CHAPTER 5

RELIABILITY ANALYSIS OF PAPER MANUFACTURING PLANT USING BOOLEAN FUNCTION WITH FUZZY LOGIC TECHNIQUE

5.1 INTRODUCTION

Reliability analysis is benefited to the industry in terms of higher productivity and lower maintenance cost. This can also help the management to understand the effect of decreasing or increasing the repair rates of a particular component or subsystem in the overall system. Several researchers have discussed during the past many years, the various facts of reliability technology of the subsystems or systems in process industries at various levels and a number of research papers have been published in this direction including Singh et al. [170] who have given reliability of a fertilizer production supply problem. Gupta et al. [37] have derived a numerical analysis of reliability and availability of the serial processes in a butter oil manufacturing plant.

The longevity of a component is assumed to be a random variable. The probability distribution of the random variable has crisp parameters. In many situations, determination of the parameters is different as a result of uncertainties and imprecision in data. So, the assumptions of the parameters are fuzzy in nature. In this chapter, the longevity and repair times of components are assumed to have Weibull distribution with fuzzy parameters.

Paper making process is a highly complex one involving many unit operations and processes; consisting of seven subsystems. The first subsystem is the chipping unit taking input in form of raw material. Cutters start cutting into fine small pieces and then the pieces are sent to the second subsystem viz. feeding. After that it goes to the third subsystem, namely, pulp preparation unit, which has an identical unit digester for standby redundancy. The pieces are then crushed and converted into the pulp by mixing with fresh water. The pulp goes to the fourth subsystem, the washing unit, which has an indistinguishable unit Decker for standby redundancy. After the filtration, the pulp goes to the screening unit which has a similar unit cleaner in parallel redundancy. The impurities are then removed and purification is completed by the bleaching process which is connected in series.

Finally, the pulp goes to the last subsystem which is the paper production or finished product. All the subsystems are connected in series as shown in Figure 5.1. The reliability concepts have been applied in various technological fields during the last five decades. This technique has been applied to a number of industrial and transportation problems too, and the detailed discussion is available in Dhillon et al. [10].

Assessment of reliability analysis of paper manufacturing plant with fuzzy logic technique has been carried out in this chapter. Fuzzy Weibull distribution and longevity of component is also determined. The paper plant is a very complex system under consideration consisting of various subsystems such as chipping, feeding, pulp preparation, washing, bleaching, screening and paper production etc. The mathematical model has been developed based on Boolean function technique. Reliability of the paper manufacturing plant is ascertained in three different cases. Reliability of each component is discussed as when all failures follow fuzzy Weibull distribution and when all repairs follow an exponential distribution and the mean time for failure is also determined.

Finally, a numerical analysis is given to illustrate the characteristics of fuzzy reliability and their α - cuts.

Weibull distribution is proved to be a flexible and a versatile among the various other distributions describing the data in failure rate is monotonic in nature. Nevertheless, many modern complex systems exhibit unimodal or bathtub shaped failure rate. The Weibull distribution presented by Cai et al. [171] seems to be inadequate approach as it gives the possibility assumption and fuzzy state assumption for replacement of the probability and binary state assumptions. Cai et al. [172] have given an introduction to engineering system failure and the use of fuzzy methodology. Sehik Uduman et al. [173] have discussed the mathematical modelling and digestive system of a paper making plant based on queuing theory. Karpisek et al. [174] have described two fuzzy reliability models on the basis of the fuzzy Weibull distribution. Baloui [175] has evaluated reliability function using fuzzy exponential lifetime distribution. Sehik Uduman et al. [176] have discussed a mathematical modelling and performance analysis of stock preparation unit in paper plant industry using a genetic algorithm. In this chapter, it is proposed a general procedure for the construction of the reliability characteristics and its α-cuts set, when the parameters are fuzzy. The parameter of the system is represented by a trapezoidal fuzzy number. Agarwal et al. [177] have analyzed the reliability analysis assessment of sugar manufacturing plant using Boolean function technique.

