.17	1.17	1.11	--	--	--	0.83
10 _g →8 _g	1.6 ^b	1.14	--	--	--	--	--	--
2 _β →2 _g	0.026(3)	0.14	0.031	0.025	0.029	0.062	0.069	--
2 _β →4 _g	0.0909(8)	0.0108	0.05	0.07	0.089	0.283	0.274	--
2 _β →0 _g	0.0046(3)	0.0172	0.005	0.007	0.002	0.019	0.022	--
0 _β →2 _g	0.176(11) ^c	0.0092	0.156	0.12	0.166	0.314	0.347	--
4 _β →2 _g	0.0053(35)	0.0027	0.004	0.005	0.001	0.003	0.006	--
4 _β →4 _g	0.037(23) ^c	0.1091	0.026	0.016	0.026	0.07	0.08	--
2 _γ →0 _g	0.0176(8)	0.0153	0.049	0.05	0.023	0.015	0.016	--
	0.028(10)							
2 _γ →2 _g	0.042(4) ^c	0.0012	0.05	0.053	0.048	0.031	0.032	--
2 _γ →4 _g	0.004(3) ^c	0.085	0.007	0.006	0.006	0.0069	0.007	--
4 _γ →2 _g	0.0035(13)	0.0052	0.034	0.026	0.009	0.0046	0.0078	--
4 _γ →4 _g	0.0037(1) ^c	0.0034	0.068	0.076	0.047	0.017	0.013	--

^aPeker (1989), ^bVenkova and Andrejtscheff (1981), ^cTamura et al. (1979), ^dGupta (1983) ^eBhardwaj et al. (1983) and Bhardwaj (1983), ^fChuu et al. (1984)

The variation of B(E2; I_g→I_g-2) values with spin I_g is shown in Fig. 5.6. It is observed that the experimental B(E2) values of Peker (1989) and Venkova and Andrejtscheff (1981), increases rapidly on increasing I_g from 2⁺ to 10⁺ indicating the sharp change in the nuclear shape (see Fig.5.6). In the IBM calculation of Chuu et al. (1988), the B(E2) values first increases when I_g increases from 2⁺ to 4⁺ and it decreases while I_g increasing from 4⁺ to 8⁺ *unlike* the observed trend. But in the present IBM work, the B(E2) values follow the observed trend and values the more

closer than other theoretical data. The BEM6 Tamura et al. (1979) data points are much below the observed data points. However, the BEM4 Tamura et al. (1979) values are close to present calculation (see Fig 5.6). Only two data points are available for DPPQ Gupta (1983), RVM1 and RVM2 Bhardwaj et al. (1983) and Bhardwaj (1983) to find any definite conclusion.

