CHAPTER 4

MATHEMATICAL MODELLING AND BEHAVIOUR OF DIGESTING SYSTEM OF PAPER PLANT BASED ON QUEUEING THEORY

4.1 INTRODUCTION

Paper, a product of major significance is essentially a product of forestry finding extensive applications in society. The palpable uses of paper are in writing and publishing. More than that, paper is a byproduct for the pulp and paper industry, which help in the manufacture of various chemicals. The manufacture of paper involves pulping wood, followed by bleaching and spreading out into sheets. Chemicals are used during the manufacturing process for certain specific purposes that include bleaching (whitening the paper) for giving specific colour "kraft pulping". This involves a combination of heat, chemical and mechanical process for the conversion of the wood into a smooth and soft pulp which is suitable for paper making.

This chapter deals with the action of the digestive system of a paper plant based on queuing theory. Several methods are used for describing the behavior of various parts of the digestive system of paper plant on the basis of linear and non linear circuits. There are four subsystems in the digestive system of a paper plant arranged in series and parallel. The mathematical local balance equations have been developed using Markov state diagram and then solved it by using normalizing condition. The action of each part of the digestive system has been deduced, SLAM – II can also be used for simulating the model. Therefore, the research finding of this work is highly useful to the paper plant management especially with respect to satisfactory maintenance decision in order to enhance the system performance.

Paper industry comprises large complex systems that are arranged in series, parallel or a combination of both the configurations. These systems include stock preparation, chipping, cooking, washing, bleaching, screening and paper production. The process of paper manufacturing is the major factor for paper quality. In paper making process, the chips from storage are introduced into a digester to form the pulp, which later goes through various subsystems called screw feeder, belt conveyor, shuttle conveyor and digester, the flow chart is shown in Figure 4.1.