5.2 ASSUMPTIONS

a) To start with, the complete system is in operable state.

b) The components of failure rates are s - independent.

c) There is total absence of facility for failed component.

d) Reliability of each component is known in advance.

e) This system is in a fuzzy arrangement.

5.3 NOTATIONS

θ_1	-	State of chipping unit
θ_2	-	State of feeding unit
θ_3	-	State of pulp preparation unit
θ_4	-	Digester
θ_5	-	State of washing unit
θ_6	-	Decker
θ_7	-	State of screening unit
θ_8, θ_9	-	Cleaners
θ_{10}	-	State of bleaching unit
θ_{11}	-	Finished product / Paper mills.
$\theta_i (i = 1,2,---,11)$	-	1 in good state 0 in bad state
$\tilde{\theta}$	-	Fuzzy number
$\tilde{S}(t)$	-	Fuzzy reliability functions of a component with time t
$\xi_{\tilde{\theta}}$	-	Membership functions of a fuzzy number
$\wedge / \vee$	-	Conjunction/Disjunction

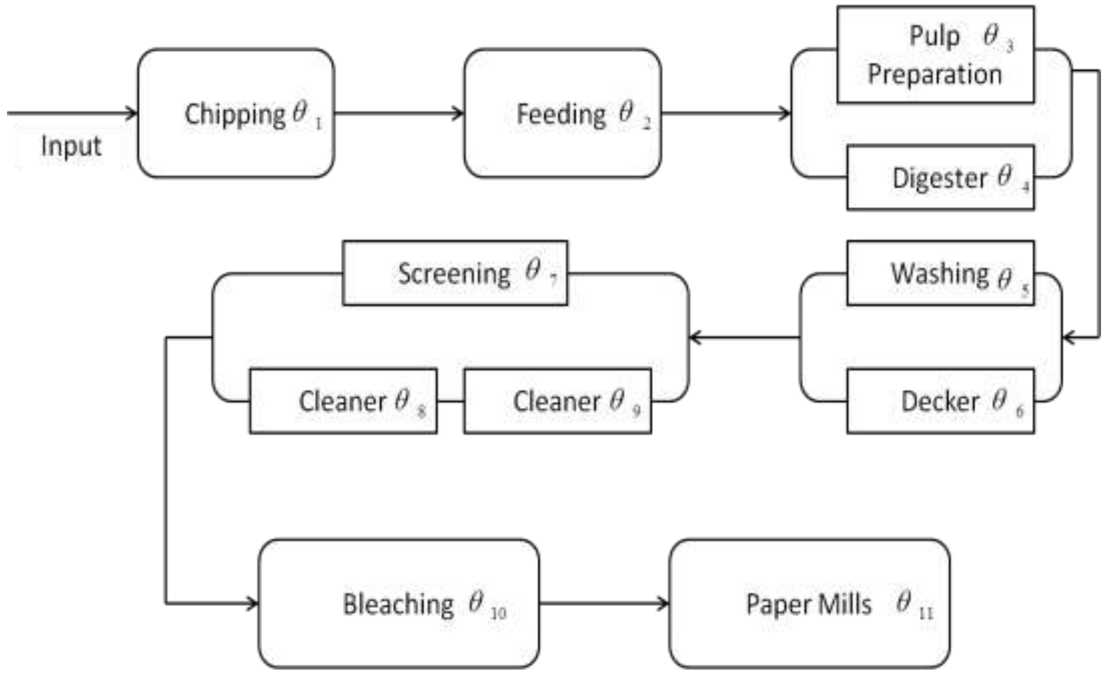

Figure 5.1: Flow of paper milling process

5.4 MATHEMATICAL FORMULATION

Through use of the Boolean function technique, the conditions of capability for the successful operation of the system in terms of logical matrix are expressed as:

