Chapter 4

Behaviour of Magnetic Fluid Based Rough Rayleigh Step Bearing

The Rayleigh step bearing is considered in one dimensional geometry. It is a well established fact that the roughness has a significant effect on the performance of the hydrodynamic lubrication of a slider bearing. In this chapter an attempt has been made to study and analyze the effect of transverse roughness in the presence of a magnetic fluid for a Rayleigh step bearing. The bearing surfaces are assumed to be transversely rough. The roughness of the surfaces has been characterized by a random variable with non-zero mean, variance and skewness. The stochastically averaged Reynolds equation is solved with suitable boundary conditions to obtain the pressure distribution in turn, which is used to get the load carrying capacity. The results presented in graphical form indicate that the effect of transverse roughness is significantly adverse. However, this research establishes that there exist sufficient scopes for improving the performance of the bearing system in the case of negatively skewed roughness especially, when negative variance is involved, in spite of the fact that the standard deviation introduces a negative effect.

4.1 Introduction

Lord Rayleigh established that when side leakage was neglected the step film shape had the greatest load capacity in a slider bearing lubricated with an incompressible fluid, employing a calculus of variations approach. Hamrock and Anderson (1968) studied the performance of an incompressible lubricated Rayleigh Step Journal Bearing. In fact, the objective of this investigation was to obtain the optimal step configuration for maximum load capacity for an incompressible lubricated one step concentric journal bearing. It is found that some bearings in practical use admit steps on sliding surface for instance Rayleigh Step bearings are invariably used for thrust bearings because of advantages in low cost and relatively easier machining. Ogata (2005) considered thermo hydrodynamic lubrication analysis of Rayleigh Step bearing. In this paper virtual clearance and its derivatives were defined in the discontinuous clearance regions that satisfy the continuity of velocity. The method adopted there allowed THL analysis of the bearings with discontinuous clearance of the sliding surface.

The fundamental aspect in a hydrodynamic slider bearing is the formation of a converging wedge of the lubricant. The hydrodynamic slider may be constructed to provide this converging wedge in a number of ways. The analysis of hydrodynamic lubrication of non-porous slider is a classical one for instance one can have a glance at Pinkus and Sternlicht (1961). Such a bearing consist of a fixed or pivoted node and a moving pad which may be plane, stepped, curved or composite shaped. Exact solutions of Reynolds' equations for slider bearing with various simple film geometries are presented in a number of books and research papers [Cameron (1966), Bogli (1947), Archibald (1956), Charnes et al. (1952), Rayleigh (1918), Basu et al. (2005)]. Lord Rayleigh applied the calculus of variation to determine the optimum film thickness by Lord Rayleigh (1918) and he found that the optimal slider profile was a step function. Plane inclined porous slider bearing was analysed by Prakash and Vij (1973 a) and it was concluded that the effect of porosity was to decrease the load carrying capacity and friction. Patel and Gupta (1983) extended this analysis by introducing slip velocity. McAllister et al. (1980) analyze the performance of slider bearings in view of the optimum design.

All the above analysis assumes the bearing surfaces to be smooth. However, we know that the bearing surfaces after run-in and wear develop roughness. In order to study and analyze the effect of roughness several investigation have proposed a approach to mathematically model the random character of the roughness by Tzeng and Seibel (1967) and Christensen and Tonder (1969 a, 1969 b, 1970). Subsequently, this approach of Christensen and Tonder formed the basis of the methods to analyze the effect of surface roughness on the performance of the bearing system in a number of investigations [Prakash and Tiwari (1983), Prajapati (1992), Guha (1993), Gupta and Deheri (1996). Andharia et al. (1997)] studied the effect of longitudinal roughness surface on hydrodynamic lubrication of slider bearing. Deheri et al.(2005) conducted an investigation into the performance of a transversely rough slider bearing with squeeze film formed by a magnetic fluid. Siddangoudu and Naduvinamani (2007) presented a theoretical study to analyze the effect of surface roughness on the hydrodynamic lubrication of porous step slider bearing with couple stress fluids. Rahmani et al. (2009) discussed the performance of the Rayleigh step bearing, including the effect of pressure at the boundaries on the optimum parameters. Here, it was observed that the optimum bearing parameters depended strictly on the variations of pressure at the boundaries. Shukla and Deheri (2011 a) studied the combined effect of surface roughness and magnetism on the behaviour of a magnetic fluid based porous rough secant shaped slider bearing. It was shown that the negative effect of transverse roughness could be compensated up to some extent at least in the case of negatively skewed roughness. Patel and Deheri (2011) discussed the effect of surface roughness on the performance of a magnetic fluid based parallel plates porous slider bearing with slip velocity. There, it was showed that the adverse effect of porosity and standard deviation could be minimized by the positive effect of magnetic fluid lubricant in the case of negatively skewed roughness while, the slip parameter deserved to be kept at minimum. Kashinath (2012) analyzed theoretically the couple stress effect on the squeeze film performance between parallel stepped plates. Here, it was shown that the influence of couple stresses enhanced the load-carrying capacity and decreased the response time compared with the classical Newtonian lubricant. Furthermore, by increasing the step height, the load-carrying capacity decreased. After receiving some run-in and wear, bearing surfaces develop roughness. Sometimes contamination by lubricants and chemical degradation of the surfaces contribute to this roughness. The roughness appears to be random in character. Surface roughness is an important factor when dealing with issues such as friction, lubrication, and wear. It also has a major impact on applications involving thermal or electrical resistance, fluid dynamics, noise and vibration control, dimensional tolerance, and abrasive processes, among others. Shukla and Deheri (2013) analysed the performance of a magnetic-fluid-based porous rough step bearing considering slip velocity. It was found that although the bearing suffers owing to transverse surface roughness, the performance of the bearing system can be improved to some extent by the positive effect of magnetization, considering the slip parameter at the minimum; at least in the case of negatively skewed roughness. A comparison of this investigation with some established investigations indicates that the reduction of load-carrying capacity due to porosity and slip velocity is comparatively less, especially, when negative variance occurs. In augmenting the performance of the bearing system, the step ratio plays a central role, even if the slip parameter is at the minimum.

4.2 Analysis

The configuration of the bearing system is shown below where in the film thickness h is constant over two regions.

$$0 \le x \le B_1, \quad h = h_1 \text{ and } B_1 \le x \le B, \quad h = h_2 \tag{4.1}$$

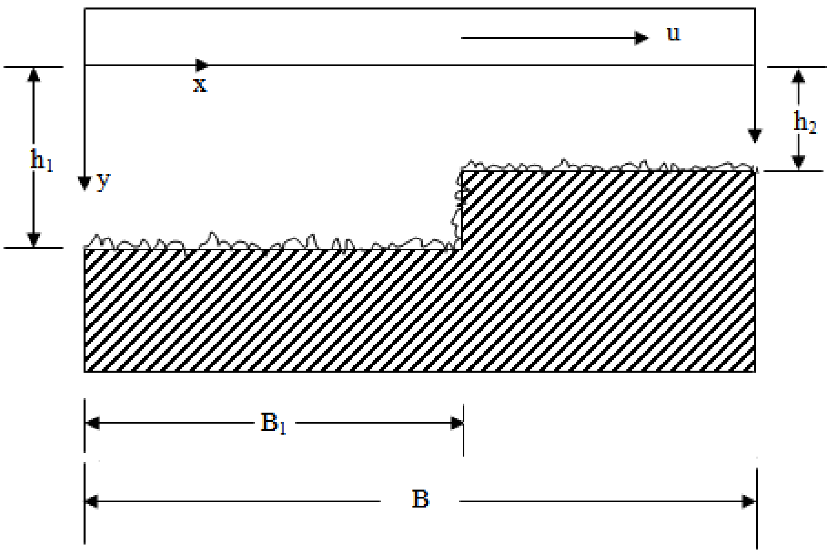

Fig.4.1 Configuration of bearing system.

Following the stochastical modeling of surface roughness by Christensen and Tonder ((1969 a, 1969 b, 1970). the thickness $h(x)$ is considered as

$$h(x) = \bar{h}(x) + h_s$$

where $\bar{h}(x)$ is the mean film thickness and h_s is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces.

h_s is assumed to be stochastic in nature and governed by the probability density function

$$f(h_s) = \begin{cases} \dfrac{35}{32c^7}\left(c^2 - h_s^{\,2}\right)^3, & -c \le h_s \le c \\ 0, & elsewhere \end{cases} \tag{4.2}$$

where c is the maximum deviation from the mean film thickness. The mean α, the standard deviation σ and the parameter ε which is the, measure of symmetry of the random variable h_s are defined by the relationships

$$\alpha = E(h_s),\ \sigma^2 = E\left\{(h_s - \alpha)^2\right\} \text{ and } \varepsilon = E\left\{(h_s - \alpha)^3\right\}$$

where, E denotes the expected value defined by

$$E(R) = \int_{-c}^{c} R\, f(h_s)\, dh_s \tag{4.3}$$

With usual assumptions of hydromagnetic lubrication theory in Bhat (2003) the governing Reynolds' equations for the pressure distributions turns out to be

$$\frac{d}{dx}\left(p - \frac{\mu_0 \bar{\mu} M^2}{2}\right) = 6\eta u \frac{h - h_m}{h^3} \tag{4.4}$$

where η is the viscosity of the lubricant and

$$M^2 = K\, x(B - x)\ ,$$

M being the magnitude of the magnetic field.

