CHAPTER -III

SYSTEMATIC DEPENDENCE OF ASYMMETRIC PARAMETER FOR EVEN Z EVEN N NUCLEI IN LIGHT AND MEDIUM MASS REGION

3.1 INTRODUCTION

The study of collective nuclear structure with N, Z, N_B (=Np+Nn) and N_pN_n provide detailed information of nuclear interactions involved. Several studies have been carried out to study the collectivity, deformation and systematic dependence of other nuclear properties on N_pN_n. de-Shalit & Goldhaber (1953) pointed out the important role of valence nucleons. Talmi (1953) noted the constancy of nuclear level structure in semi-magic isotones/isotopes. Hamamoto (1965) observed that the protons (p^+) and neutrons (n^0) both are required for producing the nuclear deformation. In interacting boson model-I (IBM-1) Casten (1990), the structure of nuclei depends on the total boson numbers $N_{\mathbf{B}}$**.** The concept of F-spin multiplets was based on this and was well explained by von Brentano et al. (1985). Casten (1985) noted that the E_{2g}+ have smooth dependence on N_pN_n. Other studies have been carried out by Casten and Zamfir (1996) to study the collectivity, deformation and systematic dependence of various nuclear observables on the product N_pN_n.

Gupta (1986) observed that 1/α was linearly dependent on N_pN_n, where the coefficient α contributes for rotational part of energy in the SU(3) symmetry limit of IBM Casten (1990) as,

$$\mathbf{E([N](\lambda, \mu)\ KLM) = \alpha\ L(L+1) + \beta\ C(\lambda, \mu).} \qquad (3.1)$$

The B(E2; $2_1^+ \rightarrow 0_1^+$) values were also related with N_pN_n. Gupta et al.(1990a) noted a systematic dependence of γ–g B(E2) ratios on the N_pN_n in different parts of the major shell space Z=50–82, N<82 and N=82–126. Casten (1985) presented a review on the evolution of nuclear structure based on N_pN_n product. The N_pN_n scheme was further modified to use P- factor Gupta et al. (1990a).

In this chapter, we study the role of valence nucleons and holes on the nuclear structure, through N, N_B and N_pN_n. Casten (1985) and Casten and Zamfir (1996) covered the various regions, viz., A=100, 130, 150 (Z<64, Z>64) and A=190. We present our results for $50 \le Z \le 82$ and $82 \le N \le 126$ region on *quadrant wise basis*. The systematic dependence of asymmetry parameter on N, N_B and N_pN_n has been studied. The role of Z=64 subshell effect for $N \le 90$ region Casten (1985) is also taken care in this work.

3.2 LITERATURE REVIEW

The values of asymmetry parameter (γ_0) are calculated for $50 \le Z \le 82$ and $82 \le N \le 126$ region and the whole data is divided into four quadrants as suggested by Gupta et al. (1990b).

3.2.1 Calculation of Asymmetric Parameter

The values of asymmetry parameter (γ_0) of asymmetric rotor model (ARM) Davydov and Filippov (1958) are evaluated using the experimental energies $E2_2^+$ and $E2_1^+$ states. The experimental data is taken from the website of Brookhaven National Laboratory *http://www.nndc.bnl.gov* (2015). The energy ratio $R\gamma = E_{22} / E_{21}$ and γ_0 is:

$$\gamma_0 = \frac{1}{3}\sin^{-1}\{\frac{9}{8}[1-\left(\frac{R_\gamma - 1}{R\gamma + 1}\right)^2\}^{1/2}. \qquad (3.2)$$

It can be evaluated using:

(a) The energy ratio $R_4 = (E_{4g}/E_{2g})$ but only the nuclei with $2.8 \le R4 \le 3.33$ will be allowed as noted by Sharma (1989) and Gupta and Sharma (1989).

(b) The B(E2) values which are very small and available with uncertainties. Therefore the values from energy ratio $R\gamma$ are more reliable. The calculated values of asymmetry parameter (γ_0) are listed in Table 3.1 and Table 3.2 for light and medium mass region, respectively.

3.3 RESULT AND DISCUSSIONS

The whole observed data of asymmetry parameter is divided into four quadrants. The Quadrant I (Q-I) is for $50 \leq Z \leq 66$ and $82 \leq N \leq 104$ shell space with particle like proton-bosons and neutron-bosons and it is forming the p-p space. The Quadrant II (Q-II) is for $66 \leq Z \leq 82$ and $82 \leq N \leq 104$ shell space, with hole like proton-bosons space and particle like neutron-bosons space and it is forming the h-p space. The Quadrant III (Q-III) is for $66 \leq Z \leq 82$ and $104 \leq N \leq 126$ region shell space, with hole like proton-bosons and neutron-bosons and it is forming h-h space. The quadrant IV (Q-IV) is for $50 \leq Z \leq 66$ and $104 \leq N \leq 126$ shell space with particle like proton-bosons and hole like neutron-bosons and it is forming the p-h space. Therefore, the quadrant I and III have p-p and h-h bosons space, respectively and quadrant II and IV for h-p and p-h bosons space respectively. It has been observed that there are no nuclei in quadrant IV. The division of the $50 \leq Z \leq 82$ and $82 \leq N \leq 126$ shell space had been suggested by Gupta et al. (1990b) to study the concept of F-spin multiplets. Further this concept of four quadrant used by Kumar (2013), Kumar et al. (2012) and Sharma and Kumar (2010) to study the systematic dependence of various nuclear observables and it was found that this concept gives deep information of nuclear structure.