$$F(\theta_1,\theta_2,---,\theta_{11}) = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \theta_5 & \theta_7 & \theta_{10} & \theta_{11} & \\ \theta_1 & \theta_2 & \theta_3 & \theta_5 & \theta_8 & \theta_9 & \theta_{10} & \theta_{11} \\ \theta_1 & \theta_2 & \theta_3 & \theta_6 & \theta_7 & \theta_{10} & \theta_{11} & \\ \theta_1 & \theta_2 & \theta_3 & \theta_6 & \theta_8 & \theta_9 & \theta_{10} & \theta_{11} \\ \theta_1 & \theta_2 & \theta_4 & \theta_5 & \theta_7 & \theta_{10} & \theta_{11} & \\ \theta_1 & \theta_2 & \theta_4 & \theta_5 & \theta_8 & \theta_9 & \theta_{10} & \theta_{11} \\ \theta_1 & \theta_2 & \theta_4 & \theta_6 & \theta_7 & \theta_{10} & \theta_{11} & \\ \theta_1 & \theta_2 & \theta_4 & \theta_6 & \theta_8 & \theta_9 & \theta_{10} & \theta_{11} \end{bmatrix} \tag{5.1}$$

By the application of algebra of logic, (5.1) may be written as,

$$F(\theta_1,\theta_2,---,\theta_{11}) = [\theta_1\theta_2\theta_{10}\theta_{11} \wedge f(\theta_1,\theta_2,---,\theta_{11})] \tag{5.2}$$

Where,

$$F(\theta_1,\theta_2,---,\theta_{11}) = \begin{bmatrix} \theta_3 & \theta_5 & \theta_7 & \\ \theta_3 & \theta_5 & \theta_8 & \theta_9 \\ \theta_3 & \theta_6 & \theta_7 & \\ \theta_3 & \theta_6 & \theta_8 & \theta_9 \\ \theta_4 & \theta_5 & \theta_7 & \\ \theta_4 & \theta_5 & \theta_8 & \theta_9 \\ \theta_4 & \theta_6 & \theta_7 & \\ \theta_4 & \theta_6 & \theta_8 & \theta_9 \end{bmatrix} \tag{5.3}$$

Then,

$$A_1 = [\theta_3 \quad \theta_5 \quad \theta_7] \tag{5.4}$$

$$A_2 = [\theta_3 \quad \theta_5 \quad \theta_8 \quad \theta_9] \tag{5.5}$$

$$A_3 = [\theta_3 \quad \theta_6 \quad \theta_7] \tag{5.6}$$

$$A_4 = [\theta_3 \quad \theta_6 \quad \theta_8 \quad \theta_9] \tag{5.7}$$

$$A_5 = [\theta_4 \quad \theta_5 \quad \theta_7] \tag{5.8}$$

$$A_6 = [\theta_4 \quad \theta_5 \quad \theta_8 \quad \theta_9] \tag{5.9}$$

$$A_7 = [\theta_4 \quad \theta_6 \quad \theta_7] \tag{5.10}$$

$$A_8 = [\theta_4 \quad \theta_6 \quad \theta_8 \quad \theta_9] \tag{5.11}$$

a) FUZZY WEIBULL DISTRIBUTION

The Weibull distribution is broadly used in reliability engineering and the probability density function is given by,

$$f(x) = \frac{\beta}{\theta}\left(\frac{x-\delta}{\theta}\right)^{\beta-1} e^{-\left(\frac{x-\delta}{\theta}\right)^{\beta}} \tag{5.12}$$

Where θ is scale parameter, β is shape parameter and δ is location parameter and they are crisps in nature.

b) FUZZY RELIABILITY FUNCTION

Fuzzy reliability is on the basis of fuzzy set theory of Zadeh [7]. The fuzzy reliability (i.e. fuzzy survival) function $(\tilde{S}(t))$ is the fuzzy probability where a unit survives beyond time t. Assuming the random variable X denotes the longevity of a system component also X as having a density function and fuzzy cumulative distribution function $\tilde{F}_X(t) = \tilde{P}(X \le t)$. Where parameter $\tilde{\theta}$ is a fuzzy number and then the fuzzy reliability function at time t is defined as,

$$\tilde{S}(t) = \tilde{P}(X > t) = 1 - \tilde{F}_X(t) = \{[1 - F_{\max}(x)[\alpha], 1 - F_{\min}(x)[\alpha]], \mu_{F(x)} = \alpha\}, t > 0 \quad (5.13)$$

