CHAPTER- II

NUCLEAR MODELS

2.1 INTRODUCTION

Different models for nucleus have been proposed each of which explains the behavior of nucleus in some specific situation. But at the same time each of these models is in noticeable contradiction with other models or with known facts about nuclear forces. We will limit ourselves only to some basic models for the nucleus that can be explaining the general characteristics.

2.2 BOHR-MOTTELSON UNIFIED COLLECTIVE MODEL

It is also called unified model, description of atomic nuclei that incorporates aspects of both the shell nuclear model and the liquid-drop model to explain certain magnetic and electric properties that neither of the two separately can explain. It views the nucleus as vibrating –rotating core capable of being deformed to various shapes i.e. prolate, oblate or tri-axial.

In the shell model, nuclear energy levels are calculated on the basis of a single nucleon (proton or neutron) moving in a potential field produced by all the other nucleons. Nuclear structure and behaviour are then explained by considering single nucleons beyond a passive nuclear core composed of paired protons and paired neutrons that fill groups of energy levels, or shells. In the liquid-drop model, nuclear structure and behaviour are explained on the basis of statistical contributions of all the nucleons (much as the molecules of a spherical drop of water contribute to the overall energy and surface tension). In the collective model, high-energy states of the nucleus and certain magnetic and electric properties are explained by the motion of the nucleons outside the closed shells (full energy levels) combined with the motion of the paired nucleons in the core. Roughly speaking, the nuclear core may be thought of as a liquid drop on whose surface circulates a stable tidal bulge directed toward the rotating unpaired nucleons outside the bulge. The tide of positively charged protons constitutes a current that in turn contributes to the magnetic

properties of the nucleus. The increase in nuclear deformation; that occurs with the increase in the number of unpaired nucleons accounts for the measured electric quadrupole moment; which may be considered a measure of how much the distribution of electric charge in the nucleus departs from spherical symmetry.

2.2.1 The Vibrational Model

A spherical nucleus can be considered as compressible liquid drop. Its excitation mode arises from small oscillations about the equilibrium spherical shape. The surface of the spherical drop can be written as (see Alder et al. (1956)):

$$R(\Theta,\Phi) = R_0 = \left[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\Theta,\Phi)\right] \quad (2.1)$$

where, R_0 is the average nuclear radius, $\alpha_{\lambda\mu}$ are the deformation variables, λ verify in mode of the nuclear motion, μ is the projection of λ on the Z-axis and $Y_{\lambda\mu}(\Theta,\Phi)$ are the spherical harmonics where Θ and Φ are the polar angles with respect to the arbitrary space-fixed axes. The $\lambda = 0$ mode corresponds to the change in the nuclear radius without any change in the shape, λ = 1 mode corresponds to the translation of the center of mass, λ = 2 is the quadrupole mode of the lowest order of mode and λ = 3 corresponds to the octupole mode related to the higher lying excitation. In the λ = 2 mode the ground state has no phonon while the first excited state has one phonon excitation and is five-fold degenerate, since the azimuthal quantum number μ can take of the integral value -2, -1, 0, 1, 2.

In vibrational model it was assumed that the nucleus performs vibrations around the spherical shape and the Hamiltonian in quadrupole mode can be written as (see Alder et al. (1956) and Bohr and Mottelson (1953):

$$H_v = \frac{1}{2}C\sum_{\mu}\left|\alpha_\mu^2\right| + \frac{1}{2}B\left|\dot{\alpha}_\mu^2\right| \quad (2.2)$$

Where, B and C are the mass parameter and the stiffness parameter respectively. A typical spectrum of the ^{118}Cd isotope, which has the vibrational characteristics, is given by Aprahamian et al. (1987) is shown in Figure 2.1.

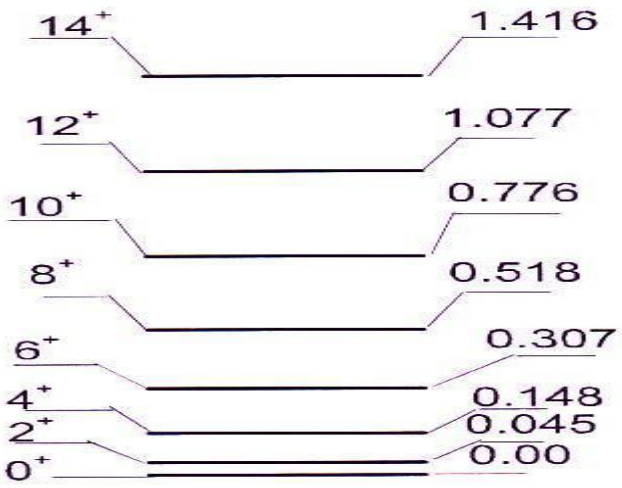

Figure 2.1: Energy levels diagram of $^{238}U_{92}$ nuclei.