3.3.1The variation of asymmetry parameter (γ_0) in quadrant- I for $50 \leq Z \leq 66$ and $82 \leq N \leq 104$ region:

The systematic variation of asymmetry parameter γ_0 versus N, N_B and NpNn for quadrant-I are shown in Fig. 3.1, Fig. 3.2 and Fig. 3.3, respectively. It is evident from Fig. 3.1 that the γ_0 decreases sharply from 30^0 to 9^0 as N increases from 82 to 90 indicating the shape phase transition from Vibrational (VM) to rotational (RM) limit of collective model of Bohr and Mottelson (1975) and also SU(5) or O(6) limit to SU(3) limit of IBM Casten (1990). If N is increased beyond 92 the γ_0 does not change and becomes almost saturated indicating that the full nuclear core deformation is achieved even at about 9^0 - 12^0 for each isotopes example for Sm, Gd and Dy. It is clear from the Fig. 3.1 that there is little scattering of data for fixed values of N i.e. the asymmetry parameter is having smooth dependence on N.

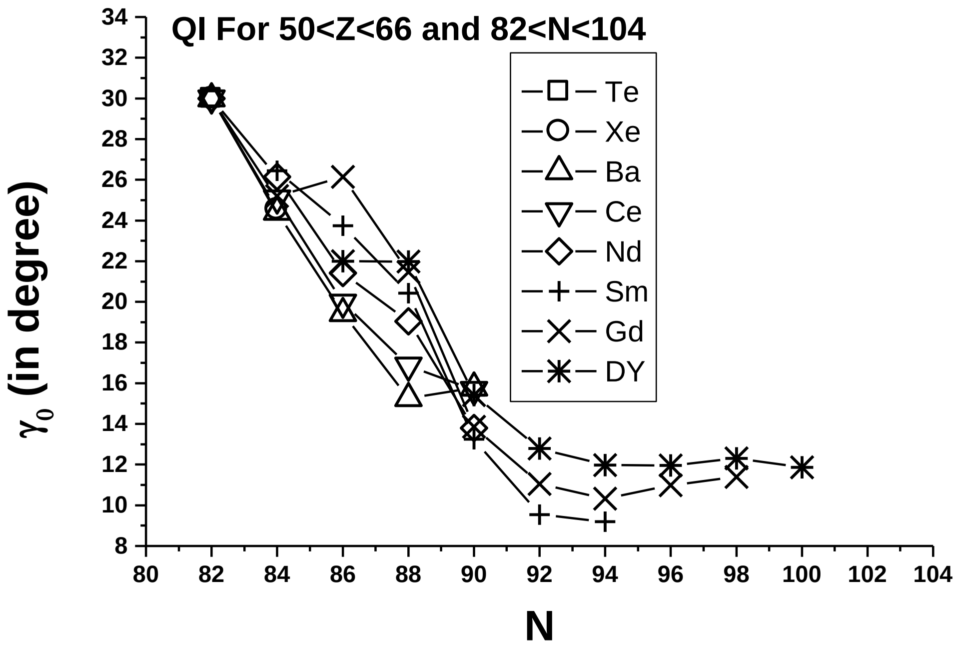

Fig.3.1 The variation of asymmetry parameter (γ_0) vs. Neutron number (N) for Quadrant I for 50≤Z≤66 and 82≤N≤104 region.

However, the data points of asymmetry parameter have much scattering for a fixed values of N_B, for example for a fixed value of N_B = 6 there is variation in the values of γ_0 from 15^0 to 26^0 (see Fig.3.2) and indicating very week dependence of asymmetry parameter on N_B. The asymmetry parameter rises for N=84, 86 and 88 with little increasing slop for N=84 and fast increasing slop for N=86 and 88 with increasing N_B and for N=90 there is a small fall instead, and which finally saturates for N≥92, that is for N_B≥12 (see Fig. 3.2). It is also evident that the asymmetry parameter decreases sharply on increasing the value of N_B from 2 to 12 for each value of Z with almost same slope for Xe, Ba, Ce, Nd, Sm, Gd and Dy elements for 82≤N≤90 region. But the individual curve of each element has been shifted towards right while Z is increased.