When the calculation of the reliability of a component is desired, such that the lifetime random variable has fuzzy Weibull distribution, parameter $\tilde{\theta}$ is represented with a trapezoidal fuzzy number as $\tilde{\theta} = (a_1, a_2, a_3, a_4)$ to enable the description of a membership function $\xi_{\tilde{\theta}}(x)$ in the following manner.

$$\xi_{\tilde{\theta}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & , \quad a_1 \le x < a_2 \\ 1 & , \quad a_2 \le x < a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & , \quad a_3 \le x < a_4 \end{cases} \quad (5.14)$$

The α -cuts $\tilde{\theta}$ is denoted as follows,

$$\tilde{\theta}[\alpha] = [a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha] \quad (5.15)$$

Then the fuzzy reliability function of a component is,

$$\tilde{S}(t)[\alpha] = \left\{ \int_c^d \frac{\beta}{\theta}\left(\frac{x-\delta}{\theta}\right)^{\beta-1} e^{-\left(\frac{x-\delta}{\theta}\right)^{\beta}} dx \mid \theta \in \tilde{\theta}[\alpha] \right\} = \left\{ e^{-\left(\frac{t-\delta}{\theta}\right)^{\beta}} \mid \theta \in \tilde{\theta}[\alpha] \right\} \quad (5.16)$$

According to that the $e^{-\left(\frac{t-\delta}{\theta}\right)^{\beta}}$ increasing θ, then the α -cuts of fuzzy reliability function is as,

$$\tilde{S}(t)[\alpha]=\left[\exp\left\{-\left(\frac{t-\delta}{a_1+(a_2-a_1)\alpha}\right)^{\beta}\right\},\exp\left\{-\left(\frac{t-\delta}{a_4-(a_4-a_3)\alpha}\right)^{\beta}\right\}\right] \tag{5.17}$$

$\tilde{S}(t)[\alpha]$ is two dimensional function in terms of α and t ($0\le\alpha\le 1$ and $t>1$), for t_0 is a fuzzy trapezoidal number and membership function of $\tilde{S}(t_0)$ is as follows,

$$\xi_{\tilde{S}(t_0)}(x)=\begin{cases}\dfrac{x-\exp\left\{-\left(\frac{t_0-\delta}{a_1}\right)^{\beta}\right\}}{\exp\left\{-\left(\frac{t_0-\delta}{a_2}\right)^{\beta}\right\}-\exp\left\{-\left(\frac{t_0-\delta}{a_1}\right)^{\beta}\right\}}, & \exp\left\{-\left(\frac{t_0-\delta}{a_1}\right)^{\beta}\right\}\le x<\exp\left\{-\left(\frac{t_0-\delta}{a_2}\right)^{\beta}\right\}\\ 1, & \exp\left\{-\left(\frac{t_0-\delta}{a_2}\right)^{\beta}\right\}\le x\le\exp\left\{-\left(\frac{t_0-\delta}{a_3}\right)^{\beta}\right\}\\ \dfrac{\exp\left\{-\left(\frac{t_0-\delta}{a_4}\right)^{\beta}\right\}-x}{\exp\left\{-\left(\frac{t_0-\delta}{a_4}\right)^{\beta}\right\}-\exp\left\{-\left(\frac{t_0-\delta}{a_3}\right)^{\beta}\right\}}, & \exp\left\{-\left(\frac{t_0-\delta}{a_3}\right)^{\beta}\right\}<x\le\exp\left\{-\left(\frac{t_0-\delta}{a_4}\right)^{\beta}\right\}\end{cases}$$

(5.18)