Stochastically averaging this equation along the model adopted by Christensen and Tonder (1969 a, 1969 b, 1970) one finds that the fluid film pressure is governed by a generalized form of the Reynolds' equation of the type

$$\frac{d}{dx}\left(p - \frac{\mu_0 \bar{\mu} M^2}{2}\right) = 6\eta u \frac{h - h_m}{g(h)} \tag{4.5}$$

where

$$g(h) = h^3 + 3\alpha h^2 + 3(\sigma^2 + \alpha^2)h + 3\sigma^2\alpha + \alpha^3 + \varepsilon \tag{4.6}$$

Considering the boundary conditions

$$p = 0 \text{ at } x = 0 \text{ and } p = p_c \text{ at } x = B_1 \tag{4.7}$$

one obtains the non dimensional pressure for

$$0 \le X \le \frac{B_1}{B}$$

as

$$P = \frac{h_1^3}{\eta u B^2} p = \frac{\mu^* X (1-X)}{2} + \frac{6}{G(1)} \frac{h_1}{B} \left(1 - \frac{B_1}{B}\right) \left(\frac{m}{m+1}\right) X \tag{4.8}$$

and for $\frac{B_1}{B} \le X \le 1$ with the boundary conditions

$$p = p_c \text{ at } x = B_1 \text{ and } p = 0 \text{ at } x = B \tag{4.9}$$

one gets the non dimensional pressure

$$P = \frac{\mu^* X(1-X)}{2} + \frac{6}{G(1)} \frac{h_2}{B} \frac{B_1}{B} m(1-X) \tag{4.10}$$

where $m = \frac{h_1 - h_2}{h_2}, X = \frac{x}{B}$ and $\mu^* = \frac{\mu_0 \bar{\mu} h_1^3 K}{\eta u}$,

μ_0 is the free space permeability and

$\bar{\mu}$ is the magnetic susceptibility,

while,

$$G(1) = 1 + 3\bar{\alpha} + 3\bar{\alpha}^2 + 3\bar{\sigma}^2 + 3\bar{\sigma}^2\bar{\alpha} + \bar{\alpha}^3 + \bar{\varepsilon}\,. \tag{4.11}$$

Here,

$$G(1) = \frac{g(h)}{h_1^3}, \quad \bar{\alpha} = \frac{\alpha}{h_1}, \quad \bar{\sigma} = \frac{\sigma}{h_1}, \quad \bar{\varepsilon} = \frac{\varepsilon}{h_1^3} \tag{4.12}$$

The non dimensional load carrying capacity is given by

$$W = \int_0^{B_1/B} P\, dX + \int_{B_1/B}^{1} P\, dX \tag{4.13}$$

$$= \frac{\mu^*}{6} + \frac{3}{G(1)} \frac{h_2}{B} m \left(1 - \frac{B_1}{B}\right) \frac{B_1}{B} \tag{4.14}$$

4.3 Results and discussion

It is clearly seen that equation (4.10) and equation (4.14) present respectively the dimensionless pressure and non-dimensional load carrying capacity. These expressions depend on various parameters such as $\mu^*, \bar{\sigma}, \bar{\alpha}, \bar{\varepsilon}, m, \frac{B_1}{B}, \frac{h_2}{B}$.

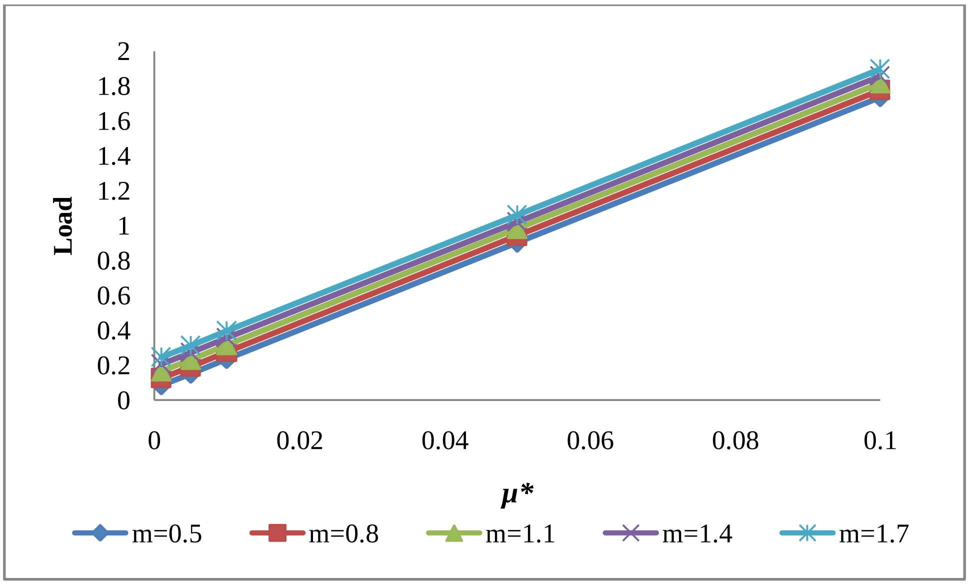

Fig.4.2 Variation of Load carrying capacity with respect to μ^* and aspect ratio *m*

We can see that magnetization increases the load carrying capacity sharply. Moreover, the effect of aspect ratio on the load with respect to magnetization parameter is not that significant.

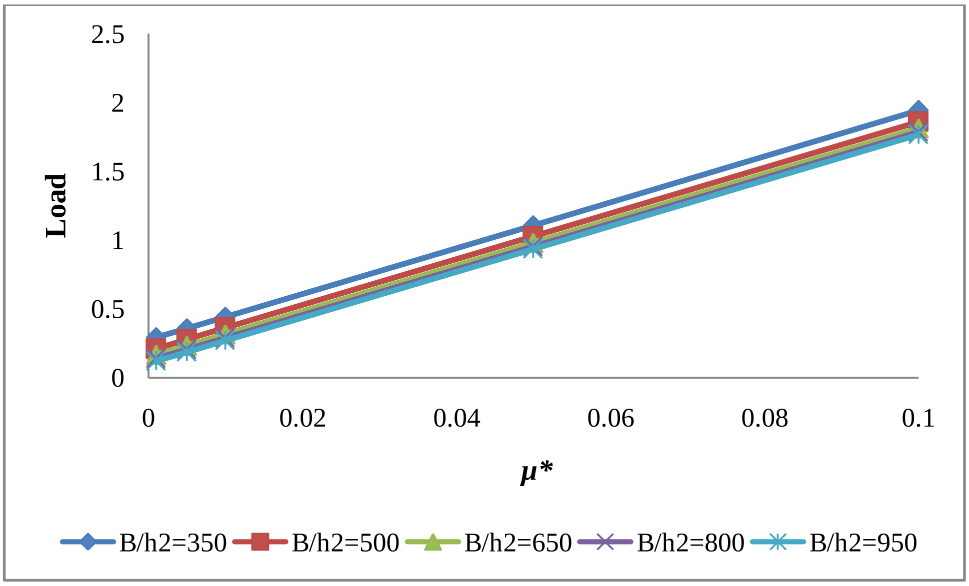

Fig. 4.3 Variation of Load carrying capacity with respect to μ^* and ratio B/h_2.

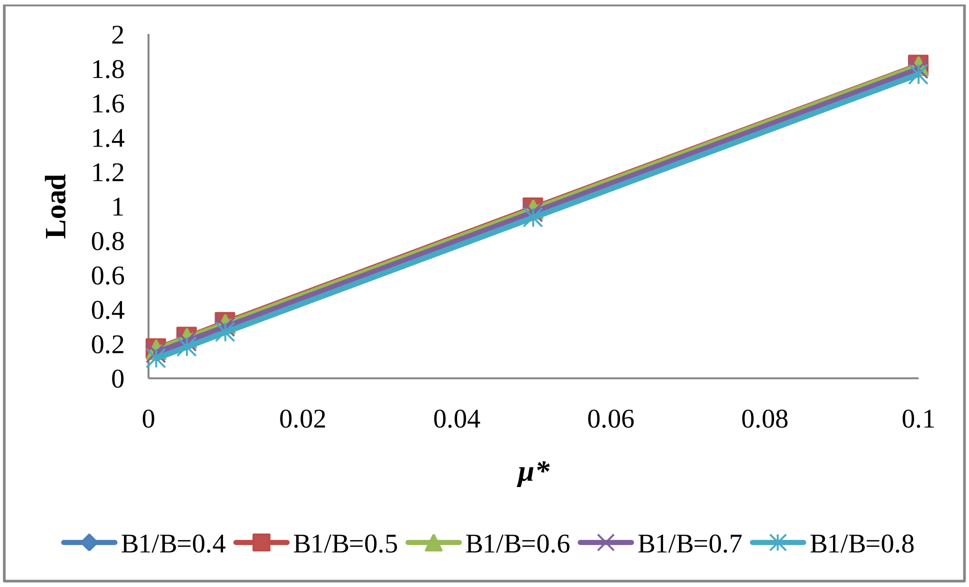

Fig.4.4 Variation of Load carrying capacity with respect to μ^* and ratio B_1/B

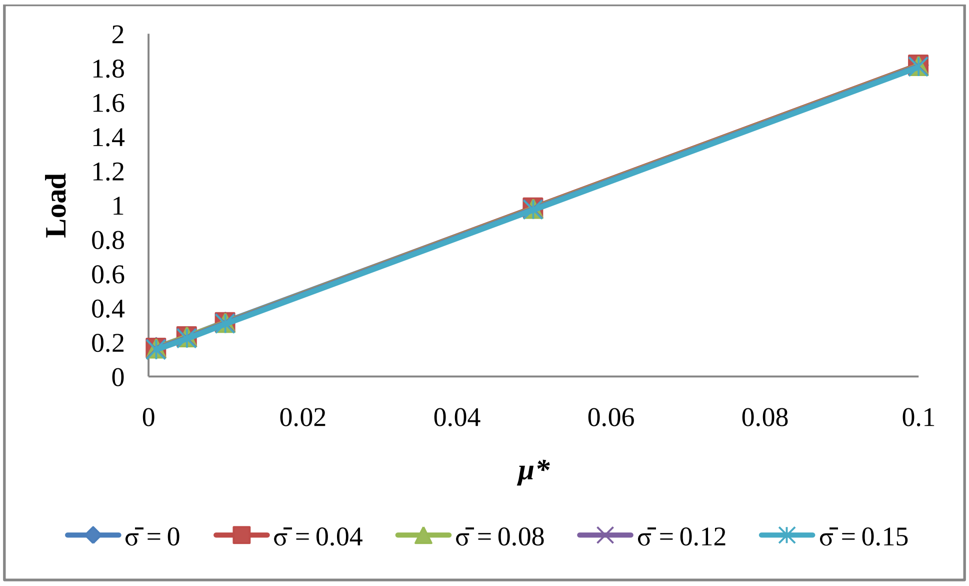

Fig. 4.5 Variation of Load carrying capacity with respect to μ^* and $\bar{\sigma}$