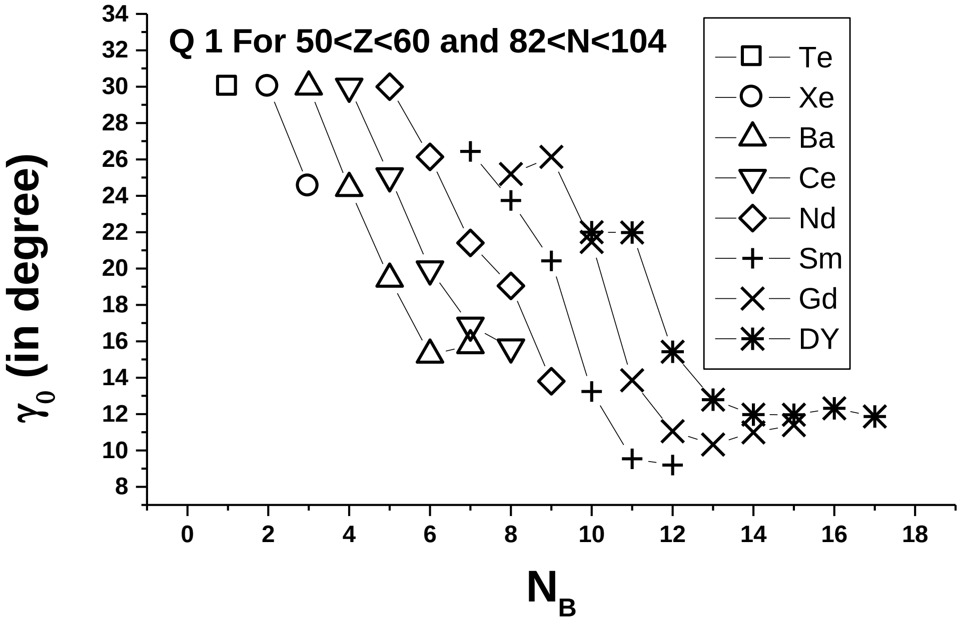

Fig.3.2 The variation of asymmetry parameter (γ_0) vs. Boson number (N_B) for Quadrant I for 50≤Z≤66 and 82≤N≤104 region.

The variation of asymmetry parameter γ_0 versus Np.Nn has been shown in Fig. 3.3. The γ_0 decreases from a maximum value of 30° for N_pN_n = 0 (i.e. SU(5) limit of IBM Casten (1990) to a minimum values of about 9° (i.e. SU(3) limit of IBM). The γ_0 saturates for $N_pN_n \geq 30$. It is evident that the asymmetry parameter also rises for N=84, 86 and 88 isotones with little increasing slop for N=84 and fast increasing slop for N=86 and 88 with increasing NpNn and for N=90 there is a small fall instead, and finally γ_0 saturates for N≥92, that is for NpNn ≥30. The same feature was observed for the E_{2g} in quadrant I by Kumar (2013) and Kumar et al. (2012). But this effect was in reverse order for the ground state moment of Inertia (θ_g= 1/ E_{2g}) and energy ratio R_4 (= E_{4g}/ E_{2g}) by Kumar (2013) and Kumar et al. (2012).

These variations in rising slopes of N=84, 86 and 88 versus the product NpNn in Fig. 3.3 arise on account of the Z=64 proton subshell gap. Ogawa et al.(1978) noted the Z=64 sub-shell effect in ^{146}Gd. The role played by the Z=64 subshell effect in Nd-Sm-Gd-Dy nuclei had been stressed earlier by Casten (1985), Casten et al. (1996) and Gupta (1993). It is evident here that the smooth dependence of asymmetry parameter γ_0 on NpNn is confined to N>90 region (see Fig. 3.3), where the Z=64 subshell effect disappears, unless one uses the effective proton bosons Np number for N<90.

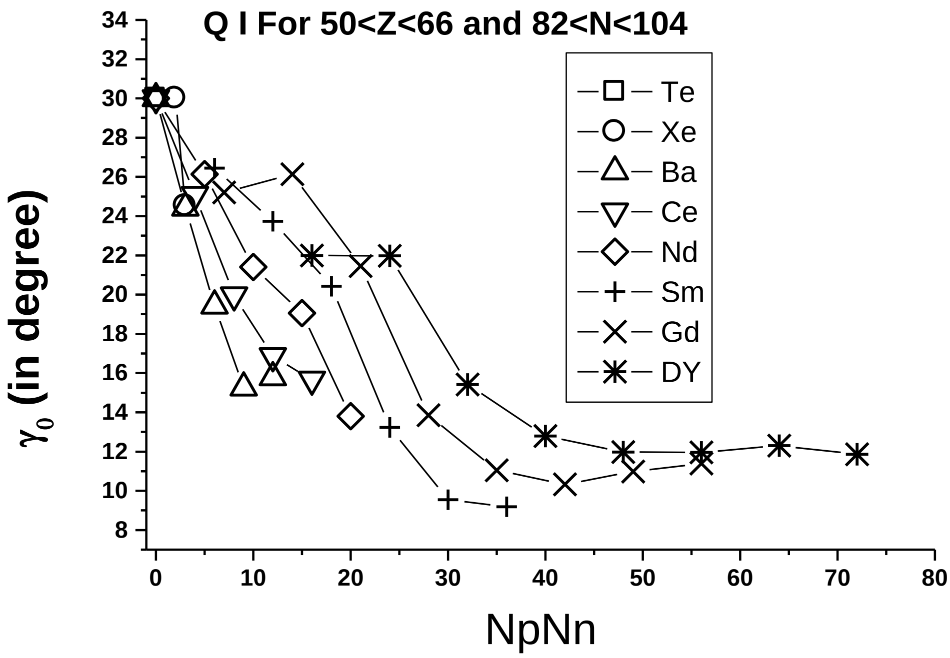

Fig.3.3 The variation of asymmetry parameter (γ_0) vs. N_pN_n for Quadrant I for 50≤Z≤66 and 82≤N≤104 region.

This shows non-dependence of γ_0 with N_pN_n because for a fixed value of N_pN_n the γ_0 is having varying values. It is clear from the Figs. 3.1-3.3 that the asymmetry parameter γ_0 vividly display the formation of isotonic multiplets in quadrant-I which supports the observation of Gupta et al. (1990b) who had illustrated it in a different way and Kumar (2013), Kumar et al. (2012).