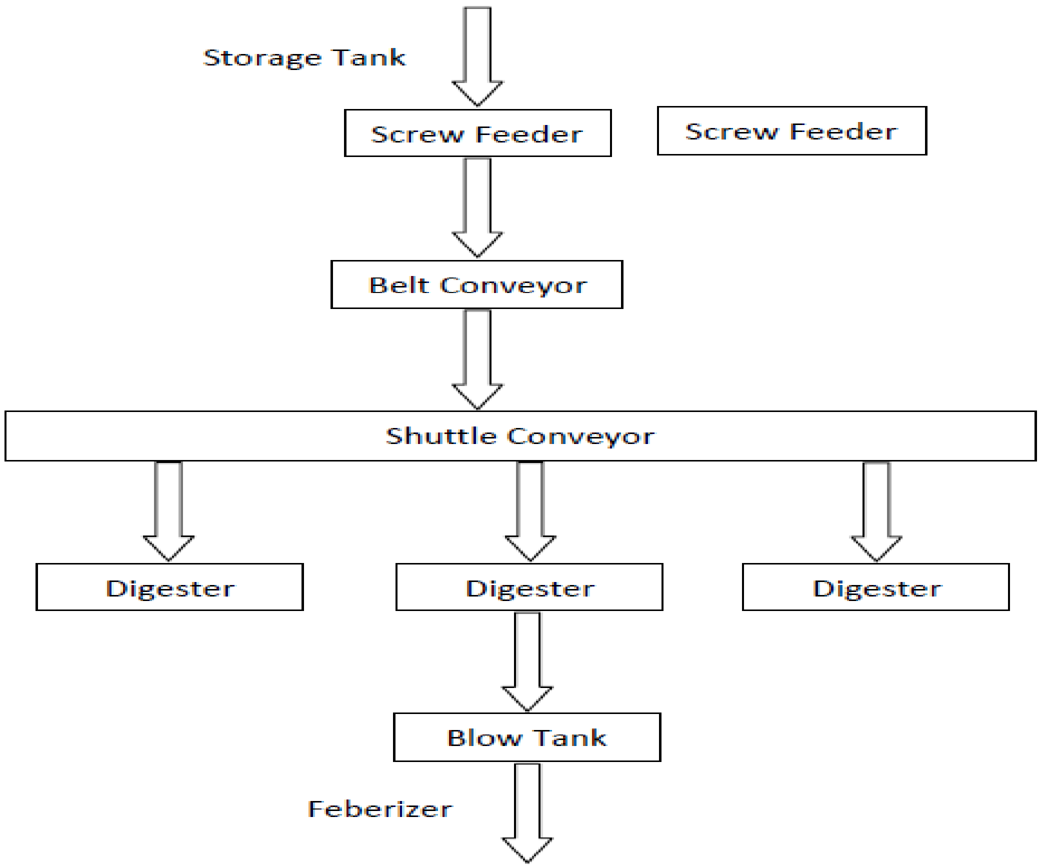

Figure 4.1: Diagram of digestive system of paper plant

In this chapter, a mathematical model has been developed for evaluating the action of the digestive system of a paper plant based on the queuing model and on assumptions. The performance optimization has been done following this technique which provides the optimum system availability levels for different combinations of arrival and processing rates of the digestive system to improve the performance of the paper plant. So, the findings of this work will be highly useful for efficient plant, specifically in maintenance planning and control to enhance the system performance.

4.2 SYSTEM DESCRIPTIONS

The process of paper making comprises of four main subsystems, having the following description:

a) **Subsystem B_1:** It consists of a screw feeder unit. The function of the screw feeder is to extract the wooden chips from storage silos and transfer to a belt conveyor.

b) **Subsystem B_2:** It consists of a belt conveyor unit for carrying the chips.

c) **Subsystem B_3:** It consists of a shuttle conveyor unit to feed the wooden chips from belt conveyor to the digester.

d) **Subsystem B_4:** It comprises of three digester units in parallel for cooking the wooden chips.

4.3 ANALYTICAL AND MATHEMATICAL MODELLING OF DIGESTIVE SYSTEM

The following are the steps for modelling of the digestive system of a paper plant:

a) Drawing a Markov state diagram.

b) Deduction of the local balance equations.

c) Deduction of the general recurrence equations.

d) Solving the recurrence equations and finding the values of each state probability.

e) Calculation of the parameters of each part of the digestive system.

- Idle time.
- Length of queue in each part.
- Waiting time of specific quantity in each part.

The capacity of each part of the digestive system of a paper plant is based on the following assumptions:

a) The capacity of the chipping system is N_1

b) The permissible capacity on an average for the cooking filled by material is N_2 which is equal to 20 N_1.

c) The average capacity of the bleaching system is N_3 which is equal to 30 N_1.

d) The permissible capacity on an average for the screening to be filled by remainder is N_4 which is equal to 30 N_1.

e) Figure 4.2 below is the block diagram of the digestive system of a paper plant.

4.4 FORMULATION OF $M/M/1/N$ QUEUE MODEL

Assumptions are made as, the arrival rate for the queue is α , the service rate is β, the states of the queue is P_i and the capacity of the queue is N, where $i = 0,1,2,\ldots,n$. The block diagram of this queue is given in Figure 4.3.