Fuzzy mean time to failure (FMTTF) is the expected time of failure. According to the classification of Buckley [178], FMTTF of this fuzzy system is a fuzzy number and can be calculated as follows,

$$M\tilde{T}TF[\alpha]=\left\{\int_0^{\infty}xf(x)dx\mid\theta\in\tilde{\theta}[\alpha]\right\}=\left\{\int_0^{\infty}S(t)dt\mid\theta\in\tilde{\theta}[\alpha]\right\} \tag{5.19}$$

When the failure random variable has fuzzy Weibull distributed then,

$$\begin{aligned}M\tilde{T}TF[\alpha]&=\left\{\theta\Gamma(1+\beta^{-1})\mid\theta\in\tilde{\theta}[\alpha]\right\}\\&=[(a_1+(a_2-a_1)\alpha)\Gamma(1+\beta^{-1}),(a_4-(a_4-a_3)\alpha)\Gamma(1+\beta^{-1})]\end{aligned} \tag{5.20}$$

Accordingly (5.20), the membership function is obtained in the following way,

$$\xi_{\tilde{S}(t_0)}(x)=\begin{cases}\dfrac{x-a_1\Gamma(1+\beta^{-1})}{(a_2-a_1)\Gamma(1+\beta^{-1})} & , \quad a_1\Gamma(1+\beta^{-1})\le x< a_2\Gamma(1+\beta^{-1})\\ 1 & , \quad a_2\Gamma(1+\beta^{-1})\le x\le a_3\Gamma(1+\beta^{-1})\\ \dfrac{a_1\Gamma(1+\beta^{-1})-x}{(a_4-a_3)\Gamma(1+\beta^{-1})} & , \quad a_3\Gamma(1+\beta^{-1})\le x< a_4\Gamma(1+\beta^{-1})\end{cases}$$

(5.21)

c) FUZZY HAZARD FUNCTION

In fuzzy reliability theory, the fuzzy hazard function plays very significant role. The concept of a fuzzy hazard function is on the basis of the fuzzy probability measure and calls its α-cuts is proposed. The fuzzy conditional probability of an item is the fuzzy hazard function $\tilde{h}(t)$ falling in the interval t to $(t+dt)$ and it has not failed by time t. Hazard function is also known as the instant failure rate. Mathematically, the fuzzy hazard function is defined as:

$$\begin{aligned}\tilde{h}(t)[\alpha] &= \lim_{\Delta t\to 0}\frac{\tilde{P}(t<X<t+\Delta t \mid X>t)}{\Delta t}\\ &= \left\{\lim_{\Delta t\to 0}\frac{S(t)-S(t+\Delta t)}{\Delta t S(t)} \mid \theta\in\tilde{\theta}[\alpha]\right\} \qquad (5.22)\\ &= \left\{\frac{-S'(t)}{S(t)} \mid \theta\in\tilde{\theta}[\alpha]\right\}=\left\{\frac{f(t)}{S(t)} \mid \theta\in\tilde{\theta}[\alpha]\right\}.\end{aligned}$$

Fuzzy Weibull distribution with $\delta=0$ has fuzzy hazard function and it defined as,

$$\tilde{h}(t)[\alpha]=\left\{\frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \mid \theta\in\tilde{\theta}[\alpha]\right\} \qquad (5.23)$$

Where $\tilde{h}(t)[\alpha]$ is a two dimensional function in terms of α and t $(0\le\alpha\le 1$ and $t>1)$.

5.5 NUMERICAL ILLUSTRATION

Assuming the longevity of the component as modeled by a Weibull distribution with fuzzy parameter $\tilde{\theta}$ and $\delta = 0$ that $\tilde{\theta}$ = (1.25, 1.30, 1.45 and 1.60). Then α -cuts of fuzzy reliability function, fuzzy hazard function and fuzzy mean time to failure are given by

a) Fuzzy reliability is:

$$\tilde{\theta}[\alpha] = (1.25 + 0.05\alpha,\ 1.60 - 0.15\alpha),$$
$$\tilde{S}(t)[\alpha] = \left[\exp\left\{ -\frac{t}{1.25 + .005\alpha} \right\}^{\beta}, \exp\left\{ -\frac{t}{1.60 - .002\alpha} \right\}^{\beta} \right] \quad (5.24)$$

$$\tilde{S}(t)[\alpha] = \left[\exp\left\{ -\left(\frac{t-\delta}{1.25 + 0.05\alpha} \right)^{\beta} \right\},\ \exp\left\{ -\left(\frac{t-\delta}{1.60 - 0.15\alpha} \right)^{\beta} \right\} \right] \quad (5.25)$$