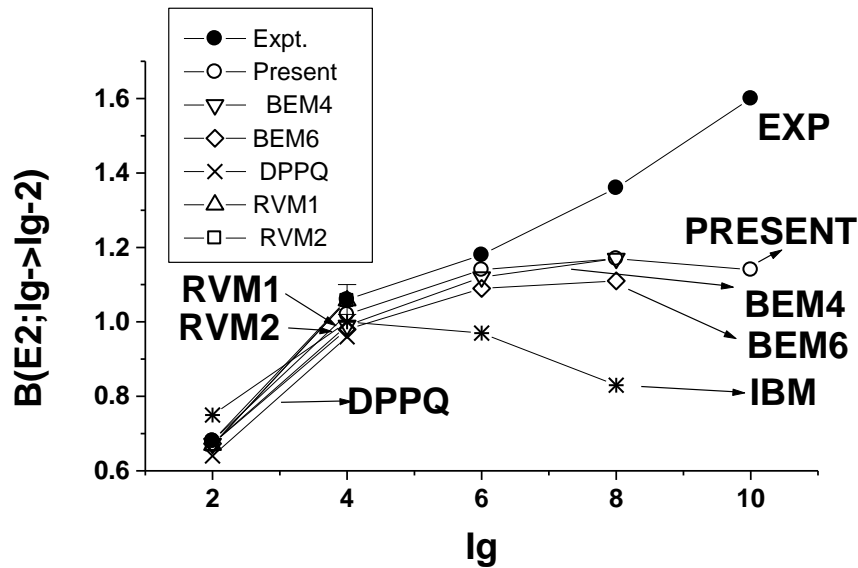


FIG 5.6: The variation of $B(E2; I_g \rightarrow I_g - 2)$ values with spin I_g for ground bands for ^{152}Sm . The experimental data points of Peker (1989) and Venkova and Andrejtscheff (1981) are shown by solid circles (●), present IBM calculation by hollow circles (○), BEM4 Tamura et al. (1979) by inverted hollow triangle (▽), BEM6 Tamura et al. (1979) by hollow diamond (◇), DPPQ Gupta (1984) by cross (x), IBM Chuu et al. (1984) by star (*), RVM1 Bhardwaj et al. (1983) and Bhardwaj (1983) by upright triangle (△) and RVM2 Bhardwaj et al. (1983) and Bhardwaj (1983) by hollow square (□).

The theoretical results of vibrational model (VM), SU(5), O(6) and SU(3) limiting values, present calculation and IBM calculation of Chuu et al. (1988) along with the experimental data for B(E2) values of Peker (1989) and Venkova and Andrejtscheff (1981) are shown in Fig. 5.7. It is clear from the Fig. 5.7 that the observed data is quite below from the VM limiting values and is lying between SU(5) and S(3) limiting values. The B(E2) values from present calculation and BEM4 Tamura et al.

(1979) are very close to the experimental data point and also present IBM calculation produces the observed slop of this ratio with I_g . There are only two data points from RVM1 and RVM2 Bhardwaj et al (1983) and Bhardwaj (1983) and DPPQ Gupta (1984) not shown in the Fig. to avoid overlapping?

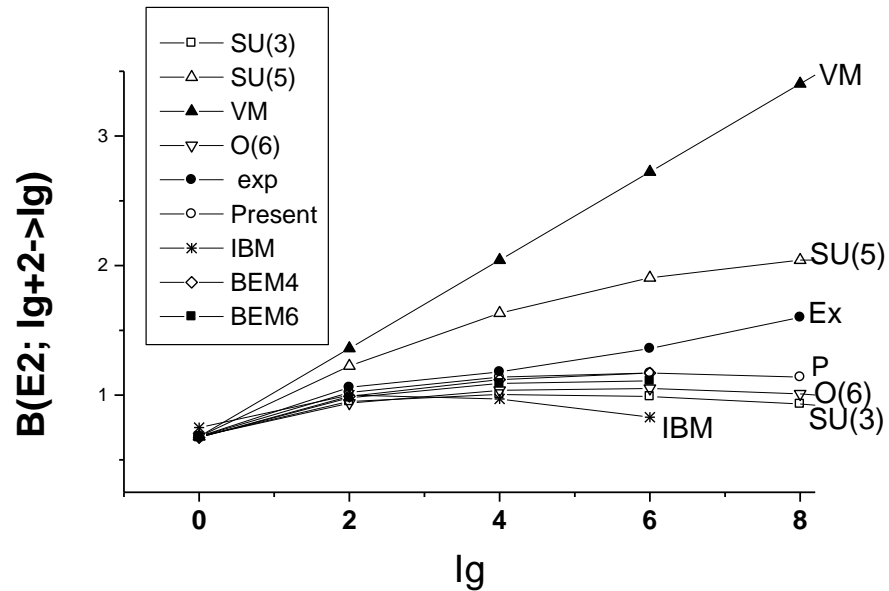


FIG 5.7: The variation of $B(E2; I_g \rightarrow I_g-2)$ values with spin I_g for ground state rotational bands for ^{152}Sm . The experimental data points of Peker (1989) and Venkova and Andrejtscheff (1981) are shown by solid circles (●), present IBM calculation by hollow circles (○), BEM4 Tamura et al. (1979) by hollow diamond (◇), BEM6 Tamura et al. (1979) by solid square (■), IBM Chuu et al. (1984) by star (*). The vibrational model (VM) is shown by solid triangle (▲), SU(5) limiting values by hollow upright triangle (△), O(6) limiting values by hollow inverted triangle (▽) and SU(3) limiting values by hollow square (□). There are only two data points from RVM1 and RVM2 Bhardwaj et al (1983) and Bhardwaj (1983) and DPPQ Gupta (1984) not shown in the Fig. to avoid overlapping.