2.2.2 Rotational Model

In the rotational model the shape of the nucleus is assumed to be fixed and the nuclear system rotates like a rigid structure. The energy associated with rotation would be purely kinetic and equal to $\frac{1}{2}\Im_0 w^2$. According to the collective model, the $\Im_0$ of nuclei can be determined from the energies of the rotational states. Rotational energy level of an axially symmetric nucleus can be described by three constants of motion: J, the total angular momentum; K, the projection of J on the nuclear symmetric axis (Z-axis); M, the projection of J on the space fixed axis (Z' –axis). The collective rotational angular momentum R is perpendicular to the symmetric axis.

If $\mathfrak{J}$ and $\mathfrak{J}$ are the moments of inertia for rotations about symmetric axis 3 (i.e. Z-axis) and about an axis perpendicular to Z-axis, J_1, J_2 and J_3 are the components of the total angular momentum operator along the body fixed axis. The Hamiltonian given by Bohr and Mottelson (1953) can be written as if $J_1 = J_2 = J$:

$$H_{rot} = \sum_{i=3}^{3} \frac{\hbar^2}{2\Im_i} J_i^2 = \frac{\hbar^2}{2\Im}(J_1^2 + J_2^2) + \frac{\hbar^2}{2\Im_3} J_3^2 \qquad (2.3)$$

$$= \frac{\hbar^2}{2\Im}(J_1^2 - J_2^2) + \frac{\hbar^2}{2\Im_3} J_3^2$$

For H_{rot} the eigenfunctions are the D functions, which are the transformation functions for spherical harmonics under finite rotations,

$$J^2 D^J_{MK} = J(J+1) D^J_{MK}$$
$$J_3 D^J_{MK} = K D^J_{MK} \qquad (2.4)$$
$$J'_z D^J_{MK} = D^J_{MK}$$

and

$$H_{rot}\ D^J_{MK} = \left[\frac{\hbar^2}{2\Im}(J(J+1) - K^2) + \frac{\hbar^2}{2\Im_3} K^2\right] D^J_{MK}\ .$$

The energy Eigen values are:

$$E = \frac{\hbar^2}{2\Im}[J(J+1) - K^2] + K^2 + \frac{\hbar^2}{2\Im_3} K^2 \qquad (2.5)$$

For $K = 0$ the energy expression becomes

$$E = \frac{\hbar^2}{2\Im} J(J+1), \quad J = 0,2,4,6 \ldots\ldots\ldots \qquad (2.6)$$

The energy levels of the ground state rotational band have the relation,

$E(2^+_1) : E(4^+_1) : E(6^+_1) : E(8^+_1)$: … = 1 :10/3 : 7 : 12 : …… (2.7)

Gupta et al. (1990) study the few good examples of rigid rotors such as ^{156}Gd, ^{170}Er, ^{170}Yb and ^{176}W isotopes. The rotational spectrum of the ^{156}Gd isotope is given in the Figure 2.2 and the values of experimental energies are taken from Sakai (1984).

In the presence of the centrifugal stretching, most of the nuclei deviate from the expression (2.8) and this effect can be taken into account by modifying to,

$$E_{rot} = \frac{\hbar^2}{2\Im} J(J+1) - B[J(J+1)]^2 \qquad (2.8)$$

where B is constant parameter for all J and $\Im$.