For all α ,

Case 1: If $t = 0.5$ then reliability is,

$$\xi_{\tilde{S}(0.5)}(x) = \begin{cases} \dfrac{x - \exp\{-(0.4)^{\beta}\}}{\exp\{-(0.3846)^{\beta}\} - \exp\{-(0.4)^{\beta}\}} & , \exp\{-(0.4)^{\beta}\} \le x < \exp\{-(0.3846)^{\beta}\} \\ 1 & , \exp\{-(0.3846)^{\beta}\} \le x \le \exp\{-(0.3125)^{\beta}\} \\ \dfrac{\exp\{-(0.3125)^{\beta}\} - x}{\exp\{-(0.3125)^{\beta}\} - \exp\{-(0.3448)^{\beta}\}} & , \exp\{-(0.3448)^{\beta}\} < x \le \exp\{-(0.3125)^{\beta}\} \end{cases}$$

(5.26)

Case 2: If $\alpha = 0$ then reliability is,

$$\tilde{S}(t)[0] = \left[\exp\{-(0.8)^{\beta}\},\ \exp\{-(0.625t)^{\beta}\} \right] \quad (5.27)$$

b) Fuzzy hazard function is:

$$\tilde{h}(t)[\alpha] = \left\{ \frac{\beta}{1.60 - 0.15\alpha} \left(\frac{t}{1.60 - 0.15\alpha} \right)^{\beta - 1},\ \frac{\beta}{1.25 + 0.05\alpha} \left(\frac{t}{1.25 + 0.05\alpha} \right)^{\beta - 1} \right\} \quad (5.28)$$

c) Fuzzy mean time to failure is:

$$\begin{aligned} M\tilde{T}TF[\alpha](\beta) &= \left\{ \theta\Gamma(1 + \beta^{-1}) \mid \theta \in \tilde{\theta}[\alpha] \right\} \\ &= [(1.125 + 0.005\alpha)\Gamma(1 + \beta^{-1}),\ (1.60 - 0.15\alpha)\Gamma(1 + \beta^{-1})] \end{aligned} \quad (5.29)$$

According to the action of the gamma function, the value of FMTTF at β = 2.166 has a minimum value.

$$M\widetilde{T}TF[\alpha](2.166) = [(1.1070 + 0.0443\alpha),\ (1.4170 - 0.0443\alpha)] \tag{5.30}$$

Accordingly the following membership function is obtained:

$$\xi_{\tilde{S}(t_0)}(x) = \begin{cases} \dfrac{x - 1.1070}{0.0443} & , \quad 1.1070 \le x < 1.1513 \\ 1 & , \quad 1.1513 \le x \le 1.2841 \\ \dfrac{1.4170 - x}{0.1328} & , \quad 1.2841 \le x < 1.4170 \end{cases} \tag{5.31}$$

5.6 CONCLUSION

This chapter is focused on the investigation of Weibull distribution, fuzzy reliability function, fuzzy hazard function and their α-cuts. The approach of reliability theory which is purely based on some of the conventional statistical methods seems to be inappropriate whenever the longevity of components and parameters have randomness and fuzziness respectively. Reliability of fuzzy system is based on the concept of fuzzy set while the fuzzy probability theory is considered in our method. In this chapter, the scale parameter is considered as a fuzzy trapezoidal number. In addition, the shape and location parameters can be measured in terms of fuzzy either separately or in combination. Further research is required for the study of some important topics in fuzzy reliability theory such as mean residual life.