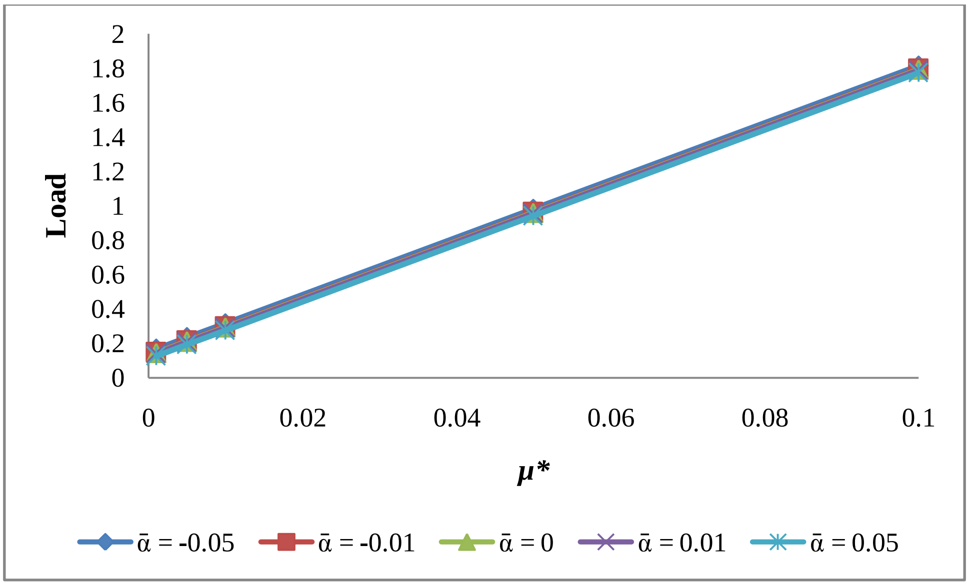

Fig.4.6 Variation of Load carrying capacity with respect to μ^* and $\bar{\alpha}$

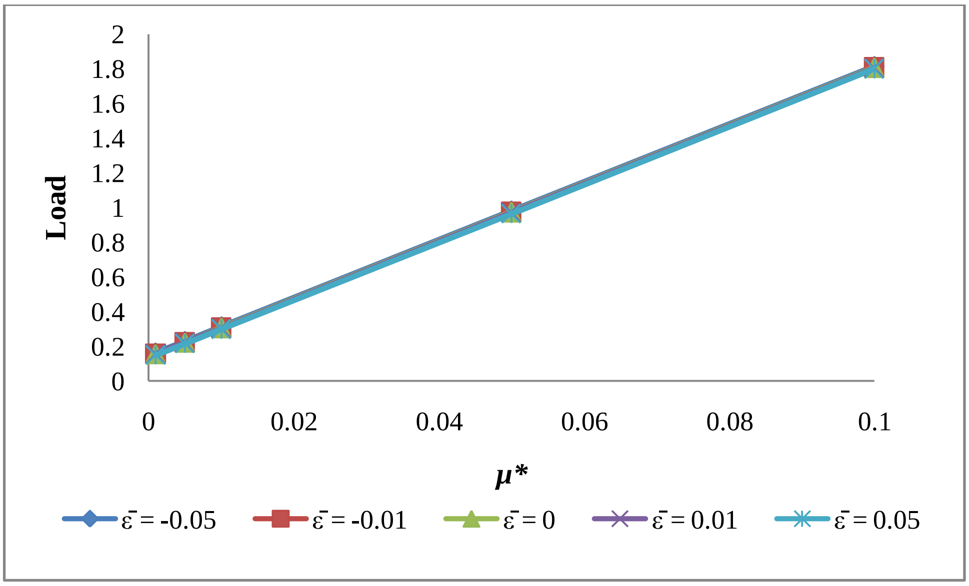

Fig.4.7 Variation of Load carrying capacity with respect to μ^* and $\bar{\varepsilon}$

It is seen that although the magnetization results in sharply increased load carrying capacity the effect of variance on load carrying capacity with respect ot magnetization is negligible, while, the effect of B/h_2 on load carrying capacity with respect to magnetization is not that much effective.

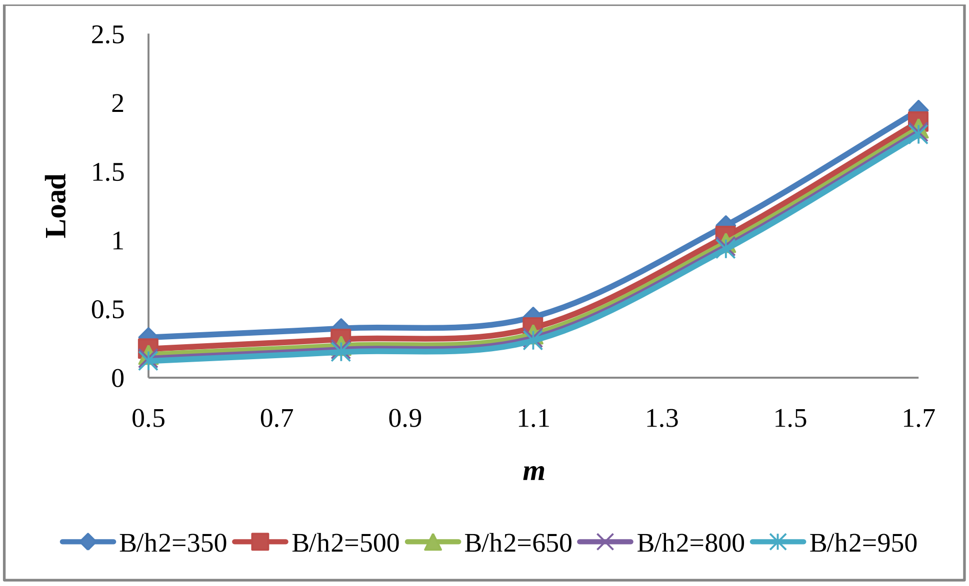

Fig.4.8 Variation of Load carrying capacity with respect to *m* and B/h_2

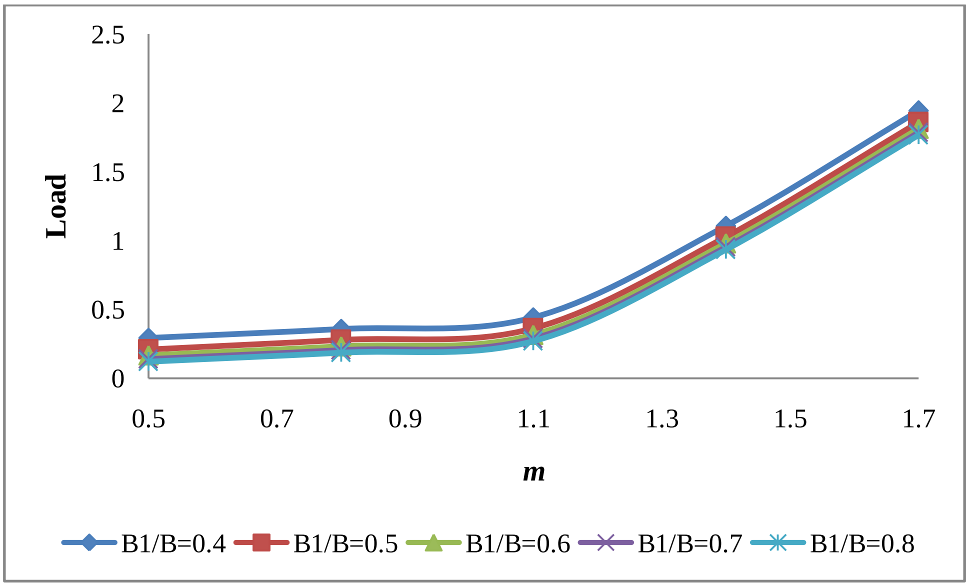

Fig.4.9 Variation of Load carrying capacity with respect to *m* and B_1/B

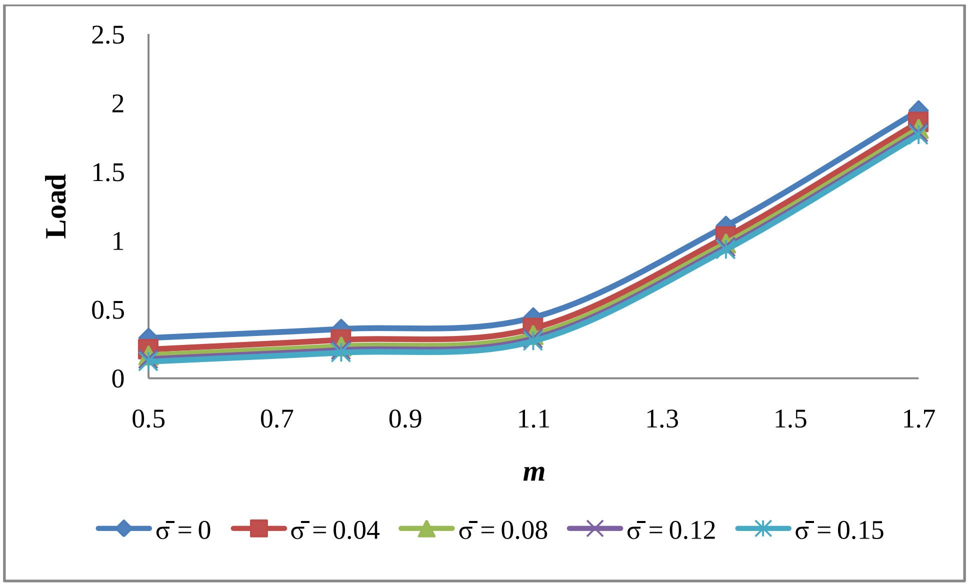

Fig.4.10 Variation of Load carrying capacity with respect to *m* and $\bar{\sigma}$