3.3.2 The variation of asymmetry parameter γ_0 for quadrant-II for 66≤Z≤82 and 82≤N≤104:

The systematic variation of asymmetry parameter γ_0 versus N, N_B and NpNn for quadrant –II are shown in Fig. 3.4, Fig. 3.5 and Fig. 3.7, respectively. It is evident from Fig. 3.4 that the γ_0 decreases sharply from 30^0 to 12^0 as N increases from 82 to 94 for Dy, Er and Yb isotopes indicating that the shape phase transition takes place from Vibrational (VM) to rotational (RM) limit of collective model of Bohr and

Mottelson (1975) and also SU(5) or O(6) limit to SU(3) limit of IBM Casten (1990) as observed in quadrant-I. If N is increased from 94 to 98, γ_0 does not change and remains saturated indicating that the full deformation is achieved even at about $\approx 12^0$ and $\approx 13^0$ for each isotopes example for Dy and Er isotopes, respectively. However, for Yb and Hf isotopes the nature of the asymmetry parameter γ_0 is different because it goes on decreasing 21^0 to 8^0 for Yb and from 18^0 to 10^0 for Hf while N increases from 90 to 104 for Yb and from 94 to 104 for Hf isotopes. It indicates that for Yb and Hf isotopes the asymmetry parameter γ_0 goes on decreasing i.e. nuclear core deformation increases when the neutrons number (N) is increased from 82 to 104, i.e. till the shell is half filled. The point of Os and Pt are away from the line of general trend. It is clear from the Fig. 3.4, that there is much scattering of data points for fixed values of N i.e. the asymmetry parameter is not having smooth dependence on N.

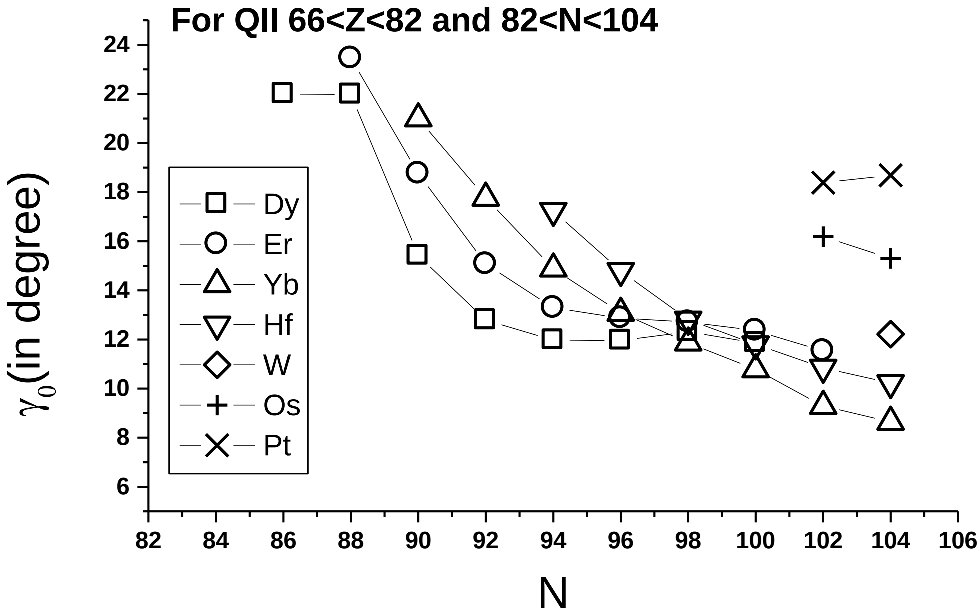

Fig. 3.4 The variation of asymmetry parameter (γ_0) vs. Neutron number (N) for Quadrant II for 66≤Z≤82 and 82≤N≤104 region.

The data points of asymmetry parameter are overlapping on each other for Dy - W isotopes (see Fig. 3.5) for N_B = 12 and 13. For N_B= 11 to16 the data points of Yb and Hf isotopes are overlapping on each other and the value of asymmetry parameter for these nuclei goes on decreasing till N_B approaches 16. The data points of Dy and

Er are also overlapping for N_B=13-17 and the value of asymmetry parameter are independent of N_B. It indicates that the Dy and Er nuclei have different nature of nuclear deformation than Yb and Hf isotopes for this region. The Os and Pt data points are above the uniform pattern curve indication different nature of these nuclei. For this region (quadrant-II), the data points of asymmetry parameter have less scattering for a fixed value of N_B in comparison to quadrant-I (see Fig. 3.2). The Fig. 3.5 is indicating a week dependence of asymmetry parameter on N_B.

The variation of asymmetry parameter γ_0 vs. Np.Nn has been shown in Fig. 3.6. The γ_0 decreases fast at first and gradually later while NpNn is increasing; and remains unchanged for NpNn≥45 for Dy and Er isotopes for which the proton boson pair Np decreases from 8 to 7; and decreases for Yb and Hf on increasing NpNn beyond 45 for which the proton boson pair Np decreases from 6 to 5. The two data points of Os and one of W are lying on the smooth curve while for Pt the data point are slightly below the uniform curve.