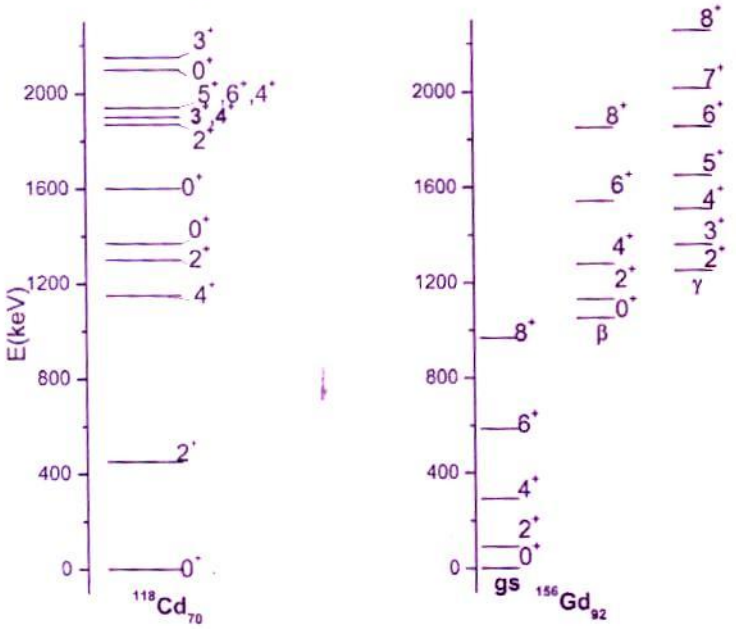

Figure 2.2: The vibrational spectrum of ^{118}Cd isotope and the rotational spectrum of ^{156}Gd isotope.

2.2.3 Rotation Vibration Interaction Model

Fasessler et al. (1965) formulated the rotation vibration interaction model that was a complete form the collective model of Bohr (1952). It assumes the nucleus to be an axially symmetric rotating body undergoing very small amplitude shape vibrations (β_0, equilibrium deformation parameter $\neq 0$; γ_0 asymmetry parameter $= 0$). This extended model allows the diagonalization of the collective Hamiltonian, and the interaction of $\beta-$vibrational, rotational motion, as well as of $\beta-,\gamma-$ vibrational interaction Hamiltonian that can be written as:

$$\hat{H} = \hat{H}_{rot.} + \hat{H}_{vib.} + \hat{H}_{rot-vib.} \tag{2.9}$$

The value of g-, $\beta-$ and $\gamma-$ band, the absolute B(E2) values and B(E2) ratios of a given nucleus can be obtained with the help of four model parameters i.e. the moment of inertia which takes into account the energy of 2_1^+ state: the equilibrium deformation β_0 of the nucleus which can be obtained from the absolute B(E2) value for the first excited 2_1^+ states; the $\gamma-$ vibrational energy E_γ which is fitted to the

energy of the 2_1^+ state, the β −vibrational energy E_β which is taken from the energy of first excited 0^+ state.

2.3 ASYMMETRIC ROTOR MODEL

Davydov and Filippov (1958) investigated the energy levels corresponding to rotation of nucleus which does not change its internal state. They established that the violation of axial symmetry of even nuclei affect the rotation spectrum of axial nucleus with appearance of some new rotational states having total angular moments of 2,3,4,…. If the deviation from axial symmetry is small than these levels lie very high and are not excited. The energy rotation of a non-spherical even-even nucleus is given, in the adiabatic approximation, by the Schrodinger equation:

$$(H - E)\psi = 0 \tag{2.10}$$

where, E is measured in units of $\hbar^2/4B\beta^2$, and the operator H is given by the formula:

$$H = \sum_{k=1}^{3} \frac{A J_k^2}{2sin^2(\gamma_0 - \frac{2\pi k}{3})} \tag{2.11}$$

Here, $A = \hbar^2/4B\beta^2$ is a quantity having dimension of energy, γ_0 varies between 0 and $\frac{\pi}{3}$ and determines the deviation of the nucleus from axial symmetry. The J_k are the operators of the angular momenta on the axis of a coordinate system connected with the nucleus. In eq. (2.29), for $\gamma \neq 0$ or $\frac{\pi}{3}$ the nucleus should be regarded as an asymmetric top. The wave function corresponding to the state with total moment J, can be represented as:

$$\psi_{JN} = \sum_{\lambda \geq 0} |J\lambda > A_\lambda| \tag{2.12}$$

where,

$$|J\lambda> = \left[\frac{2j+1}{16\,\pi^2(1+\delta_{\lambda 0})}\right]^{1/2} \{D^J_{N\lambda} + (-1)^J\ D^J_{N,-\lambda}\} \ldots\ldots\ldots \tag{2.13}$$

The function $D^J_{N\lambda}$ in eq. (2.13) are the functions of the Euler angles that determine the orientation of the principal axis of the nucleus with respect to the laboratory space. It can be shown that the wave functions (2.12) from the basis of totally

symmetric representation of the group D_2 the element of which are the rotation through 180° around each of three principal axes of the nucleus (see Davydov and Filippov (1958); Davydov and Rostovsky (1959)). The wave function of the rotational 2_1^+ states of the non-axial nucleus can be rewritten as (see Davydov and Filippov (1958)):

$$\psi_{21n} = \sqrt{5/8\pi^2}\left[A_1 D^2_{n0} + B_1(D^2_{n2} + D^2_{n,-2})/\sqrt{2}\right], \tag{2.14}$$