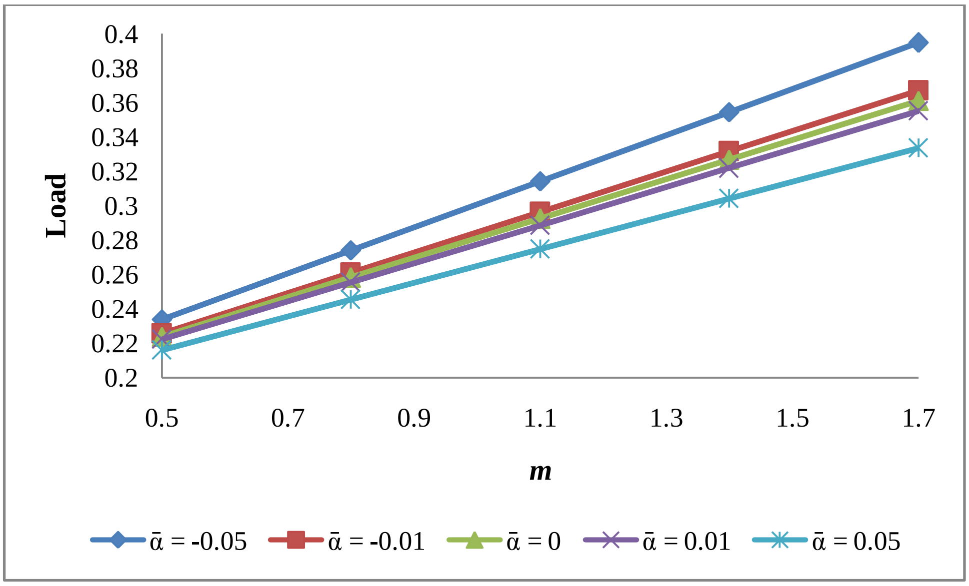

Fig.4.11 Variation of Load carrying capacity with respect to *m* and $\bar{\alpha}$

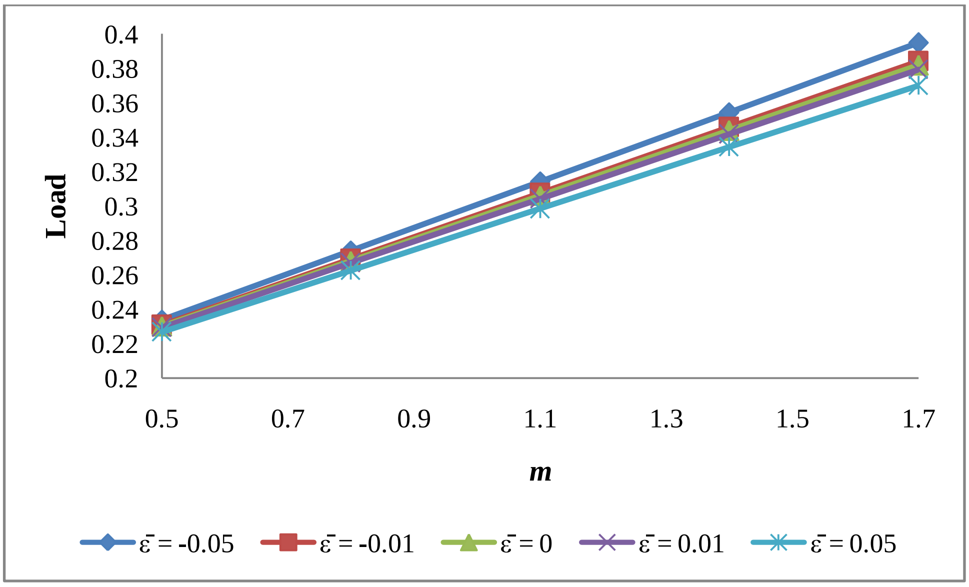

Fig.4.12 Variation of Load carrying capacity with respect to *m* and $\bar{\varepsilon}$

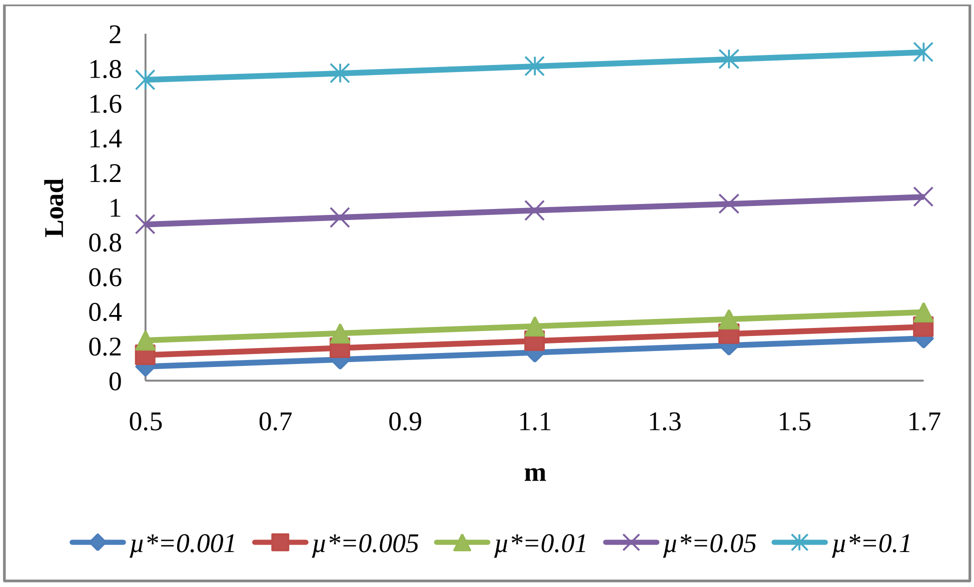

Fig.4.13 Variation of Load carrying capacity with respect to m and μ^*

For all most all values of aspect ratio the load carrying capacity gets increase but, higher values of the aspect ratio results in significantly increase load carrying capacity. At the same time the parameters B/h_2, B_1/B, $\bar{\sigma}$ fail to register any significant increase in load carrying capacity when considered with magnetization parameter. Interestingly, the effect of variance on load carrying capacity with respect to aspect ratio is quite tangible.