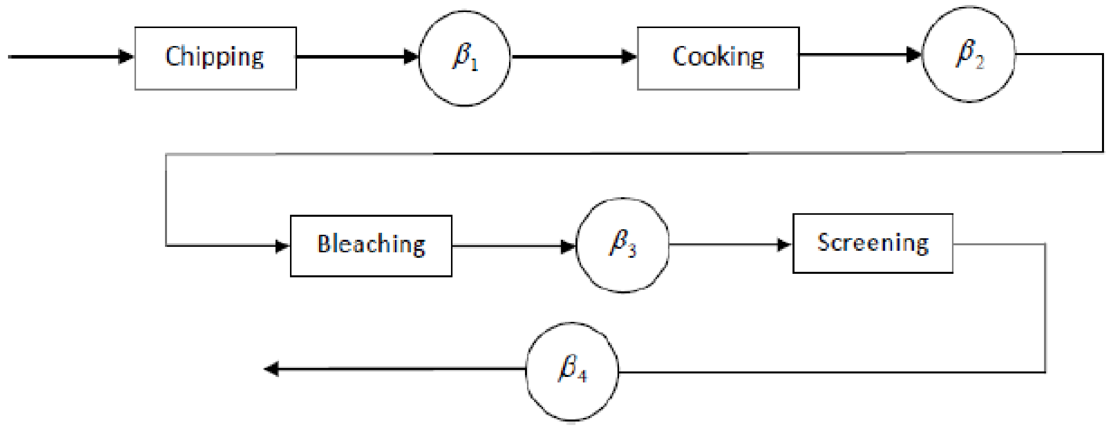

Figure 4.2: Block diagram of the digestive system of paper plant

Figure 4.3: Block diagram for the $M/M/1/N$ queue

The local balance equation of the queue can be deduced on the basis of the above transition block diagram:

For $n = 0,$

$$\beta P_1 - \alpha P_0 = 0 \tag{4.1}$$

For $n = 1,2,3,\ldots,N-1,$

$$-(\alpha + \beta)P_n + \alpha P_{n-1} + \beta P_{n+1} = 0 \tag{4.2}$$

For $n = N,$

$$-\beta P_N + \alpha P_{N-1} = 0 \tag{4.3}$$

Let

$$\phi = \frac{\alpha}{\beta} \tag{4.4}$$

Where ϕ is called the utilization factor.

Solving equations (4.1, 4.2 and 4.3) by considering equation (4.4),

$$P_n = \phi^n P_0 \qquad for \qquad 1 \le n \le N \tag{4.5}$$

Now, by using the normalizing condition i.e. the summation of all probabilities is equal to one, and it can be defined as,

$$\sum_{n=0}^{N} P_n = 1 \tag{4.6}$$

From equation (4.5) and (4.6), it is concluded,

$$P_0 = \frac{1-\phi}{1-\phi^{N+1}} \tag{4.7}$$

From equation (4.5) and (4.7), P_n can be calculated for $n = 1,2,\ldots,N$ as,

$$P_n = \frac{\phi^n(1-\phi)}{1-\phi^{N+1}} \tag{4.8}$$

An average length of the system L is,

$$L = L_s + L_q \tag{4.9}$$

Where, L_s is the expected length in the serving queue and L_q is the expected length in the queue.

Also L can be calculated as,

$$L = \sum_{n\ge 0}^{N} nP_n \tag{4.10}$$

Substituting equation (4.5) into equation (4.10), it is obtained,

$$L = \frac{1-\phi}{1-\phi^{N+1}} - \sum_{n\ge 0}^{N} n\phi^n \tag{4.11}$$

Solving equation 4.5 it is concluded,

$$L = \frac{\phi[1 + N\phi^{N+1} - (N+1)\phi^n]}{(1-\phi)(1-\phi^{N+1})} \tag{4.12}$$

Also L_s and L_q can be calculated as given in the following equations,

$$L_s = 1 - P_0 \tag{4.13}$$

$$L_q = L - L_s \tag{4.14}$$

Substituting equation (4.7) into equation (4.13) then,

$$L_s = \frac{\phi(1-\phi^N)}{1-\phi^{N+1}} \tag{4.15}$$

Substituting equation (4.11) and (4.15) into equation (4.14) then,

$$L_q = \frac{\phi^2[1+(N-1)\phi^N - N\phi^{N-1}]}{(1-\phi)(1-\phi^{N+1})} \tag{4.16}$$