$$\psi_{22n} = \sqrt{5/8\pi^2}\left[A_2 D^2_{n0} + B_1(D^2_{n2} + D^2_{n,-2})/\sqrt{2}\right], \tag{2.15}$$

where, the value of A_λ coefficients in the wave function of eq.(2.14, 2.15) can be obtained using the value of γ_0:

$$\begin{aligned}
&A_1 M_1 = -\left[sin\gamma_0 sin3\gamma_0 + 3cos\gamma_0 cos3\gamma_0 + (9 - 8sin^2 3\gamma_0)^{1/2}\right]\\
&B_1 M_1 = 3sin\gamma_0 cos3\gamma_0 - cos\gamma_0 sin3\gamma_0,\\
&M_1^2 = 2\sqrt{(9 - 8sin^2 3\gamma_0)} \times \left[\sqrt{(9 - 8sin^2 3\gamma_0)} + sin\gamma_0 sin3\gamma_0 + 3cos\gamma_0 cos3\gamma_0\right],\\
&A_2 M_2 = \sqrt{(9 - 8sin^2 3\gamma_0)} - sin\gamma_0 sin3\gamma_0 - 3cos\gamma_0 cos3\gamma_0\ , \qquad (2.16)\\
&B_2 M_2 = 3sin\gamma_0 cos3\gamma_0 - cos\gamma_0 sin3\gamma_0,
\end{aligned}$$

$$M_2^2 = 2\sqrt{(9 - 8sin^2 3\gamma_0)} \times \left[\sqrt{(9 - 8sin^2 3\gamma_0)} - sin\gamma_0 sin3\gamma_0 - 3cos\gamma_0 cos3\gamma_0\right]$$

Similarly for 3_1^+ state the wave function can be written as

$$\psi_{3n} = \sqrt{7/16\pi^2}\left(D^3_{n2} - D^3_{n,-2}\right).$$

And spin 4_1^+ wave function is

$$\psi_{41} = \sqrt{9/8\pi^2}D^4_{n0},$$

$$\psi_{42} = \sqrt{9/16\pi^2}\left(D^4_{n2} + D^4_{n,-2}\right)$$

$$\psi_{43} = \sqrt{9/16\pi^2}\left(D^4_{n2} + D^4_{n,-4}\right), \text{ etc.}$$

Putting the eq.(2.30)in eq.(2.28) and making use the value of matrix element of the operator of the rotational energy eq. (2.29) acting on the wave function eq.(2.31)

$$< J\boldsymbol{\lambda}|\mathrm{H}|\mathrm{J}\boldsymbol{\lambda} > = \frac{(\alpha+\beta)}{4}[J(J+1) - \lambda^2] + \frac{\delta_\lambda^2}{2}$$

$$< J\lambda + 2|\mathrm{H}|\mathrm{J}\lambda> = \frac{(\alpha-\beta)}{4}[(1+\delta_{\lambda 0})(J-\lambda)\times(J-\lambda-1)(J+\lambda+1)(J+\lambda+2)]^{1/2}$$

……(2.17)

$$\alpha = sin^{-2}\left(\gamma_0 - \frac{2\pi}{3}\right), \qquad \beta = sin^{-2}\left(\gamma_0 + \frac{2\pi}{3}\right),$$

$$\delta = \quad sin^{-2}\gamma_0 \qquad \delta_{\lambda 0} = \begin{cases} 0, & for\ \lambda \neq 0 \\ 1, & for \lambda = 0 \end{cases}$$

(2.18)

One obtains for each value of J a system of algebraic equations for the coefficients A_λ in the wave function (2.11). For $J = 4$, the Schrodinger eq. (2.10) is reduced to a system of equation as (see Davydov and Rostovsky (1959))

$$\begin{bmatrix} 5(\alpha+\beta)-E & 3/2.\sqrt{5}(\alpha-\beta) & 0 \\ 3/2.\sqrt{5}(\alpha-\beta) & 4(\alpha+\beta)+2\delta-E & \frac{\sqrt{7}}{2}.(\alpha-\beta) \\ 0 & \frac{\sqrt{7}}{2}.(\alpha-\beta) & (\alpha+\beta)+8\delta-E \end{bmatrix} \begin{bmatrix} A_0 \\ A_2 \\ A_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(2.19)