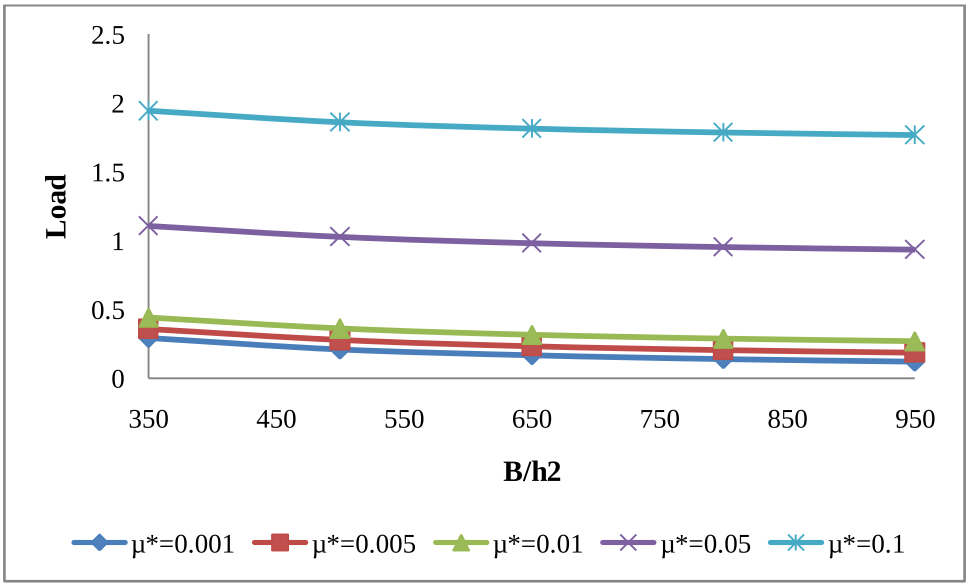

Fig.4.14 Variation of Load carrying capacity with respect to B/h_2 and μ^*

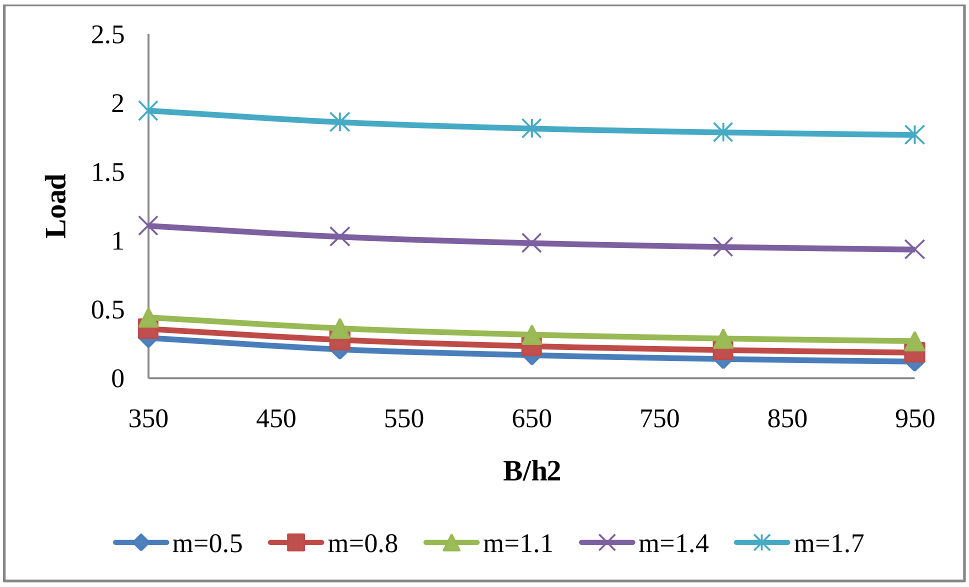

Fig.4.15 Variation of Load carrying capacity with respect to B/h_2 and m

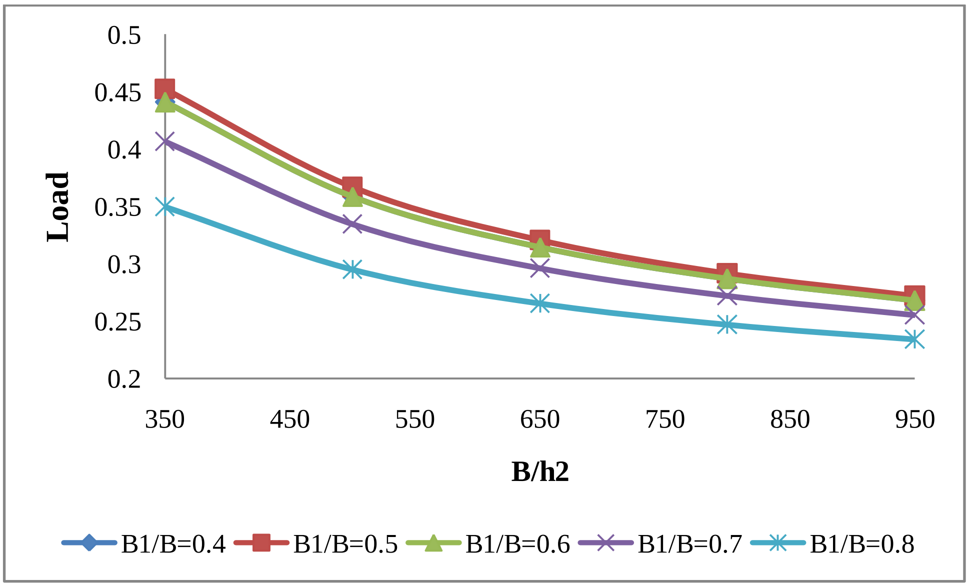

Fig.4.16 Variation of Load carrying capacity with respect to B/h_2 and B_1/B

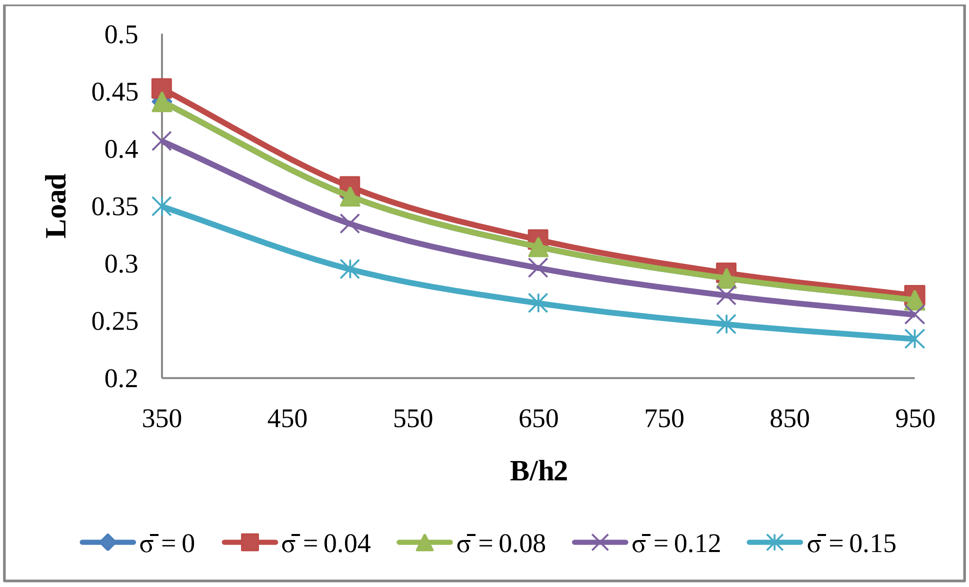

Fig.4.17 Variation of Load carrying capacity with respect to B/h_2 and $\bar{\sigma}$

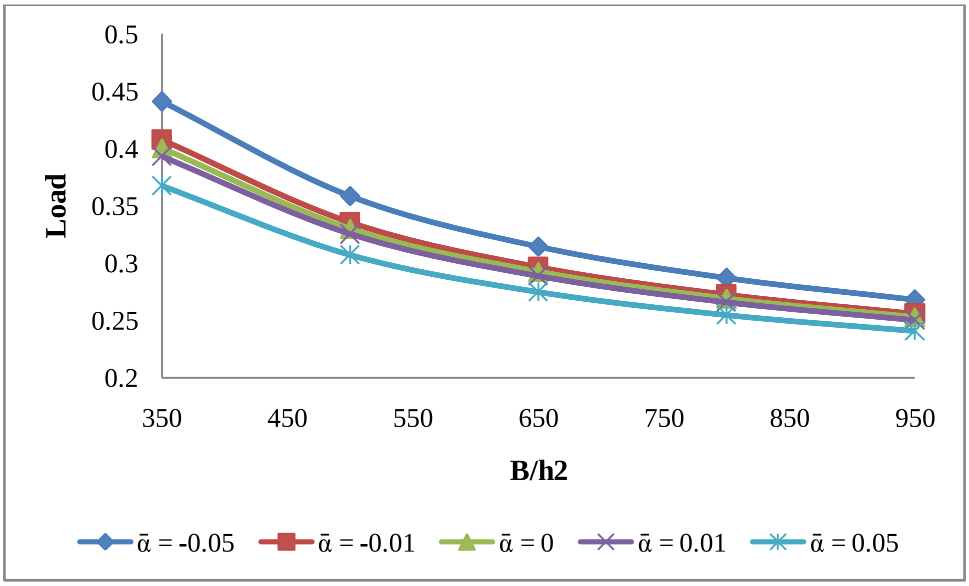

Fig.4.18 Variation of Load carrying capacity with respect to B/h_2 and $\bar{\alpha}$

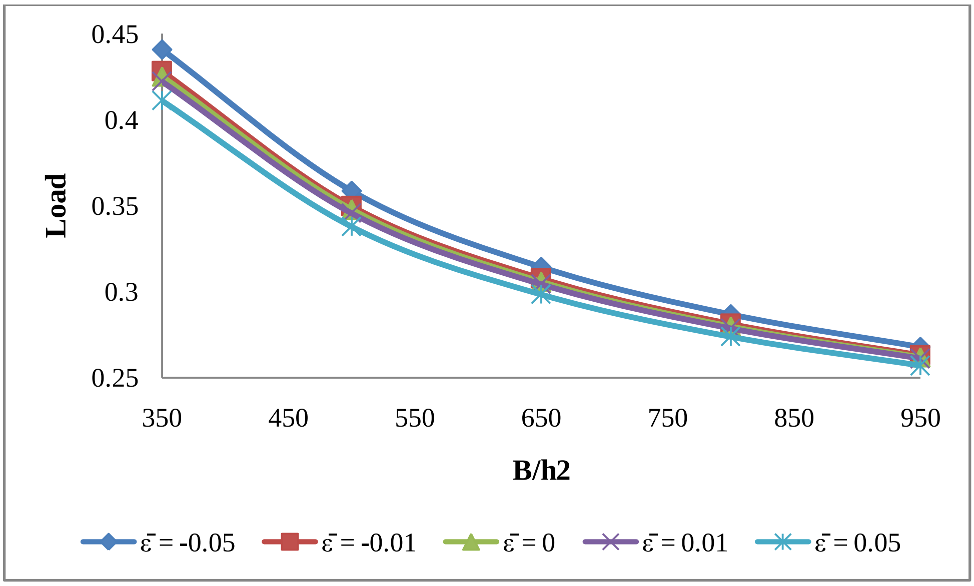

Fig.4.19 Variation of Load carrying capacity with respect to B/h_2 and $\bar{\varepsilon}$

The ratio $\frac{B}{h_2}$ tends to introduce the decrease in the load carrying capacity, the decrease being least in the case of the magnetization. In addition, the decrease in load carrying capacity due to $\frac{B}{h_2}$ is more in the case of skew-ness $\bar{\varepsilon}$. The positively skewed roughness decreases the load carrying capacity while, the load carrying capacity gets increased owing to negatively skewed roughness. The trends of the variance $\bar{\alpha}$ are identically that of skewness so far as the distribution of load is concerned.