The mean number of material arrival and enter the digestive system per unit time is called α effective α_E which is equal to,

$$\alpha_E = \alpha(1-P_N) \tag{4.17}$$

Therefore,

$$\alpha_E = \frac{\alpha\phi^N(1-\phi)}{(1-\phi^{N+1})} \tag{4.18}$$

By using little's theorem, the average waiting time W_q is calculated as,

$$W_q = \frac{L_q}{\alpha_E} \tag{4.19}$$

4.5 FORMULAS OF DIGESTIVE SYSTEM OF PAPER PLANT USING M/M/1/N QUEUE MODEL

The operation in every part of the digestive system of paper plant can be assumed as exponential distribution. Hence, the equations (4.1 - 4.19) can be applied for a study of the actions of digestive system of a paper plant and can be summarized as follow:

- The utilization factor for the chipping is,

$$\phi_1 = \frac{\alpha_1}{\beta_1} \tag{4.20}$$

- The utilization factor for the cooking is,

$$\phi_2 = \frac{\alpha_1}{\beta_2} \tag{4.21}$$

- The utilization factor for the bleaching is,

$$\phi_3 = \frac{\alpha_1}{\beta_3} \tag{4.22}$$

- The utilization factor for the screening is,

$$\phi_4 = \frac{\alpha_1}{\beta_4} \tag{4.23}$$

The probability of idleness for all parts can be calculated by substituting equations (4.20 - 4.23) into equation (4.7). Also, the probability to be at any state for any part of the digestive system of paper plant can be deduced from equation (4.8).

Independent of the operation of all parts is assumed. Therefore, the probability of idleness of digestive systems can be represented as,

$$P_{01,02,03,04} = P_{01}P_{02}P_{03}P_{04} \tag{4.24}$$

and the probability of any state is represented as,

$$P_{n1,n2,n3,n4} = P_{n1}P_{n2}P_{n3}P_{n4} \tag{4.25}$$

4.6 PERFORMANCE ANALYSIS

Following a detailed discussion with the paper plant management, it is concluded that appropriate arrival and serving rates of all the subsystems are found. Subsequently, the performance analysis is prepared by putting these arrival and serving rate values to form the above four different expressions of the utilization factor. This deals with the qualitative analysis of the factors, e.g. courses of action and the state of actions, which influence the decision associated with maintenance of the plant. Development of these values is done under a real decision making environment, i.e. decision making under a risk model and used for implementing the appropriate maintenance decisions for the paper plant.

Table 4.1: Performance analysis of the subsystem of paper plant

Performance analysis of the subsystem of chipping unit					
β_1 \ α_1	0.05	0.10	0.15	0.20	0.25
0.01	0.20	0.10	0.07	0.05	0.04
0.03	0.60	0.30	0.20	0.15	0.12
0.05	1.00	0.50	0.34	0.25	0.20
0.07	1.40	0.70	0.47	0.35	0.28
0.09	1.80	0.90	0.60	0.45	0.36
Performance analysis of the subsystem of cooking unit					

β_2 \ α_1	0.10	0.20	0.30	0.40	0.50
0.02	0.20	0.10	0.07	0.05	0.04
0.04	0.40	0.20	0.14	0.10	0.08
0.06	0.60	0.30	0.20	0.15	0.12
0.08	0.80	0.40	0.27	0.20	0.16
0.10	1.00	0.50	0.34	0.25	0.20
Performance analysis of the subsystem of bleaching unit					
β_3 \ α_1	0.15	0.30	0.45	0.60	0.75
0.05	0.34	0.17	0.12	0.09	0.07
0.10	0.67	0.34	0.23	0.17	0.14
0.15	1.00	0.50	0.34	0.25	0.20
0.20	1.34	0.67	0.45	0.34	0.27
0.25	1.67	0.84	0.56	0.42	0.34

Performance analysis of the subsystem of screening unit					
β_4 \ α_1	0.20	0.40	0.60	0.80	1.00
0.10	0.50	0.25	0.17	0.13	0.10
0.20	1.00	0.50	0.33	0.25	0.20
0.30	1.50	0.75	0.50	0.38	0.30
0.40	2.00	1.00	0.67	0.50	0.40
0.50	2.50	1.25	0.83	0.63	0.50