The energy of the corresponding rotational states can be determined from the condition that the system (2.19) has a solution. The three values of E can be obtained the cubic equation,

$$X^3 - \frac{45X^2}{2Sin^2 3\gamma_0} - \left(39t^2 + 117t\frac{cos3\gamma_0}{Sin^2 3\gamma_0} - \frac{81}{Sin^4 3\gamma_0} - \frac{78}{Sin^2 3\gamma_0}\right)\times -70t^2 cos3\gamma_0 + 5\left(42 - \frac{9}{2Sin^2 3\gamma_0}\right)t^2 + 5t\left(81 - \frac{cos3\gamma_0}{Sin^4 3\gamma_0} + 42\right) - \frac{270}{Sin^4 3\gamma_0} - \frac{70}{Sin^2 3\gamma_0} = 0$$

Where

$$X = \left(\frac{E}{\hbar^2 B\beta^2}\right) \quad and \quad t = 4T\beta/(\hbar^2 B\beta^2).$$

For a rough estimate of t, the value of $\gamma = 40MeV$, $\hbar^2 B\beta^2 = 400keV$ and $\beta = 0.2$ gives the value of $t = 80$. Similarly, the energy of $E(2_1^+)$ states can be determined from the,

$$\begin{vmatrix} \frac{3}{2}(a+b) - 6T\beta cos\gamma_0 - E & 6T\beta sin\gamma_0 + (a-b)\sqrt{3/2} \\ 6T\beta sin\gamma_0 + (a-b)\sqrt{3/2} & 6T\beta cos\gamma_0 + \frac{(a+b)}{2} + 2c - E \end{vmatrix} = 0$$

(2.20)

where

$$a = \hbar^2[4B\beta^2 sin^2(\gamma_0 - 2\pi/3)]^{-1},$$
$$b = \hbar^2[4B\beta^2 sin^2(\gamma_0 + 2\pi/3)]^{-1} \quad , \qquad (2.21)$$

$$c = \hbar^2[4B\beta^2 sin^2(\gamma_0)]^{-1}. \tag{2.22}$$

Substituting the values of a, b, c and expending the determinant (2.20) we obtained the second degree equation,

$$X^2 - \frac{9X}{2sin^2 3\gamma_0} - \frac{9t^2}{4} - \frac{27cos3\gamma_0}{4sin^2 3\gamma_0}t + \frac{9}{2sin^2 3\gamma_0} = 0 \tag{2.23}$$

The roots of eq. (2.23) can be written as:

$$E_{21} = \frac{9\left[1 - \sqrt{\left\{1 - \frac{8sin^2(3\gamma_0)}{9}\right\}}\right]}{Sin^2(3\gamma_0)} \tag{2.24}$$

$$E_{22} = \frac{9\left[1 + \sqrt{\left\{1 - \frac{8sin^2(3\gamma_0)}{9}\right\}}\right]}{Sin^2(3\gamma_0)} \tag{2.25}$$

The energy levels of $J = 3$ state is given by Davydov and Filippov (1958):

$$E(3) = \sum_{K=1}^{3} \frac{2}{sin^2\left(\gamma_0 - \frac{2\pi}{3}\lambda\right)} = \frac{18}{sin^2(3\gamma_0)} \tag{2.26}$$

And energies of $J = 5$ states are given

$$E_\tau = \frac{[45 \pm \sqrt{(9 - 8Sin^2 3\gamma_0)}]}{Sin^2 3\gamma_0} \tag{2.27}$$

In eq. (2.26), $\boldsymbol{\tau}$=1 for the minus sign on the square root and $\boldsymbol{\tau}$=2 for the plus sign. The value of asymmetry parameter (γ_0) can be obtained using the eq. (2.24) and (2.25):

$$\gamma_0 = \frac{1}{3}Sin^{-1}\frac{9}{8}\left[1 - \left(\frac{1-R_\gamma}{1+R_\gamma}\right)^2\right]^{1/2} \quad ; \qquad R_\gamma = \frac{E(2_2^+)}{E(2_1^+)} \ . \tag{2.28}$$

In stationary states of the asymmetric top not one of the projections of the total momentum on axes 1, 2, 3 of the body-fixed coordinate system has a definite value and hence the energy levels cannot be specified the values of $K = J_3$. Each value of the total angular momentum in the asymmetric top corresponds to $2J$+1 different energy levels.