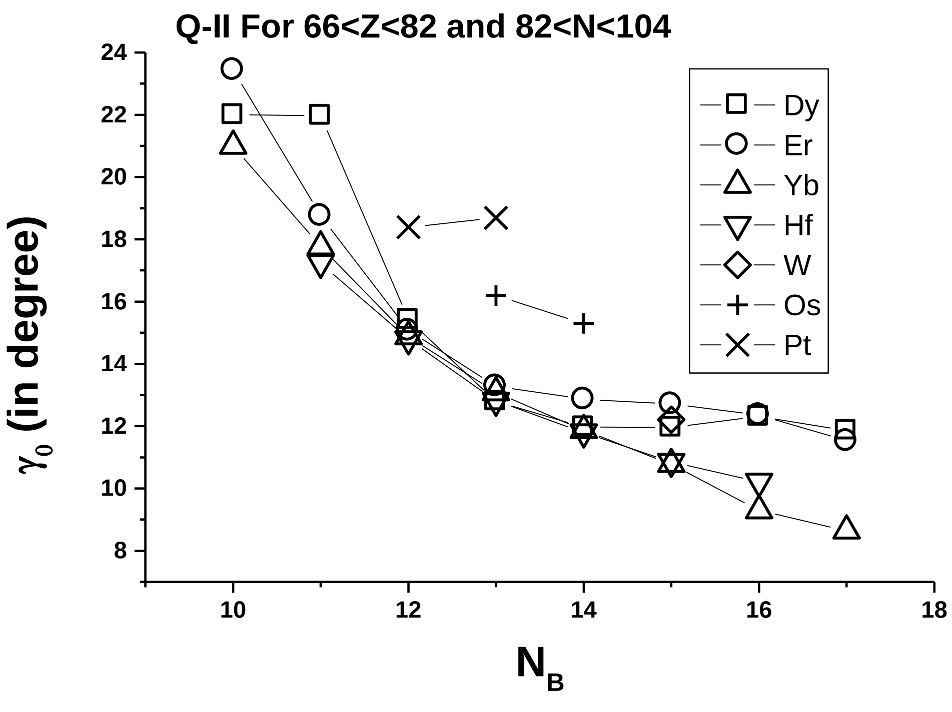

Fig. 3.5 The variation of asymmetry parameter (γ_0) vs. Boson number (N_B) for Quadrant II for 66≤Z≤82 and 82≤N≤104 region.

One important point is to be noted here that the asymmetry parameter γ_0 is calculated from the values of E_{2g} and $E_{2\gamma}$ and the nature of variation of $E_{2\gamma}$ verses N is different for Dy and Er isotopes than Yb and Hf as shown in Fig. 3.6. The $E_{2\gamma}$ remains almost constant for Dy and Er isotopes for N=88-102 but for Yb and Hf it increases sharply as N increases from N=94-104 and becomes maximum for N=104. This effect is reflected here and the value of asymmetry parameter remains constant and (i.e. above the usual trend) for Dy and Er isotopes for NpNn≥45 as stated above. The same feature of $E_{2\gamma}$ state had been observed with NpNn by Kumar (2013). There is a smooth dependence of asymmetry parameter γ_0 with NpNn in quadrant –II.

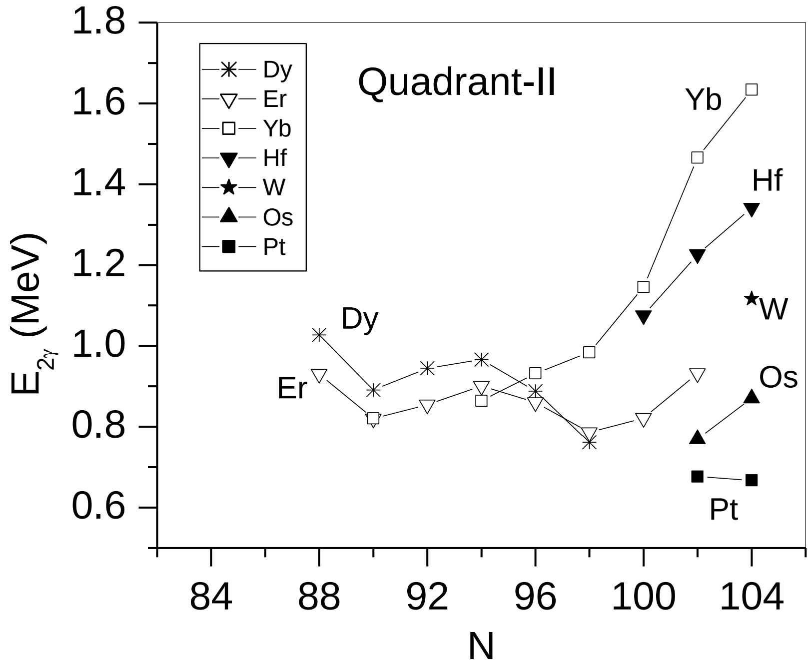

Fig. 3.6 The variation of the energy of E2γ state vs. N for Quadrant II for 66≤Z≤82 and 82≤N≤104 region.