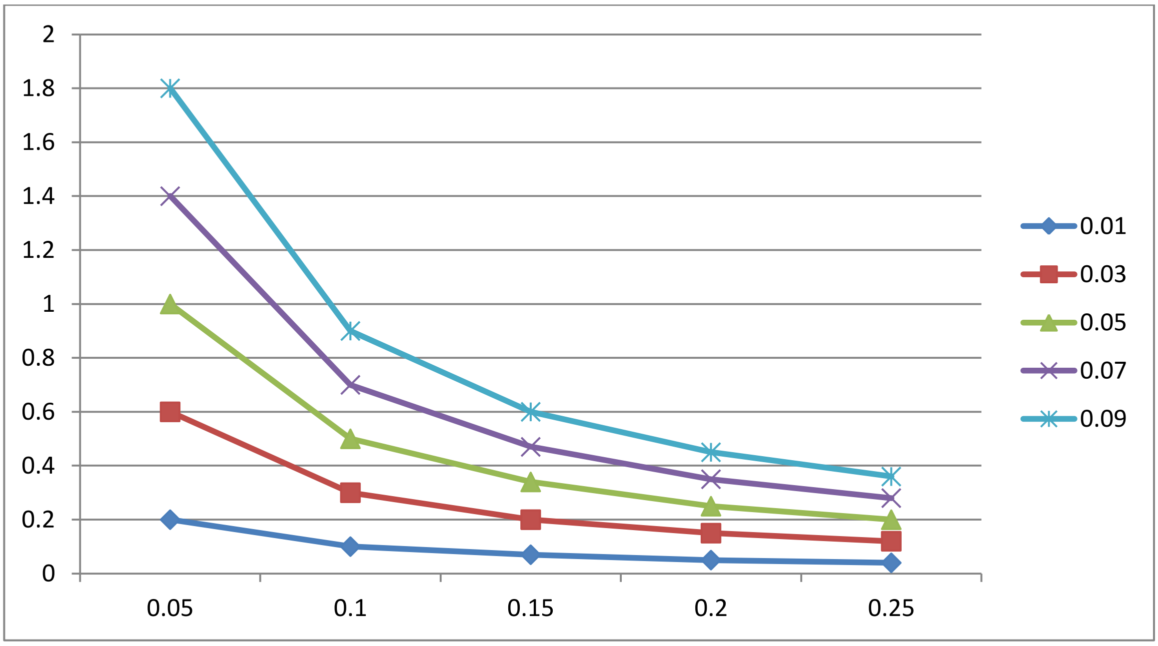

Figure 4.4: Performance analysis of the subsystem of chipping unit

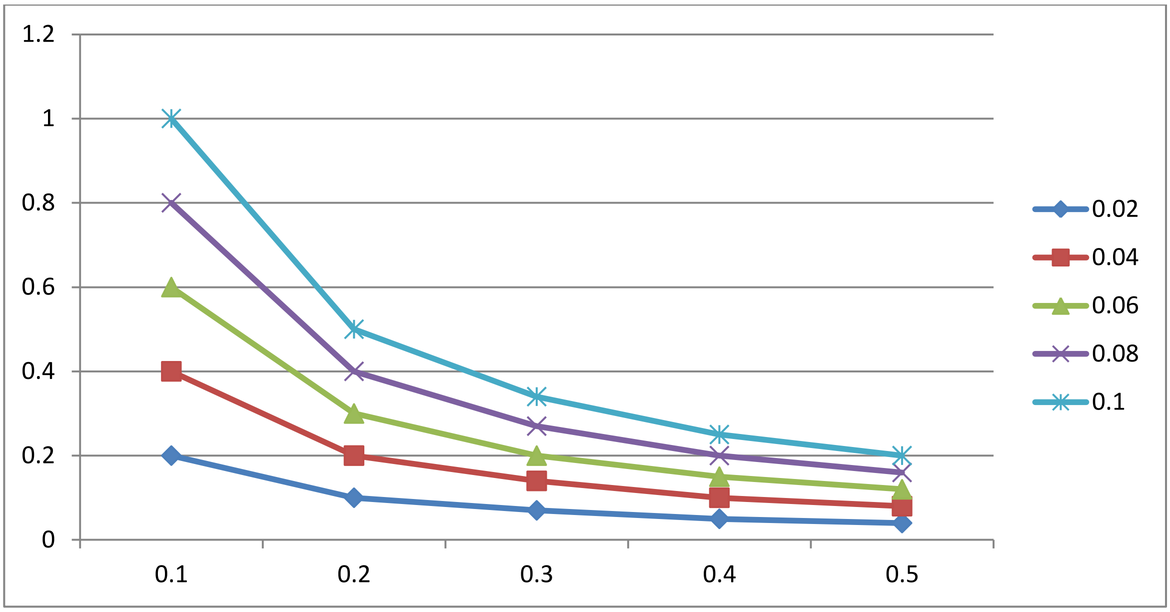

Figure 4.5: Performance analysis of the subsystem of cooking unit

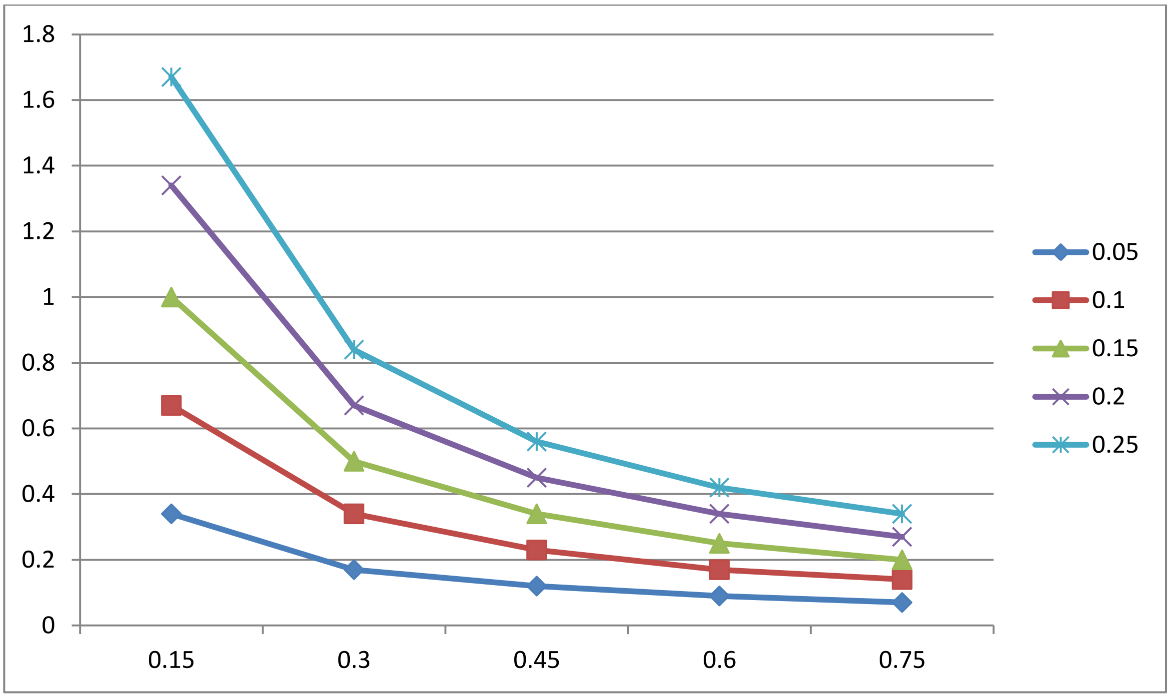

Figure 4.6: Performance analysis of the subsystem of bleaching unit

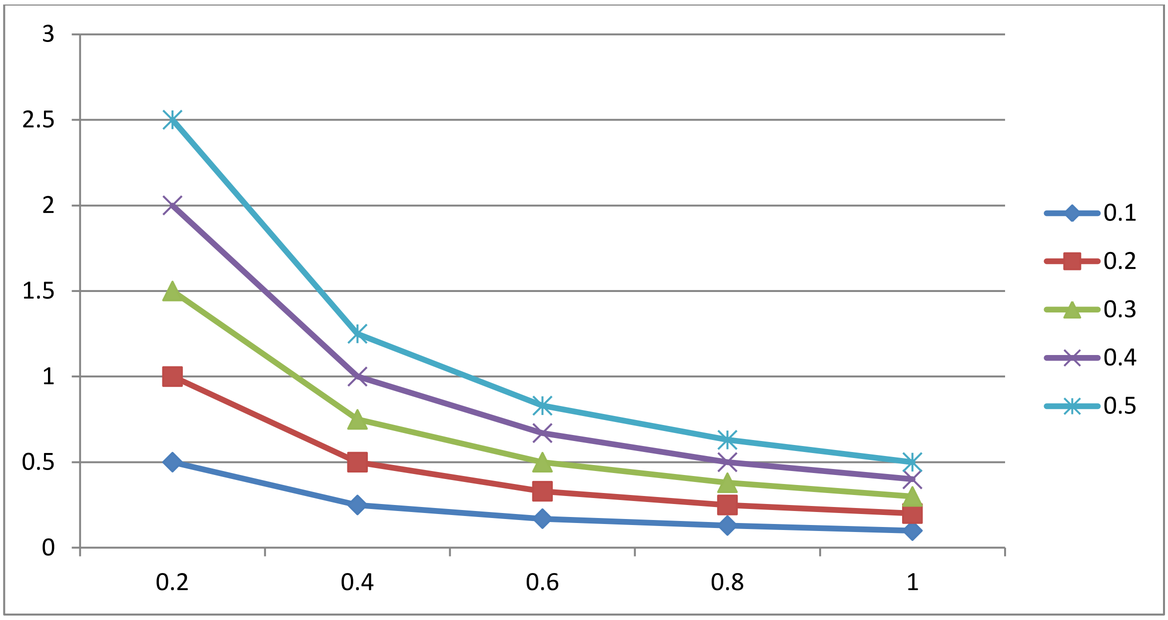

Figure 4.7: Performance analysis of the subsystem of screening unit

The performance analysis for various subsystems of the paper plant is shown in the Table 4.1 and Figure 4.4, Figure 4.5, Figure 4.6 and Figure 4.7. These matrices reveal the various performance levels for different combinations of arriving and serving rates. It also depicts the effect of arriving and serving rates parameters of all the systems on paper plant performance.

In general, the operations involving in chipping, cooking, screening and bleaching cases, the SLAM – II model is ensured high efficiency in calculation.

4.7 SIMULATION OF THE DIGESTIVE SYSTEM OF PAPER PLANT USING SLAM-II

The simulation of the Figures (4.5 - 4.7) using SLAM – II nodes and symbols at the first stage provides a satisfactory approach for putting different cases for cooking without any need for a change in the algorithm. Thus the method of modelling and simulation is straight forward, direct and needs very less computation time of a smaller magnitude.

4.8 CONCLUSION

The mathematical modelling and action of the digestive system of paper plant has been implemented. The digestive system of paper plant is modeled using several queues in tandem with different rates of serving, assuming the serving rate of first part as lower than the serving rate of the last and taking into consideration the capacity of each part as different. The simulation of this model is done using SLAM – II. The best action of the digestive system 'chipping' is deduced. Thus, the findings of this model have been discussed with the management of paper industry. Such results are found highly beneficial for the behavior of some parts of the digestive system of the paper plant.