These levels can be classified with respect to the irreducible representations of group D_2. In virtue of the symmetry conditions on the wave function in even nuclei of the $2J + 1$ different levels only those energy levels with a given J can exist which correspond symmetric representation of group D_2. Rotation states of the required symmetry will not exist if $J = 1$. Two such states will exist for $J = 2$, one for $J = 3$, three for $J = 4$, two for $J = 6$ etc. The energy of two levels of required symmetry for $J = 2$ are defined by the expressions

$$\varepsilon_1(2) = \frac{9\left[1-\sqrt{\left\{1-\frac{8}{9}sin^2(3\gamma_0)\right\}}\right]}{Sin^2(3\gamma_0)} \tag{2.29}$$

$$\varepsilon_1(2) = \frac{9\left[1+\sqrt{\left\{1-\frac{8}{9}sin^2(3\gamma_0)\right\}}\right]}{Sin^2(3\gamma_0)} \tag{2.30}$$

Energy of a level for J = 3 is given by

$$\varepsilon(3) = \frac{18}{Sin^2(3\gamma_0)} \tag{2.31}$$

The three spin 4 energy levels are the roots of the third degree equation:

$$\varepsilon^3 - \frac{90}{Sin^2(3\gamma_0)}\varepsilon^2 + \frac{48}{Sin^4(3\gamma_0)}[27 + 16Sin^3(3\gamma_0)]\varepsilon - \frac{640}{Sin^4(3\gamma_0)}[27 + 7Sin^2(3\gamma_0)] = 0 \tag{2.3.2}$$

The two spin 5 energy levels are given by the formula

$$E_\tau(5) = \frac{[45 \pm \sqrt{(9-8sin^2(3\gamma_0)}]}{sin^2 3\gamma_0} \qquad \text{where } \boldsymbol{\tau} = 1,2 \tag{2.33}$$

where $\boldsymbol{\tau} = 1$ with negative sign and $\boldsymbol{\tau} = 2$ with positive sign. For $\gamma_0 = 0$ the energy spectrum is identical to that of an axially-symmetric nucleus. For a fixed value of β violation of axial symmetry of the nucleus leads to an increase energy levels belonging to the axial nucleus. This increase in the energy levels corresponds to a decrease of the effective moment of inertia of the nucleus or the effective deformation parameter β_{eff}. For the first excited state of spin 2 the effective deformation parameter can be determined as

$$\beta_{eff} = \left[\frac{4sin^2(3\gamma_0)}{9-\sqrt{(81-72sin^2(3\gamma_0))}}\right] \tag{2.34}$$

The small change of the level energies of an axially symmetric nucleus, violation of axial symmetry of the nucleus leads to the appearance of some new energy levels $\varepsilon_2(2)$, $\varepsilon_2(3)$, $\varepsilon_2(4)$ etc. By using the dependence of $\varepsilon_2(2)$, $\varepsilon_1(2)$ on γ_0 one can determine the corresponding value of γ_0 from the experimental value of the ratio.

2.4 INTERACTING BOSON MODEL

As we move away from closed nuclei, proton and neutron number increases the shell model basis states increases and calculations and explanation becomes complicated. Using quadrupole interactions, basis states reduce and calculations become simple.

Feshbach & Iachello (1973, 1974) described some properties of light mass nuclei in terms of interacting boson. Whereas, Janseen et al. (1974) described the collective quadrupole in terms of SU(6). Arima and Iachello (1975) added s-boson to the d-boson collectively to explain the structure of nuclei as a boson treated as nucleon pair & gives microscopic explanation of collective quadrupole states with large theoretical information Iachello & Arima (1987) and Bonatsos (1989).

Collective excitation of nuclei is explained by boson on the basis of boson creation and annihilation operator of multi-polarity l and z component of m, $b^{\dagger}_{l,m}$ and $b_{j,m}$. On the basis of boson operator, boson model is explained. The low lying collective states of nuclei described in the form of monopole boson having angular momentum an parity $J^{\pi}=0^{+}$ as s-boson and quadrupole boson with $J^{\pi}=2^{+}$ called as d-boson.

$s^{\dagger}d_{\mu}$ (μ= 0, ±1, ±2) (2.35)

$s\ d_{\mu}$ (μ= 0, ±1, ±2) (2.36)

Above relation is following Bose communication relation as:

$$[s\ s^{\dagger}]=1 \quad (2.37)$$

and

$$[d_{\mu}\ d^{\dagger}_{\mu}]=\delta_{\mu\mu'} \quad (2.38)$$

Spherical tensor T^{k}_{k} is created with Boson operator, that transformed basis vectors of (2k+1) dimensions, give Clebsch Gorden Coefficients with product of two operators. Total boson number N is the sum of number of s and d-bosons. i. e., $N= n_s+n_d$, which is conserved.