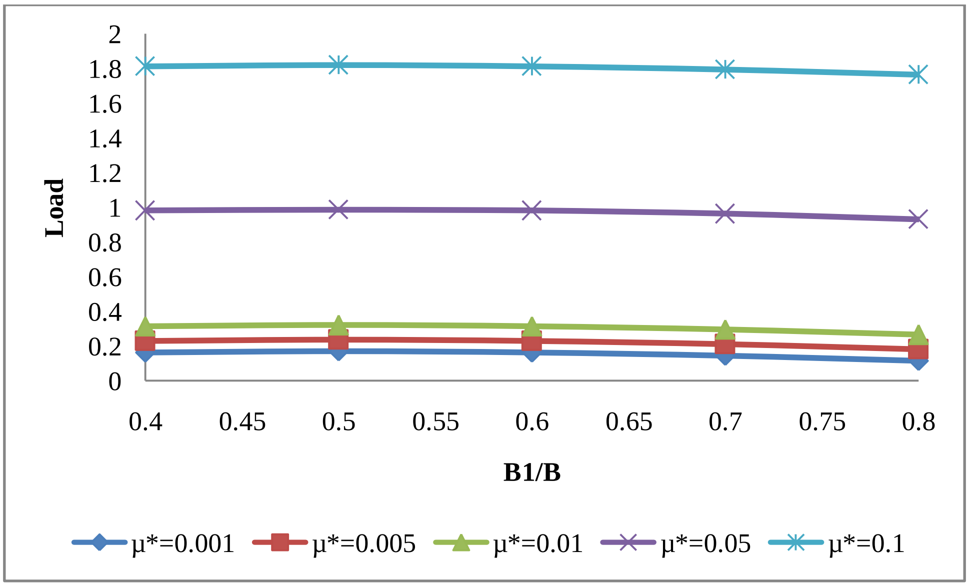

Fig.4.20 Variation of Load carrying capacity with respect to B_1/B and μ^*

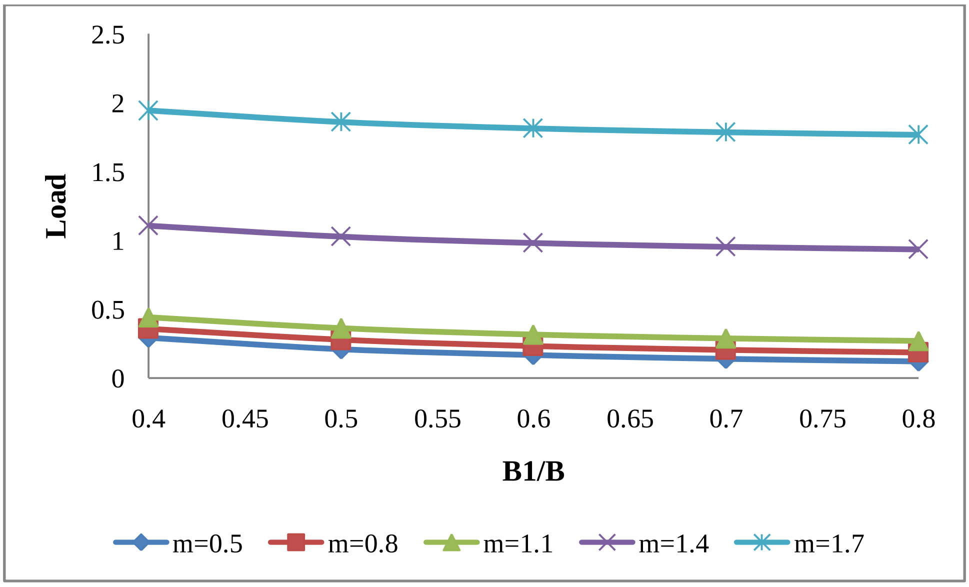

Fig.4.21 Variation of Load carrying capacity with respect to B_1/B and m

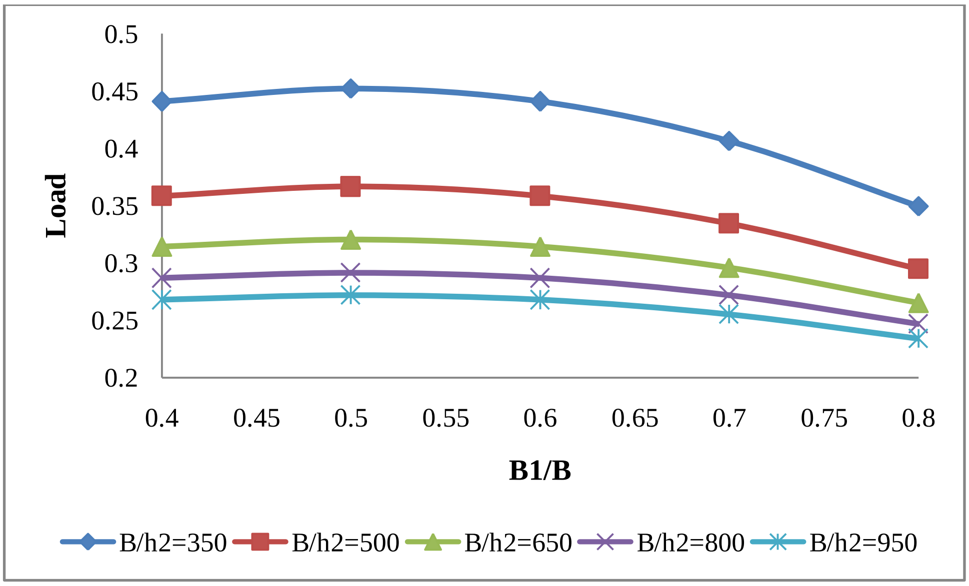

Fig.4.22 Variation of Load carrying capacity with respect to B_1/B and B/h_2

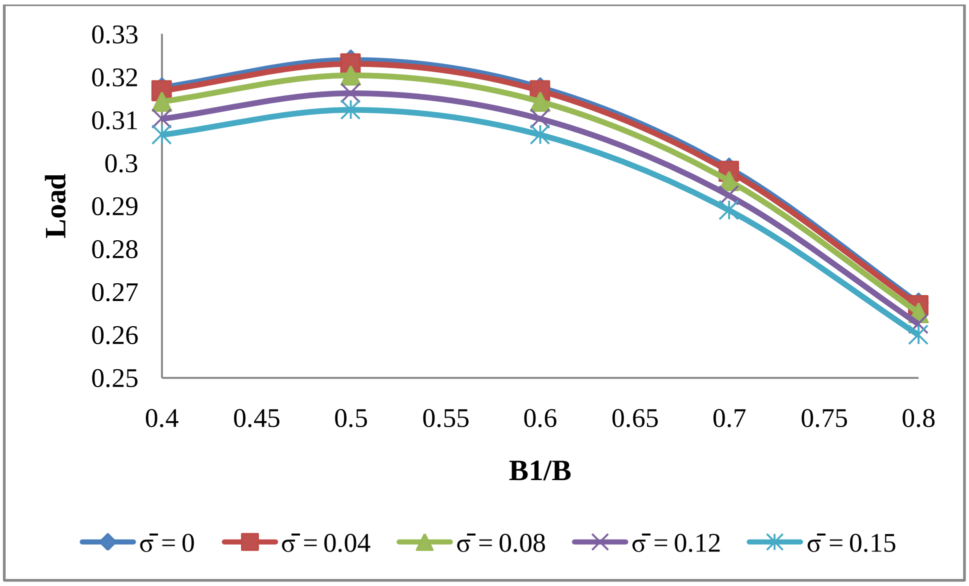

Fig.4.23 Variation of Load carrying capacity with respect to B_1/B and $\bar{\sigma}$

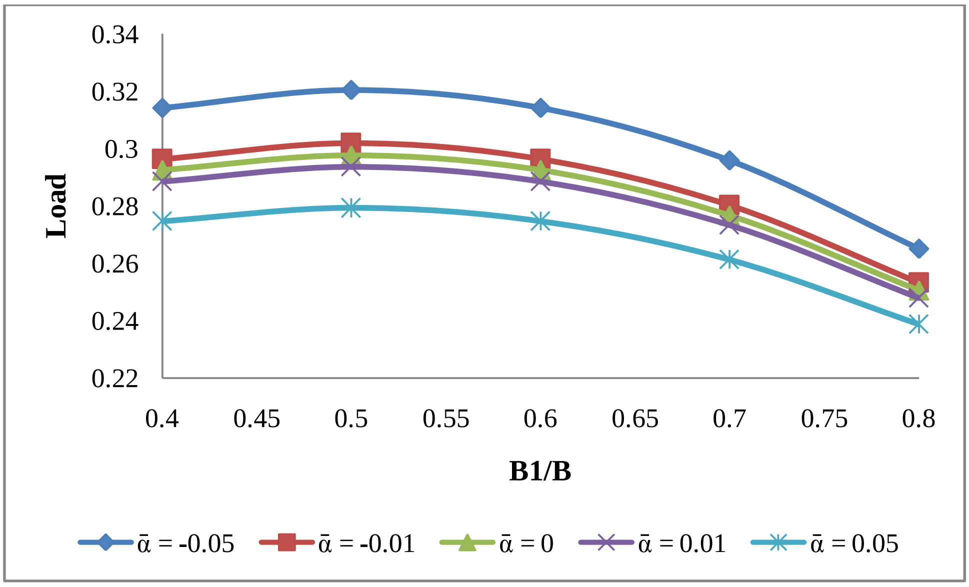

Fig.4.24 Variation of Load carrying capacity with respect to B_1/B and $\bar{\alpha}$

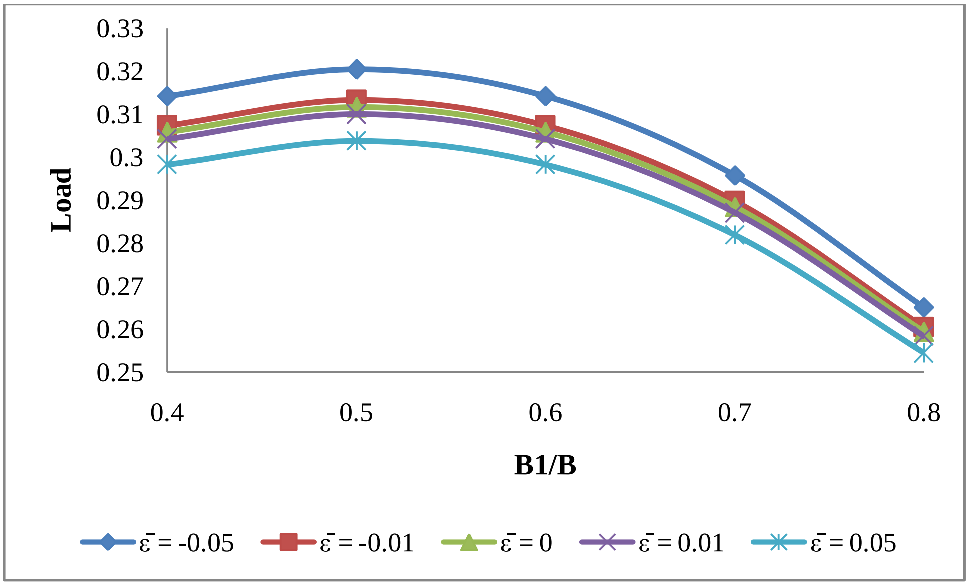

Fig.4.25 Variation of Load carrying capacity with respect to B_1/B and $\bar{\varepsilon}$

The fact that the step ratio $\frac{B_1}{B}$ leads to decreased load carrying capacity, is exhibited in Figures 4.20-4.25. However, the effect of $\frac{B_1}{B}$ is negligible when considered with magnetization. Also, Figure – 4.23 certifies the strong combined adverse effect of standard deviation and $\frac{B_1}{B}$.