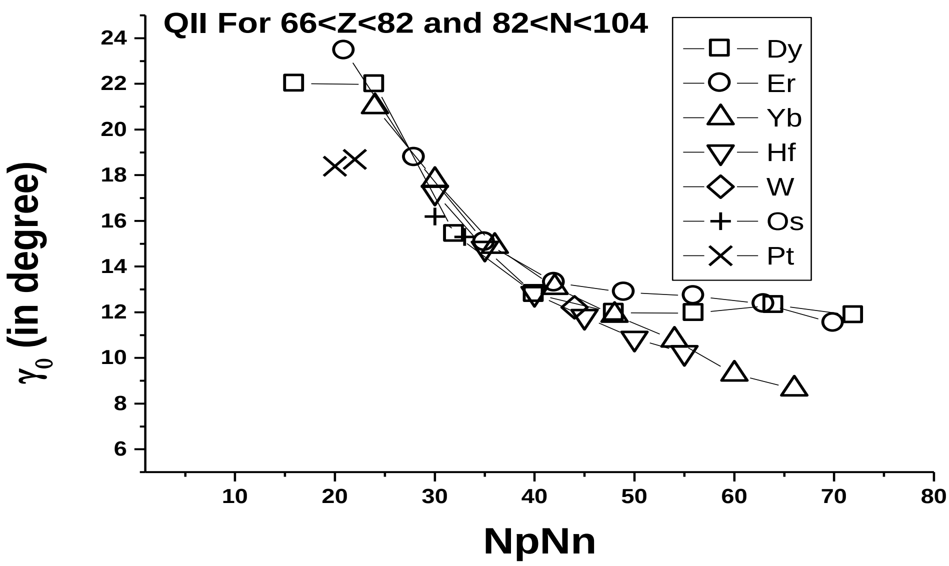

Fig. 3.7 The variation of asymmetry parameter (γ_0) vs. N_pN_n for Quadrant II for 66≤Z≤82 and 82≤N≤104 region.

3.3.3 The variation of asymmetry parameter γ_0 for quadrant-III for 66≤Z≤82 and 104≤N≤126:

The variation of asymmetry parameter γ_0 versus N, N_B and NpNn for quadrant –III are shown in Fig. 3.8, Fig. 3.9 and Fig. 3.10, respectively. In Fig. 3.8, the asymmetry parameter increases/decreases with increasing N in different style for different value of proton number Z. For Yb and Hf the curves are almost horizontal with little curvature. In W (Z=74, Np=4) and Os (Z=76, Np=3) the curve fall very slowly for N=104 to 108 (attains minimum value of γ_0 at N=108) and rises significantly when N increases beyond 108. In case of Os the asymmetry parameter decreases while N increases from 116 to 118. The nature of curve is different for Pt (Z=78, Np=2) it initially increases sharply while N increases from 104 to 112 and becomes horizontal for N=110-122. For Hg (Z=80, Np=1) the curve is almost horizontal with little curvature for N=108-116 beyond that it significantly decreases while N increasing from 116 to 120 and again increases while N increases from 120 to 122. The feature of E_{2g} vs. N had been reported for W isotopes for quadrant –III by Kumar (2013).

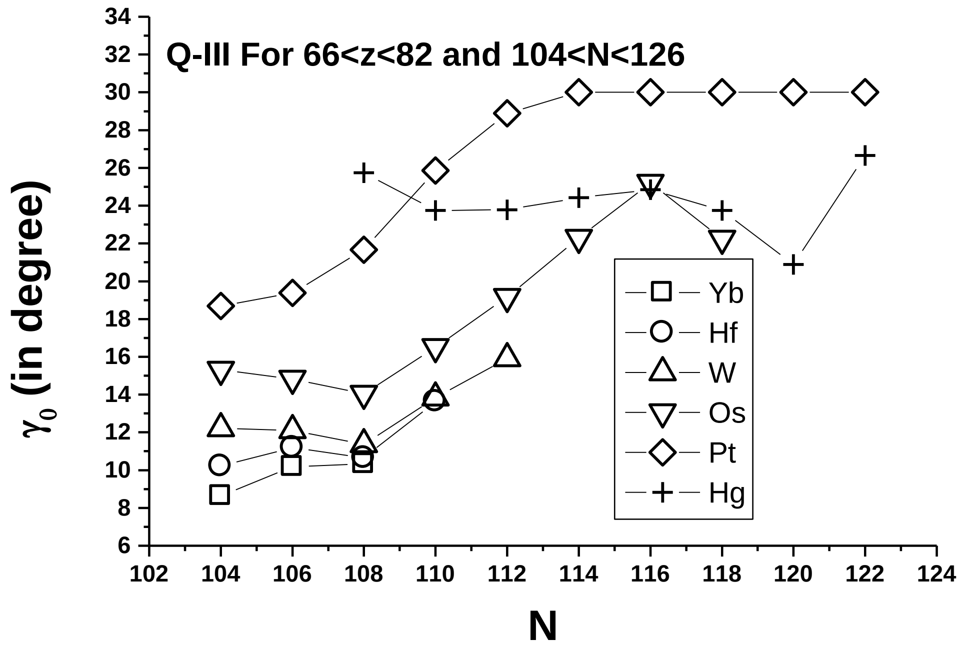

Fig. 3.8 The variation of asymmetry parameter (γ_0) vs. Neutron number (N) for Quadrant III for 66≤Z≤82 and 104≤N≤126 region.

The variation of asymmetry parameter γ_0 versus N_B is shown in Fig. 3.9. The γ_0 decreases with increasing N in different style for different value of proton number Z. For Pt, the γ_0 is independent of N_B for $4 \leq N_B \leq 10$ the curve is horizontal (with $\gamma_0 = 30^0$) and γ_0 decreases sharply on increasing N_B beyond 10. The curve for Pt is lying above the observed curve for other isotopes except Hg for which the curve is almost horizontal with little curvature except for N_B =4. For Yb – Os isotopes the asymmetry parameter has N_B dependence with little scattering for Os and W isotopes as discussed above.