The Hamiltonian is used to obtain the information about spectrum, which is a combination of energy term (E_0), one and two body interactions term, here creation

operator is equal to the annihilation operator. The Hamiltonian is hermition operator $H^{\dagger}=H$.

$$H = E_0 + \sum \epsilon_s (b_l^{\dagger}.b_l) + \sum \left(\frac{1}{2}\right) u^{L}_{l,l',l'',l'''} \left[\left(b_l^{\dagger} \times b_l^{\dagger}\right)^{L} \times (b_l \times b_l)^{L} \right]^{0}_{0} + \cdots \ldots \ldots \quad (2.39)$$

The H can also expressed as

$$H = E_0 + \Sigma \epsilon s(s^{\dagger}.s) + \Sigma \epsilon_d (d^{\dagger}.d) + \Sigma (1/2)(2L+1)^{\frac{1}{2}} c_L [(d^{\dagger} \times d^{\dagger})^{L} \times (d \times d)^{L}]^{0}_{0} + \frac{v_2}{\sqrt{2}} [\{(d^{\dagger} \times d^{\dagger})^{2} \times (d \times s)^{2}\} + \{(d^{\dagger} \times s^{\dagger})^{2} \times (d \times d)^{2}\}]^{0}_{0} + \frac{v_2}{2} [\{(d^{\dagger} \times d^{\dagger})^{0} \times ((s \times s)^{0}\} + \{(s^{\dagger} \times s^{\dagger})^{0} \times (d \times d)^{0}\}]^{0}_{0} + u_2 [(d^{\dagger} \times s^{\dagger})^{2} \times (d \times s)^{2}]^{0}_{0} + \frac{u_2}{2} [(s^{\dagger} \times s^{\dagger})^{0} \times (s \times s)^{2}]^{0}_{0} \quad (2.40)$$

It consist of two one body terms and seven two body terms c_L(L= 0, 2, 4), v_L(L= 0, 2) and u_L(L= 0, 2).

Electromagnetic transition of multi-polarity in the forms of s and d boson one body interaction is written as.

$$T_0^{E2} = \gamma_0 + \alpha_0 [(s^{\dagger} \times s)]^{0}_{0} + \beta_0 [(d^{\dagger} \times d_l)]^{0}_{0} \quad (2.41)$$

$$T_u^{M1} = \beta_1 [(d^{\dagger} \times d)]^{1}_{u} \quad (2.42)$$

$$T_0^{E2} = \alpha_2 [(d^{\dagger} \times s) + (s^{\dagger} \times d)]^{2}_{u} + \beta_2 [(d^{\dagger} \times d)]^{2}_{u} \quad (2.43)$$

$$T_u^{M3} = \beta_3 [(d^{\dagger} \times d)]^{3}_{u} \quad (2.44)$$

$$T_u^{E4} = \beta_4 [(d^{\dagger} \times d)]^{4}_{u} \quad (2.45)$$

The s and d boson have positive polarity. Operator with multi-polarity one has positive parity as a M1 operator and for negative parity E1 operator. One can construct transition operator with multi polarity four. The constant γ_0, α_L(L=0,2) and β_L(L=0,1,2,3,4) are parameters magnitude and scale of corresponding operator. The cubic term in Hamiltonian consist of three creations and three annihilation operator.

The Hamiltonian is also written as:

$$H = \varepsilon'' n_d + a_0 P^{\dagger} P + a_1 L^2 + a_2 Q^2 + a_3 T_3^{\,2} + a_4 T_4^{\,2} \quad (2.46)$$

Here P, L, Q, T_3 and T_4 are as pairing, angular momentum, quadrupole, octupole and hexadecapole operator. For microscopic calculation same Q is used in transition operator and in Hamiltonian and is known as Q- formalism.

The energy of nuclear states and reduced transition probability in the interacting boson model is calculated by PHINT program which is written by Scholten (1976) where coefficients of Hamiltonian correspond to input parameter as

ε''=EPS (2.47)

a_0= 2 PAIR (2.48)

a_1=ELL/2 (2.49)

a_2=QQ/2 (2.50)

a_3=5 OCT (2.51)

a_4=5 HEX (2.52)

The boson creation ($s^\dagger$ or $d^\dagger$) and annihilation (s or d) operator gives a set two linear operator. As $G_{\alpha\beta}=b_\alpha^\dagger b_\beta$ for (α, β=1, 2,….6) gives 36 operator which satisfies communication unitary algebra in six dimension U(6). The group of transformations is related with each relation. During communication constant is equal to one or zero, called as lie structure constant. Using Racah approach, G operator is as

$G_k^k = \left(b_l^\dagger \times b_l^\dagger\right)_k^k$ for l, l'=0, 2. In terms of s and d boson operator G expanded as $G_0^0(s,s)$, G_0^0 (d,d), G_u^1 (d,d), G_u^2 (d,d), G_u^3 (d,d), G_u^4 (d,d), G_u^2 (d,s) and G_0^2 (s,s) gives 1, 1, 3, 5, 7, 9, 5 and 5 components respectively. The U(6) algebra is classified into three sub group U(5), SU(3) and O(6).