The $B(E2)$ values for six ($\beta \rightarrow g$) and five ($\gamma \rightarrow g$) transitions are also compared (see Table 5.5) with the present calculation, boson expansion model (BEM4 and BEM6 version) of Tamura et al (1979), dynamic pairing plus quadrupole (DPPQ) model of Gupta (1983), rotational vibrational model (RVM1 and RVM2) of Bhardwaj et al

(1983) and Bhardwaj (1983) and interaction boson model –1 (IBM-1) of Chuu et al. (1988). It is evident that the present calculation gives the satisfactory results.

5.4.2.3 The B(E2) branching ratios for β - band

In the β -decay of ^{152}Eu , 13 new transitions and 5 new levels were reported by Stewart et al. (1990), which are included in the present work for useful discussion.

In Table of Isotopes of Lederer and Shirley (1978), the B(E2) ratio for $(2_{\beta} \rightarrow 0_g/2_g)$ transition is 0.84 which is more than the SU(3) limiting value 0.7. This ratio may be large due to 0.2% M1 and 4% E0 mixing in the $(2_{\beta} \rightarrow 2_g)$ 0.6886 MeV γ -ray. In a recent compilation work of Peker (1989) this ratio is 0.17(1) compared to the theoretical value 0.12 & DPPQ value Gupta (1983) 0.076 (see Table 5.6).

The $(2_{\beta} \rightarrow 4_g)$ 0.444 MeV γ -ray was overlapping with $(2^- \rightarrow 3_{\gamma})$ transition and gives $B(E2; 2_{\beta} \rightarrow 2_g/4_g) = 0.35$ (the intensity of $(2^- \rightarrow 3_{\gamma})$ γ -ray was 12 which gives this ratio 0.56) Lederer and Shirley (1978). But in the recent compilation of Peker (1989) this ratio is 0.30(3) and in decay of ^{152}Eu work of Stewart et al. (1990) ratio is 0.030(1).

In the Table of Isotopes Lederer and Shirley (1978); the $(4_{\beta} \rightarrow 4_g)$ 0.6565 MeV γ -ray had 16% M1 and 5% E0 mixing, which gave the $B(E2; 4_{\beta} \rightarrow 2_g/4_g) = 0.11$ and $B(E2; 4_{\beta} \rightarrow 4_g/6_g) = 0.76$, but Peker (1989) gave these ratios equal to 0.21(2) and 3.6(22); Stewart et al (1990) gave 0.11(2) and 0.08(2); and in the present work these ratios are 9.3 and 231 respectively (see Table 5.6).

For $(6_{\beta} \rightarrow 4_g/6_g)$, $(8_{\beta} \rightarrow 6_g/8_g)$ and $(10_{\beta} \rightarrow 8_g/10_g)$ transitions; the observed B(E2) ratios lie away from the respective Alaga values and theoretical values are close to the observed values. It is also evident that the $(\beta \rightarrow \beta)$ transitions are stronger than $(\beta \rightarrow g)$ which is supported by present IBM calculation (see Table 5.6).

Table 5.6: The B(E2) ratios for ^{152}Sm .