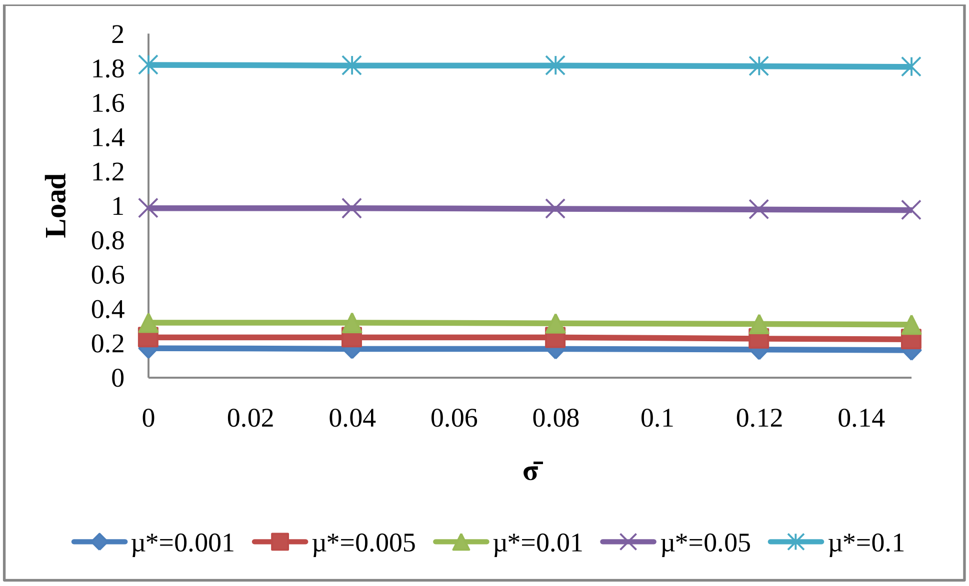

Fig.4.26 Variation of Load carrying capacity with respect to $\bar{\sigma}$ and μ^*

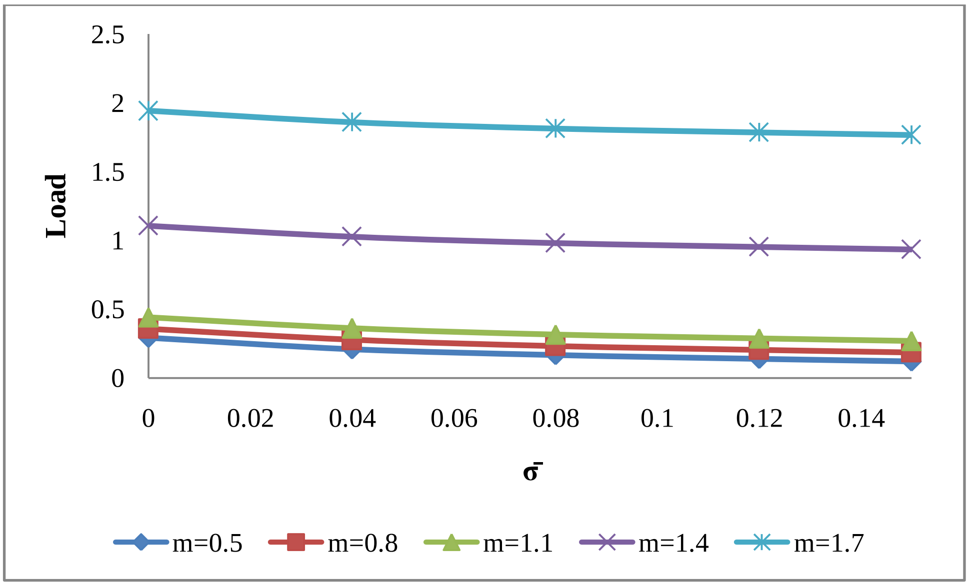

Fig.4.27 Variation of Load carrying capacity with respect to $\bar{\sigma}$ and m

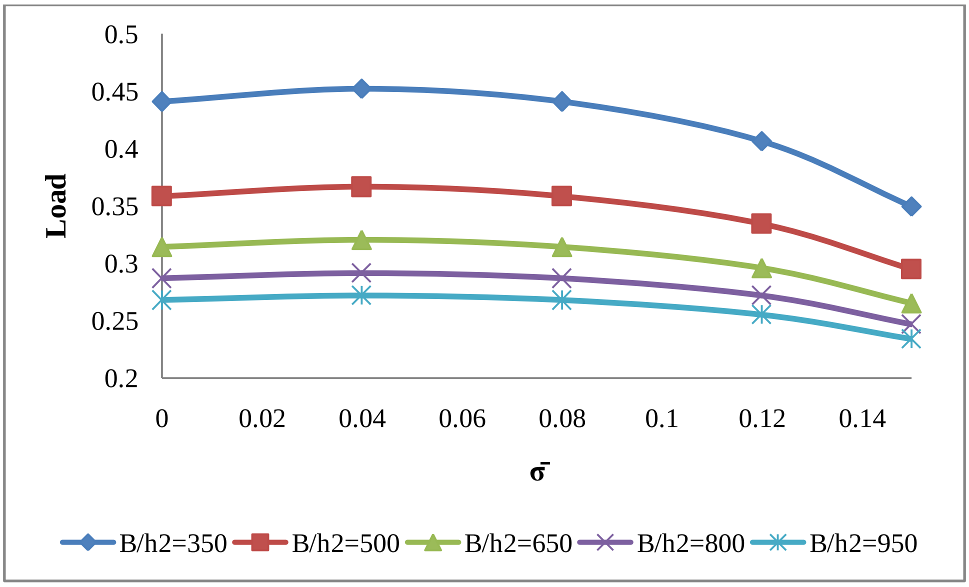

Fig.4.28 Variation of Load carrying capacity with respect to σ and B/h_2

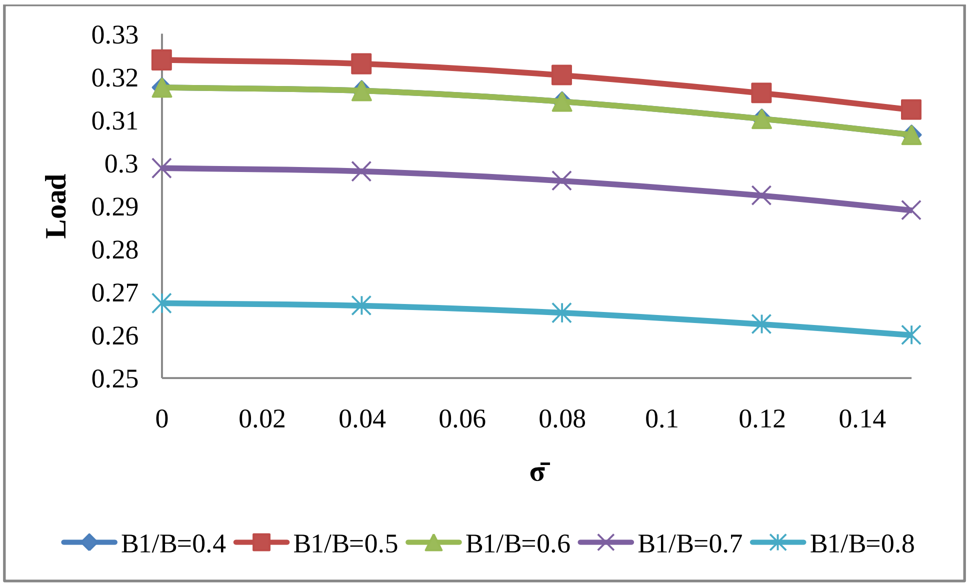

Fig.4.29 Variation of Load carrying capacity with respect to $\bar{\sigma}$ and B_1/B

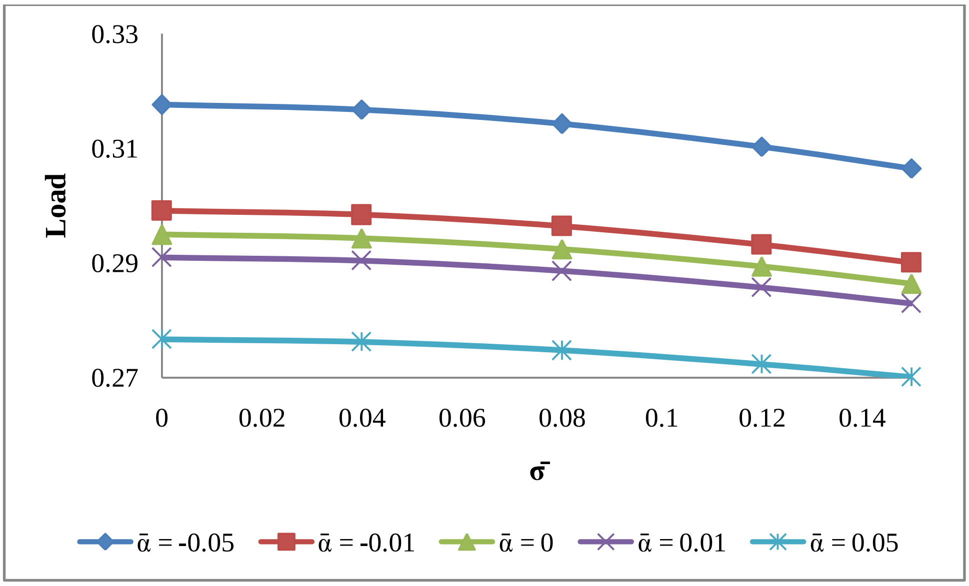

Fig.4.30 Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\alpha}$

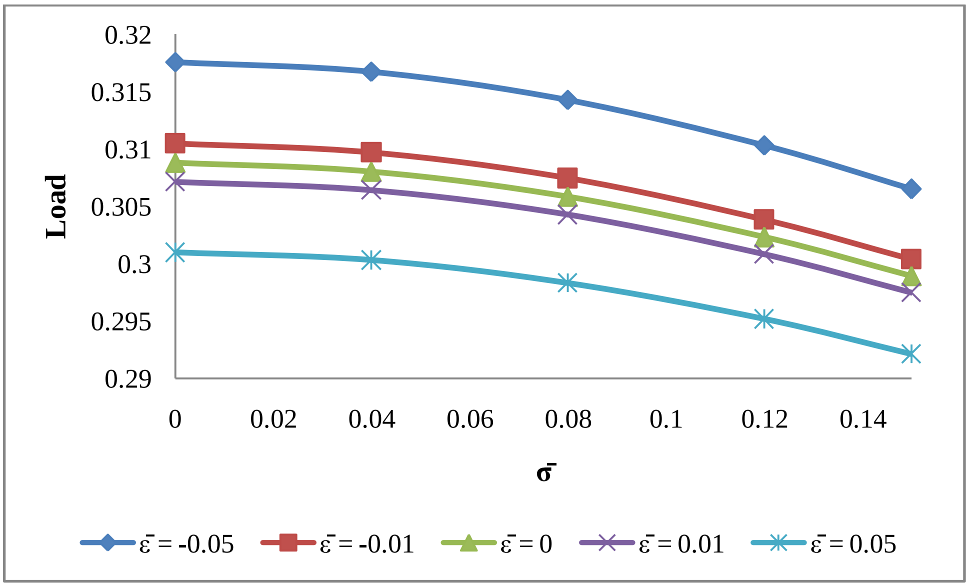

Fig.4.31 Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\varepsilon}$

The figures 4.26–4.31 suggest that the standard deviation's effect on the load carrying capacity is strongly adverse, in the sense that load carrying capacity decreases sharply with the increase in standard deviation. But, this decrease in load carrying capacity is relatively less in the case of magnetization.