2.4.1 Sub-group U(5)

The operators G_0^0d,d), G_u^1 (d,d), G_u^2 (d,d), G_u^3 (d,d) and G_u^4 (d,d) gives 25 component as a O(5) group. The operator G_u^1 (d,d) gives 3 component s a O(3). The operator G_u^3 (d,d) gives one component as a O(2) rotation group. The chain of Boson subalgebra is U(6) ⊃ U(5) ⊃ O(5) ⊃ O(3) ⊃ O(2). The quantum number for the chain is $N, nd, (v, n\Delta), L$ and M_L.

2.4.2 Sub-group SU(3)

Boson sub algebra II consist 9 components having a linear combination of G_0^0, G_u^1, G_u^2 for s and d pair with 1,3 and 5 component. The G_u^2 term is proportional to the electrical quadrupole operator gives information about deformations of the

nucleus with positive (negative) quadrupole moments. The operator $G_u^1$and G_u^2 give sub group SU(3) of U(6) having 3 and 5 components. The G_u^1 give sub-group O(3) of SU(3) with 3 components and G_0^1gives 1 component as a O(2) rotation group. The chin of boson sub-algebra II is is U(6) ⊃ U(5) ⊃ O(3) ⊃ O(2). The quantum number for the chain is $N, (\lambda, v)\chi$, L and M_L.

2.4.3 Sub-group O(6)

Boson sub-algebra III consistG_u^1, G_u^3 and G_u^2 having 3,7 and 5 terms as a sub group O(6) of U(6) with 15 terms. The operators G_u^1andG_u^3gives 10 terms as a sub-group O(5). The operator G_u^1gives 3 terms as a sub-group O(3) and G_u^1gives one term as a sub-group O(2). The chain of boson sub-algebra is U(6) ⊃ O(6) ⊃ O(5) ⊃ O(3) ⊃ O(2). The quantum number for the chain is $N, \sigma, (\tau, v\Delta), L \text{ and } M_L$.

The Hamiltonian in the form of Casimier operators gives energy spectrum, electric quadrupole operator for each symmetry with individual properties. Most of the nuclei do not show properties as like symmetry exactly. Then Hamiltonian H can be written by the operator of two chains. The classification of nuclei as (shown in Fig. 2.3):

1. Class A, nuclei with properties intermediate between I and II.
2. Class B, nuclei with properties intermediate between II and III.
3. Class C, nuclei with properties intermediate between III and I.
4. Class D, nuclei with properties intermediate between all three limits.

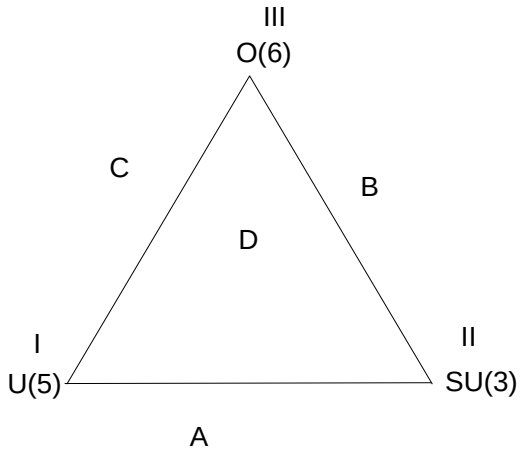

Fig. 2.3 Casten's symmetry triangle.

2.5 VARIOUS INDEPENDENT PARAMETERS

2.5.1 NpNn Product

It is the product of number of valence protons Np and the number of valence neutrons Nn. On taking it as independent parameter; we studied the variation of other dependent quantities on NpNn product.

2.5.2 P-factor

P-factor defined as it is the ratio of product of Np and Nn to the sum of number of valence proton (Np) and the number of valence neutrons (Nn). It is the normalised value of NpNn. It is represented by:

$$P=\frac{NpNn}{Np+Nn}$$

2.5.3Energy Ratio (R_4)

It is the ratio of energy of (4^+_1) and (2^+_1) levels of ground state bands. For vibration nuclei, it lies from $2 \leq R_4 \leq 2.4$, for transitional nuclei it lies as $2.4 \leq R_4 \leq 3$ and rotational nuclei it lies as $3 \leq R_4 \leq 3.33$.This ratio is also observed with other calculated quantities.