$I_i \rightarrow I_f / I_f$	Expt	Expt	Present	DPP	BEM	BEM	RV	RV	
Ref.	a	b	t	Q	4	6	M (I)	M (II)	RM
				c	d	d	e	e	
$2_{\beta} \rightarrow 0_g / 2_g$	0.17(1)	0.169(7)	0.12	0.08	0.16	0.26	0.31	0.32	0.7
$2_{\beta} \rightarrow 2_g / 4_g$	0.30(3)	0.030(1)	12.94	0.33	0.62	0.36	0.22	0.25	0.56
$2_{\beta} \rightarrow 0_{\beta} / 0_g$	406(77)	829(113)	12.5	291	91	69	--	--	--
$2_{\beta} \rightarrow 4_g / 0_g$	19.8(23)	--	0.63	40.5	10.1	10.8	--	--	0.8
$4_{\beta} \rightarrow 2_g / 4_g$	0.21(2)	0.11(2)	9.32	0.023	0.17	0.32	0.04	0.08	1.1
$4_{\beta} \rightarrow 4_g / 6_g$	3.6(22)	0.08(2)	231	--	0.37	0.34	--	--	0.57
$4_{\beta} \rightarrow 2_{\beta} / 2_g$	291(27)	--	186	--	180	126	--	--	--
$4_{\beta} \rightarrow 2_{\beta} / 4_g$	1385.7	--	20	43	27.7	3.9	10.9	9.12	1.1
$4_{\beta} \rightarrow 2_{\beta} / 3_{\gamma}$	350(170)	--	1.3	123	1831	--	--	--	--
$6_{\beta} \rightarrow 4_g / 6_g$	0.078(5)	--	0.012	--	--	--	--	--	1.24
$6_{\beta} \rightarrow 4_{\beta} / 4_g$	50(17)	--	694	--	--	--	--	--	--
$8_{\beta} \rightarrow 6_g / 6_g$	0.012(1)	--	0.008	--	--	--	--	--	1.3
$8_{\beta} \rightarrow 6_{\beta} / 6_g$	2374(309)	--	1415	--	--	--	--	--	--
$10_{\beta} \rightarrow 8_g / 10_g$	0.05(1)	--	0.007	--	--	--	--	--	--
$10_{\beta} \rightarrow 8_{\beta} / 8_g$	440(55)	--	2106	--	--	--	--	--	--
$2_{\gamma} \rightarrow 0_g / 2_g$	0.38(1)	0.40(1)	12.8	0.48	0.98	0.94	0.49	0.5	0.07
$2_{\gamma} \rightarrow 2_g / 4_g$	12.4(6)	9.8(4)	0.014	8	7.57	9.14	4.5	4.57	19.0
$2_{\gamma} \rightarrow 2_{\beta} / 2_g$	1208(68)	3.9(6)	267	2.5	--	2.64	0.25	0.28	7
$2_{\gamma} \rightarrow 0_{\beta} / 0_g$	--	--	16.8	0.42	0.37	0.42	--	--	--
$3_{\gamma} \rightarrow 2_g / 4_g$	0.94(3)	0.93(3)	0.78	--	2.83	2.68	--	--	2.5
$3_{\gamma} \rightarrow 2_{\beta} / 2_g$	0.025(3)	0.05(1)	9.74	0.026	0.33	0.4	--	--	--
$3_{\gamma} \rightarrow 2_{\gamma} / 2_g$	69(5)	80(10)	8.1	25	9.06	8.9	--	--	--
$3_{\gamma} \rightarrow 2_{\gamma} / 2_{\beta}$	2779(575)	1555(553)	0.83	961	27.5	22.3	--	--	--
$4_{\gamma} \rightarrow 2_g / 4_g$	0.096(8)	0.095(9)	1.53	0.19	0.5	0.34	0.26	0.60	0.34
$4_{\gamma} \rightarrow 4_g / 6_g$	4.36(55)	5.9(26)	0.03	--	8.5	38	--	--	11.3
$4_{\gamma} \rightarrow 2_{\beta} / 2_g$	0.31(8)	--	0.92	0.2	0.31	1.65	--	--	--
$4_{\gamma} \rightarrow 2_{\gamma} / 2_g$	97(17)	110(50)	98	--	--	--	--	--	--
$4_{\gamma} \rightarrow 2_{\gamma} / 2_{\beta}$	314(96)	--	106	--	--	--	--	--	--
$5_{\gamma} \rightarrow 4_g / 6_g$	0.33(2)	--	0.39	--	--	--	--	--	1.75
$5_{\gamma} \rightarrow 3_{\gamma} / 4_g$	25.8(88)	--	20.9	--	--	--	--	--	0.6
$5_{\gamma} \rightarrow 3_{\gamma} / 6_g$	8.5(29)	--	8.2	--	--	--	--	--	1.05
$6_{\gamma} \rightarrow 4_g / 6_g$	0.04(2)	--	0.8	--	--	--	--	--	0.27
$7_{\gamma} \rightarrow 6_g / 8_g$	0.24(2)	--	0.25	--	--	--	--	--	1.5
$7_{\gamma} \rightarrow 5_{\gamma} / 6_{\gamma}$	0.164(7)	--	4.91	--	--	--	--	--	--
$7_{\gamma} \rightarrow 6_{\gamma} / 6_g$	455(41)	--	10	--	--	--	--	--	2.15
$9_{\gamma} \rightarrow 8_g / 10_g$	0.14(5)	--	0.17	--	--	--	--	--	1.37
$9_{\gamma} \rightarrow 7_{\gamma} / 10_g$	23.6(37)	--	14.6	--	--	--	--	--	--
$2_{\beta 2} \rightarrow 0_g / 2_g$	1.74(17)	1.69(45)	0.13	--	--	--	--	--	--
$2_{\beta 2} \rightarrow 2_g / 4_g$	0.042(3)	--	16.7	--	--	--	--	--	--
$2_{\beta 2} \rightarrow 2_{\beta} / 2_{\beta}$	0.18(1)	--	0.24	--	--	--	--	--	--
$2_{\beta 2} \rightarrow 2_{\beta} / 2_g$	63.7(74)	56.3(188)	3.7	--	--	--	--	--	--
$2_{\beta 2} \rightarrow 4_{\beta} / 2_g$	14.8(4)	--	253	--	--	--	--	--	--