The variation of asymmetry parameter γ_0 versus NpNn is shown in Fig. 3.10. The value of γ_0 decreases with increasing NpNn (going towards the mid shell) and provide a single broken curve (except for Hg isotopes with Z=80, Np=1) indication that the values of γ_0 are slightly different for different Z. For quadrant –III, the Nn and Np both are the hole boson pairs and the value of both decreases as we go towards closed shell i.e., right to left in Fig. 3.9. The strong dependence of asymmetry parameter with NpNn is evident in the quadrant-III.

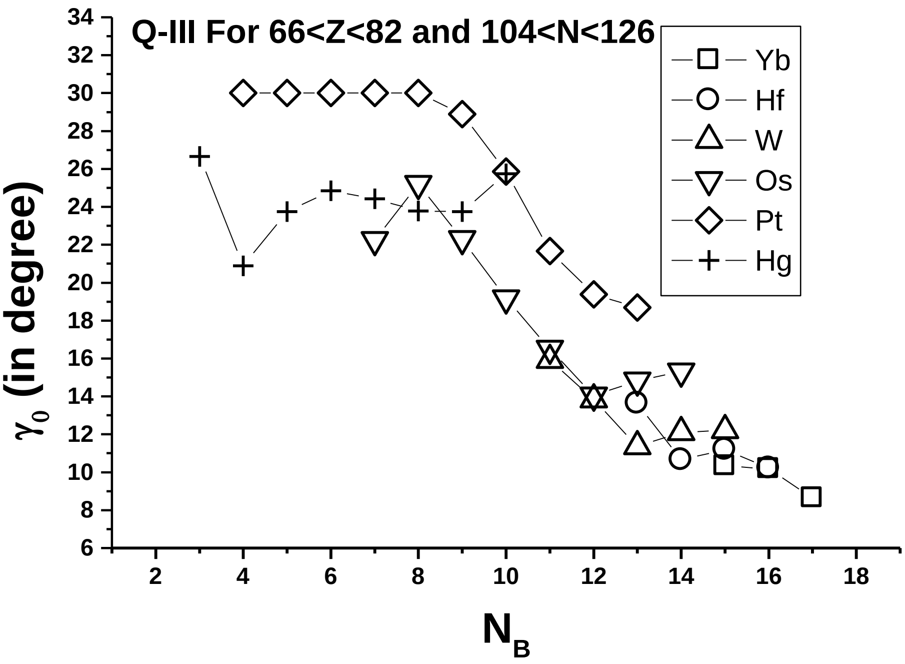

Fig. 3.9 The variation of asymmetry parameter (γ_0) vs. Boson number (N_B) for Quadrant III for 66≤Z≤82 and 104≤N≤126 region.

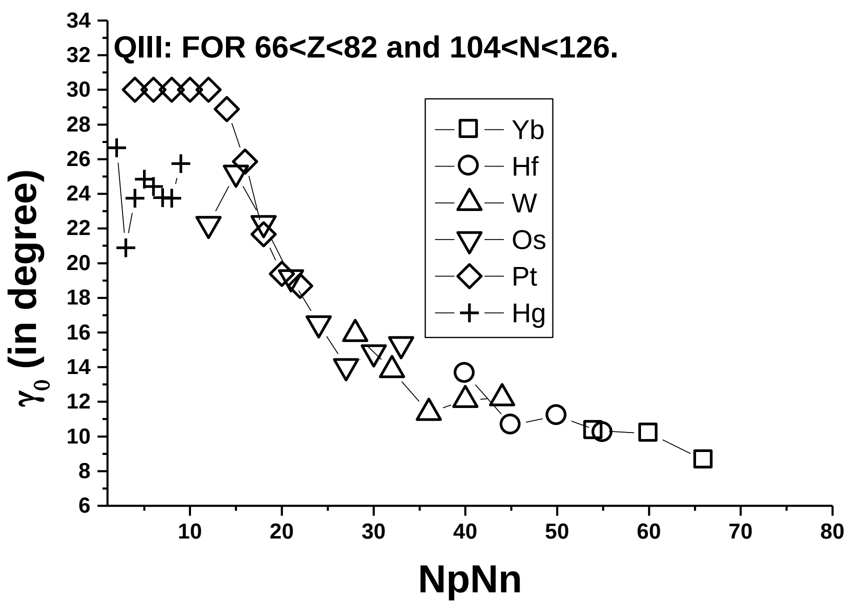

Fig. 3.10 The variation of asymmetry parameter (γ_0) vs. N_pN_n for Quadrant III for 66≤Z≤82 and 104≤N≤126 region.

3.4 CONCLUSION

The NpNn scheme is very useful in considering the systematic behavior of asymmetry parameter (γ_0) which gives the information of nuclear structure of atomic nuclei in a medium and light mass region, i.e. change in product NpNn are correlated with the change in nuclear structure. The NpNn product is a good measure of its effect in producing the deformation in atomic nuclei. This product is also an indicator of the n-p interaction among the valance proton and/or neutron nucleons causing the deformation of nuclear core.