^aPeker (1989), ^bStewart et al. (1990), ^cGupta (1983), ^dTamura et al. (1979),
^eBhardwaj et al. (1983) and Bhardwaj (1983)

5.4.2.4 The B(E2) branching ratios for γ -band

The experimental data was available for 21 ratios, for transition from γ -band (see Table 5.6). It is evident from the observed data that the ($\gamma \rightarrow \beta$) transitions are stronger than ($\gamma \rightarrow g$); and ($\gamma \rightarrow \gamma$) transitions are stronger than ($\gamma \rightarrow \beta$). Theory supports these aspects. Most of the B(E2) ratios lie on the transition from SU(5) to SU(3).

The theoretical B(E2) ratios for the transition from 5_γ , 6_γ , 7_γ and 9_γ states were not available from the earlier from any previous work Gupta (1983), Tamura et al. (1979), Bhardwaj et al. (1983) and Bhardwaj (1983). The present study is extended for these four states along with three other lower states i.e. 2_γ , 3_γ and 4_γ states for calculating the B(E2) ratios. The B(E2) ratios for the transition from 2_γ , 3_γ , 4_γ , 5_γ , 6_γ , 7_γ and 9_γ states are compared with the present work and found that most of the theoretical values are close to the observed values (see Table 5.6).

5.4.2.5 The B(E2) branching ratios for $K^\pi = 0^+_{3, \beta_2}$ -band

The five B(E2) ratios were available for transition from $2\beta_2$ state, the experimental data is compared for all these transitions and there is agreement between theory and experiment (see Table 5.6).

5.4.3 The ^{154}Sm isotope

5.4.3.1 Energy spectrum

In Table 5.7 the energy values are compared with the present work and DPPQ model. The calculated spectrum is good and the band-head of β - and γ -bands are close to the observed spectrum.

Table 5.7: The values of energy (in MeV) for ^{154}Sm .

State	Expt. ^a	Present	DPPQ ^b
2g	0.08198	0.0752	0.086
4g	0.2667	0.2492	0.270
6g	0.5443	0.5188	
8g	0.9031	0.8811	
10g	1.3333	1.3330	
0 _{β}	1.0997	1.0927	1.096
2 _{β}	1.1782	1.1995	1.198
2 _{γ}	1.4404	1.4420	1.537
3 _{γ}	(1.5400)	1.5361	
4 _{γ}	(1.6606)	1.6573	

^aSakai (1984)^bGupta (1983)

5.4.3.2 B(E2) values

There are 10 B(E2) values available from the experiment for (g \rightarrow g), ($\beta\rightarrow$ g), and ($\gamma\rightarrow$ g) transitions. The 24 B(E2) values are listed and compared with the previous work i.e. DPPQ of Gupta (1983), BEM of Tamura et al. (1979), effective IBA of Chuu et al. (1988) and RVM1 and RVM2 of Bhardwaj et al. (1983) and Bhardwaj (1983) (see Table 5.8).

The observed B(E2) values of Tamura et al. (1979) and Peker (1987) for ($\beta\rightarrow$ g) and ($\gamma\rightarrow$ g) transitions are also compared with the present work and previous work of Chuu et al. (1988), Han et al. (1990), Kumar (1974), Kumar (1976) and Gupta (1983) for useful comparison in Table 5.8. The IBM-1 yields satisfactory results.

5.4.3.3 The B(E2) branching ratios for β -band

The experimental data of Tamura et al. (1979) and Peker (1987) for (2 $\beta\rightarrow$ 0g/2g), (2 $\beta\rightarrow$ 4g/2g) and (4 $\beta\rightarrow$ 2g/4g) transitions indicate that ^{154}Sm lies close to the SU(3)

limit. The present calculation gives these ratios close to observed values. Theory gives satisfactory results (see Table 5.9).

5.4.3.4 B(E2) branching ratios for γ –band

The B(E2) ratios for ($2\gamma \rightarrow 0g/2g$) is 0.60(11) compared to the Alaga value 0.7. For ($2\gamma \rightarrow 2g/4g$), ($3\gamma \rightarrow 2g/4g$) and ($4\gamma \rightarrow 2g/4g$) transitions the Alaga values are 19.07, 2.5 and 0.34; and theoretical values are 11.2, 1.58 and 0.24 respectively (see Table 5.9). For other transitions the theoretical values are compared with BEM-4 and BEM-6 of Tamura et al. (1979), DPPQ of Gupta (1983), effective IBA of Han et al. (1990) and RVM of Bhardwaj et al. (1983) and Bhardwaj (1983) calculations. There is agreement between theory & previous work.

Table 5.8: The absolute B(E2) values (in $e^2 b^2$ unit) for ^{154}Sm .

$I_i \rightarrow I_f$	Expt.	Present	DPPQ	BEM4	BEM6	IBM-1	IBM-1
	a		b	c	c	d	e
$2_g \rightarrow 0_g$	0.86(4)	0.808	0.79	0.909	0.881	0.978	1.026
$4_g \rightarrow 2_g$	1.38(22)	1.141.17	1.26	1.25	1.364	1.445	
$6_g \rightarrow 4_g$	1.37	1.22	---	1.37	1.35	--	1.547
$8_g \rightarrow 6_g$	1.50	1.219	--	1.41	1.38	1.416	1.549
$10_g \rightarrow 8_g$	1.49	1.175	--	--	--	1.333	1.492
$0_\beta \rightarrow 2_g$	--	0.015	0.094	0.054	0.054	--	--
$2_\beta \rightarrow 0_g$	0.006(41) ^c	0.001	0.0055	0.008	0.001	--	0.0068
$2_\beta \rightarrow 2_g$	0.012 ^c	0.004	0.021	0.014	0.01	--	--
$2_\beta \rightarrow 4_g$	0.024 ^c	0.009	0.062	0.033	0.008	--	0.007
$4_\beta \rightarrow 2_g$	--	0.0004	0.003	0.011	0.018	--	--
$4_\beta \rightarrow 4_g$	--	0.0058	0.020	0.01	0.003	--	--
$4_\beta \rightarrow 6_g$	--	0.0074	--	0.029	0.014	--	--
$2_\gamma \rightarrow 2_g$	0.02 ^c	0.0242	0.039	0.037	0.047	0.018	0.003
$2_\gamma \rightarrow 4_g$	0.0008 ^c	0.0022	0.0046	0.001	0.00001	0.0012	--
$4_\gamma \rightarrow 2_g$	--	0.0062	0.0093	0.020	0.021	--	--
$2_\gamma \rightarrow 4_g$	--	0.0256	0.043	0.043	0.040	--	--
$4_\gamma \rightarrow 6_g$	--	0.0191	--	0.002	0.002	--	--
$2_\beta \rightarrow 0_\beta$	--	0.057	0.084 ^f	0.68	0.62	--	--
$4_\beta \rightarrow 2_\beta$	--	0.81	1.3 ^f	0.96	0.86	--	--
$3_\gamma \rightarrow 2_\gamma$	--	1.08	--	1.30	1.28	--	--
$4_\gamma \rightarrow 3_\gamma$	--	0.76	--	0.98	0.98	--	--
$5_\gamma \rightarrow 4_\gamma$	--	0.57	--	0.69	0.62	--	--

^aPeker (1987)

^bVenkova and Andrejtscheff (1981)

^cTamura et al. (1979)

^dHan et al. (1990)

^eChuu et al. (1984),^fKumar (1974) and Kumar (1976)

Table 5.9: The B(E2; $I_i \rightarrow I_f/I_f$) ratios for ^{154}Sm .