In quadrant-I for $50 \leq Z \leq 66$ and $82 \leq N \leq 104$ and quadrant-II for $66 \leq Z \leq 82$ and $82 \leq N \leq 104$ region, the asymmetry parameter γ_0 decreases from 30^0 in Q-I and from 22^0 in Q-II to 9^0- 10^0 with increasing N from 82 to 104 (i.e. the mid of N=82 to 126 neutron shell), signifying that the nuclear deformation (β) is increasing, while the energy ratio R_4 increase from 2.0 (for harmonic vibrators or SU(5) type nuclei) to 3.33 (for good rotors or SU(3) type nuclei). This indicates that in this region the nuclear structure depends much more on Z. The values of asymmetry parameter in Q-I, shows shape phase transition at N=88-90 and regions (QII-III) have a systematic dependence with N, but having different patterns. Partial results of this study have been presented in the DAE Symposium on Nuclear Physics Kaushik and Sharma (2014).

In quadrant-I, the asymmetry parameter is having more correlated dependence on the neutron number N, rather than on the product NpNn. In this quadrant- I, the Z=64 sub-shell effect for $N \leq 90$ nuclei affect the variation of asymmetry parameter with N and NpNn product. Casten (1985) and Casten et al. (1996) obtained a smooth dependence of various observables with NpNn by adopting effective numbers proton bosons Np for $N \leq 90$ nuclei. This was a very useful procedure for obtaining the universal smooth curves for various regions with NpNn. The present studies also confirm the observations of Gupta et al. (1990b), Kumar (2013), Kumar et al. (2012) and Sharma and Kumar (2010) i.e. the existence of isotonic multiplets in quadrant-I.

The existence of X(5) symmetry in N=90 isotones established in recent works supports the formation of isotonic multiplets in this work. The calculated values of asymmetry parameter are almost constant for N=90 isotones e.g. 13.8^0 for Nd, 13.24^0 for Sm and 13.86^0 for Gd; which support the findings of Gupta (2012a); who gave the microscopic explanation for the constant structure of N=90 isotones. This is certainly different from the universal NpNn scheme Casten (1985) and Casten and Zamfir (1996) and found to be very useful for most of the atomic nuclei throughout the periodic table as noticed by Casten and Zamfir (1996). This special condition for N=90 isotones is made more explicit in the present work for Q-I and supports the findings of Casten and Zamfir (1996).

The systematic dependence of asymmetric parameter on NpNn has strong dependence in quadrant-II. In Q-II, the line of β- stability runs nearly diagonally, i.e. parallel to N_B (where, N_B is the sum of proton hole bosons and neutron particle bosons) and leading to the formation of F-spin multiplets. The same feature had been observed earlier for E_{2g} by Kumar et al. (2012) and Sharma and Kumar (2010).

In quadrant-III, for $66 \leq Z \leq 82$ and $104 \leq N \leq 126$ region, the variation of asymmetry parameter is different from quadrant I and II because the asymmetry parameter γ_0 increases sharply from 9^0 - 10^0 to 30^0 with increasing N from 104 to 126. This is signifying that the nuclear deformation (β) is decreasing and the nuclear structure changes from pure rotor SU(3) type to vibrational SU(5) or γ-unstable O(6) type. Further, the asymmetry parameter for different elements has smooth curve with NpNn with almost same slopes except for Hg isotopes.

The graphs of asymmetry parameter against N_pN_n vividly display the formation of isotonic multiplets in quadrant-I, strong dependence on N_pN_n in quadrant-II and weak constancy with Z in quadrant-III is illustrated and support the findings of Gupta (2012b). Also in every case the role of N, Z is well evident. This also agrees with known variation of nuclear deformation in the light and medium mass region. The quadrant wise presentation of asymmetry parameter is very useful as in case of other observables of collectivity and deformation i.e. the energy of first excited state E_{2g}, the energy ratio R_4, the B(E2; $0_1^+ \rightarrow 2_1^+$) value and ground state band moment of inertia ($\theta_g = 3/E_{2g}^+$) as noted by Kumar (2013), Kumar et al. (2012) and Sharma and Kumar (2010).

Table 3.1: The calculated values of asymmetric parameter (γ_0) for Te to Ce nuclei using equation 3.2.

N	Te	Xe	Ba	Ce
82	30	30	30	30
84		24.52	24.42	25.07
86			19.43	19.95
88			15.26	16.86
90			15.78	15.66

Table 3.2: The calculated values of asymmetric parameter (γ_0) for Nd to Pt nuclei using equation 3.2.

N	Nd	Sm	Gd	Dy	Er	Yb	Hf	W	Os	Pt	Hg
82	30.0										
84	26.14	26.44	25.2								
86	21.41	23.74	26.14	22							
88	19.05	20.42	21.46	21.98	23.45						
90	13.8	13.24	13.86	15.42	18.76	21					
92		9.54	11.05	12.79	15.07	17.77					
94		9.19	10.32	11.97	13.29	14.88	17.24				
96			10.98	11.96	12.87	13.08	14.79				
98			11.39	12.31	12.71	11.87	12.81				
100				11.86	12.36	10.78	11.8				
102					11.53	9.29	10.85		16.19	18.39	
104						8.64	10.22	12.21	15.3	18.69	
106						10.18	11.19	12.1	14.83	19.39	
108						10.33	10.66	11.37	14.04	21.67	25.74
110							13.63	13.83	16.5	25.87	23.74
112								15.91	19.16	28.89	23.78
114									22.3	30	24.42
116									25.21	30	24.85
118									22.24	30	23.74
120										30	20.89
122										30	26.65