$I_i \rightarrow I_f/I_f$	Expt.	Present	DPPQ	BEM4	BEM6	RVM1	RVM2	IBM1
	<i>a</i>		<i>b</i>	<i>c</i>	<i>c</i>	<i>d</i>	<i>d</i>	<i>e</i>
$2_\beta \rightarrow 0_g/2_g$	0.44	0.25	0.26	0.61	010	0.49	0.82	--
$2_\beta \rightarrow 4_g/2_g$	2.04	2.33	2.95	2.36	0.86	2.77	2.71	--
$2_\beta \rightarrow 0_\beta/0_g$	--	569	159	82.3	616	--	--	--
$4_\beta \rightarrow 2_g/2_g$	0.9(5)	0.07	0.16	1.10	6.48	0.42	0.42	--
$2_\beta \rightarrow 2_\beta/4_g$	--	140	66	99	318	--	--	--
$4_\beta \rightarrow 2_\beta/2_g$	--	2032	6500	87.3	47.8	--	--	--
$4_\beta \rightarrow 2_\beta/3_\gamma$	--	18.7	4.15	--	--	--	--	--
$2_\gamma \rightarrow 0_g/2_g$	0.60(11)	0.6	0.56	0.72	0.45	0.56	0.59	0.38
$2_\gamma \rightarrow 2_g/4_g$	4.8(32)	11.2	8.48	37	39.6	9.5	8.5	--
$2_\gamma \rightarrow 0_\beta/0_g$	--	0.45	0.001	0.63	0.8	---	---	---
$2_\gamma \rightarrow 2_\beta/2_g$	---	2.2	1.11	0.7	0.7	---	---	---
$2_\gamma \rightarrow 0_\beta/2_\beta$	---	0.12	---	0.63	0.51	---	---	---
$3_\gamma \rightarrow 2_g/4_g$	1.45(77)	1.58	1.53	3.57	3.45	---	---	---
$3_\gamma \rightarrow 2_\beta/2_g$	---	0.41	33.0	0.54	0.63	---	---	---
$3_\gamma \rightarrow 2_\gamma/2_g$	--	44.5	0.003	26.0	33.7	---	---	---
$3_\gamma \rightarrow 2_\beta/2_\gamma$	--	0.009	11000	0.02	0.019	---	---	---
$4_\gamma \rightarrow 2_g/4_g$	0.055	0.24	0.22	0.47	0.51	0.24	0.308	0.11
$4_\gamma \rightarrow 2_\gamma/2_g$	--	57.1	0.18	--	--	---	---	---
$4_\gamma \rightarrow 2_\beta/2_g$	--	0.048	--	0.99	2.77	---	---	---
$4_\gamma \rightarrow 4_\beta/2_\beta$	--	146.7	--	1.26	017	---	---	---
$4_\gamma \rightarrow 3_\gamma/2_\gamma$	--	2.14	--	2.33	2.63	---	---	---
$4_\gamma \rightarrow 4_\beta/4_g$	--	1.71	--	0.56	0.24	---	---	---

^aPeker (1987)

^bGupta (1983)

^cTamura et al. (1979)

^d Bhardwaj et al. (1983) and Bhardwaj (1983)

^eHan et al. (1990)

^f Kumar (1974) and Kumar (1976)

5.5 CONCLUSIONS

In this Chapter, the systematic study has been carried out for the lower and higher states of lower and higher bands, absolute B(E2) values and B(E2) branching ratios of $^{152-154}\text{Sm}$ nuclei. The mass-dependent IBM-1 Hamiltonian is used to test its validity for explaining the large amount of experimental data for energy spectra, B(E2) values and B(E2) ratios. The present IBM-1 calculation is quite successful in explaining the observed properties.

In β -decay of ^{152}Eu , 13 new transitions and 5 new levels were reported by Stewart et al. (1990) for ^{152}Sm , which were included in the present work and present IBM-1 calculated results for the B(E2) branching ratios for β - band are close to observed data points (see Table 5.6). Present IBM-1 calculation also supports the X(5) character of ^{152}Sm (N=90).

The observed B(E2: $I_g+2 \rightarrow I_g$) values increases rapidly on increasing I_g from 2^+ to 10^+ indicating the sharp change in the nuclear shape of ^{152}Sm which is supported by present IBM work (see Fig.5.6). The BEM6 Tamura et al. (1979) data points are much below the observed data points. However, the BEM4 Tamura et al. (1979) values are close to present calculation. But, IBM calculation of Chuu et al. (1988) gives *opposite* trend.

The calculated energy spectrum, B(E2) values and B(E2) ratios present a coherent and varied picture of the change in nuclear shape and dynamics with n^0 number N in $^{152-154}\text{Sm}$ isotopes. It is found that the inclusion of energy states up to higher spins in the PHINT programme provides the proper transition from SU(5) to SU(3) limit. The results of our phenomenological calculations indicate that the mass-dependent Hamiltonian in IBM-1 is an encouraging approach than the effective boson approach with or without inclusion of $Z = 64$ subshell effect.