Further, figures 4.30 and 4.31 allow us to conclude that variance and skewness have almost identical effects.

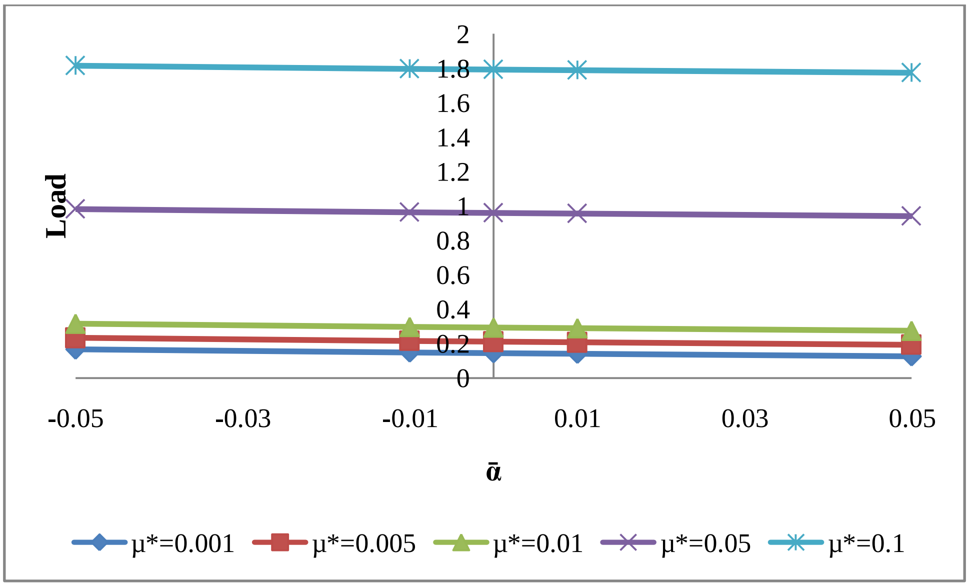

Fig.4.32 Variation of Load carrying capacity with respect to $\bar{\alpha}$ and μ^*

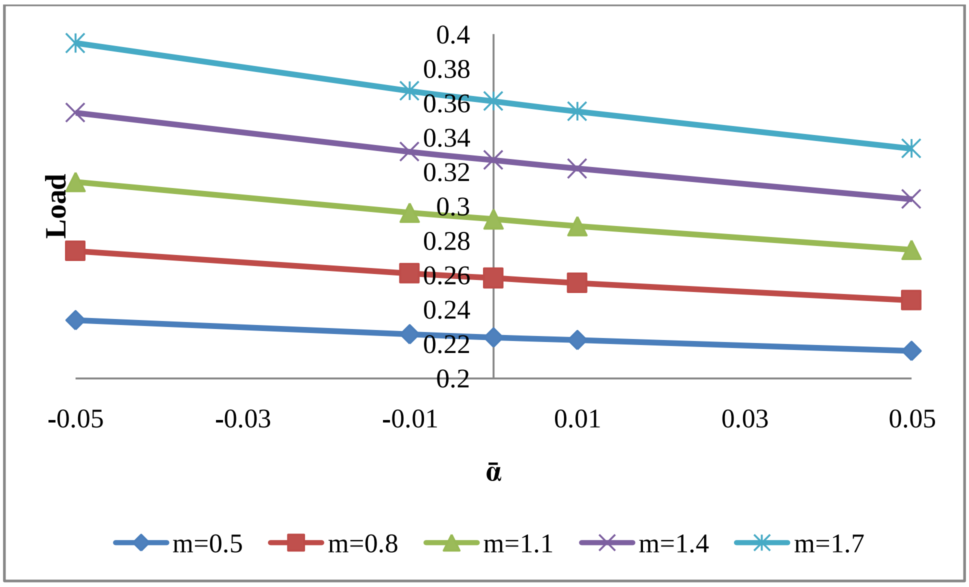

Fig.4.33 Variation of Load carrying capacity with respect to $\bar{\alpha}$ and m

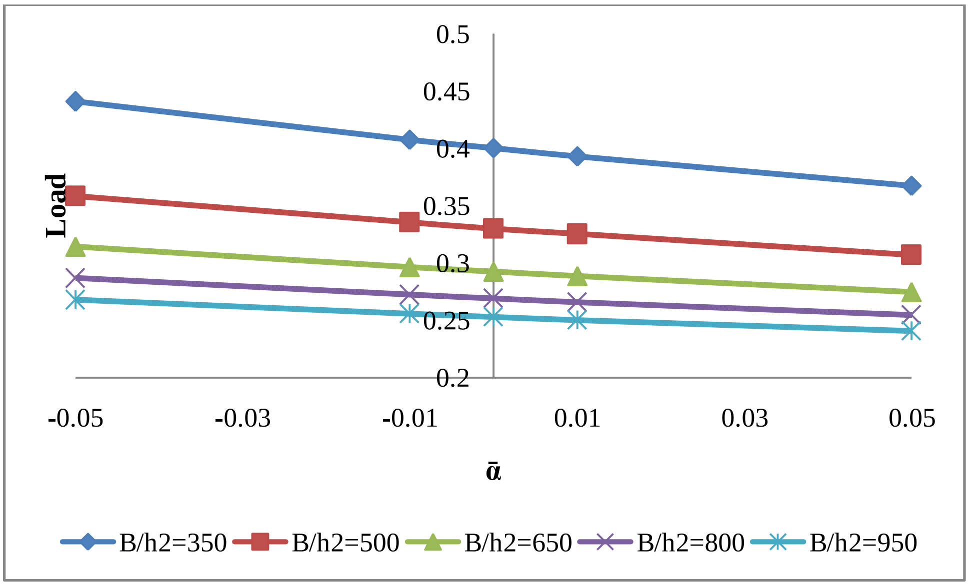

Fig.4.34 Variation of Load carrying capacity with respect to $\bar{\sigma}$ and B/h_2

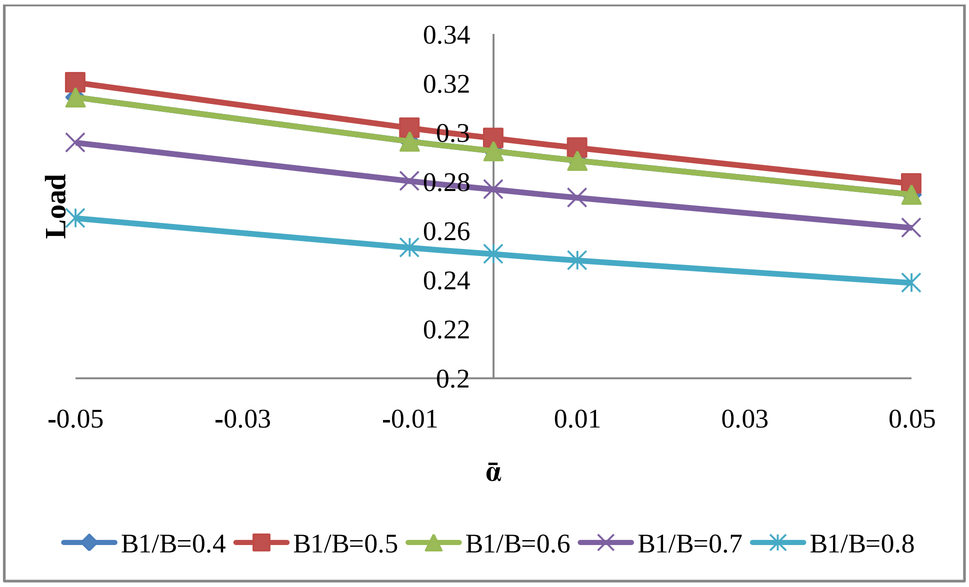

Fig.4.35 Variation of Load carrying capacity with respect to $\bar{\alpha}$ and B_1/B

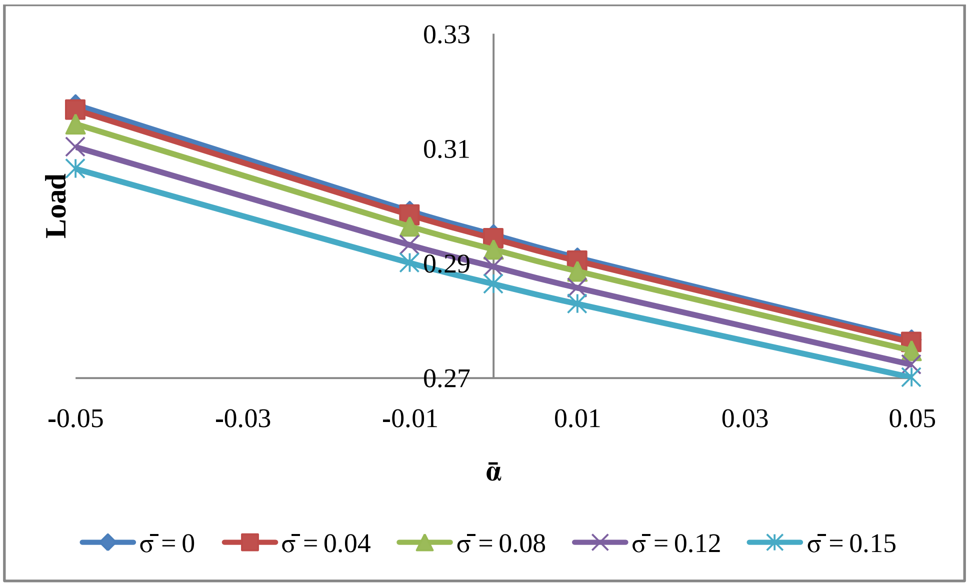

Fig.4.36 Variation of Load carrying capacity with respect to $\bar{\alpha}$ and $\bar{\sigma}$

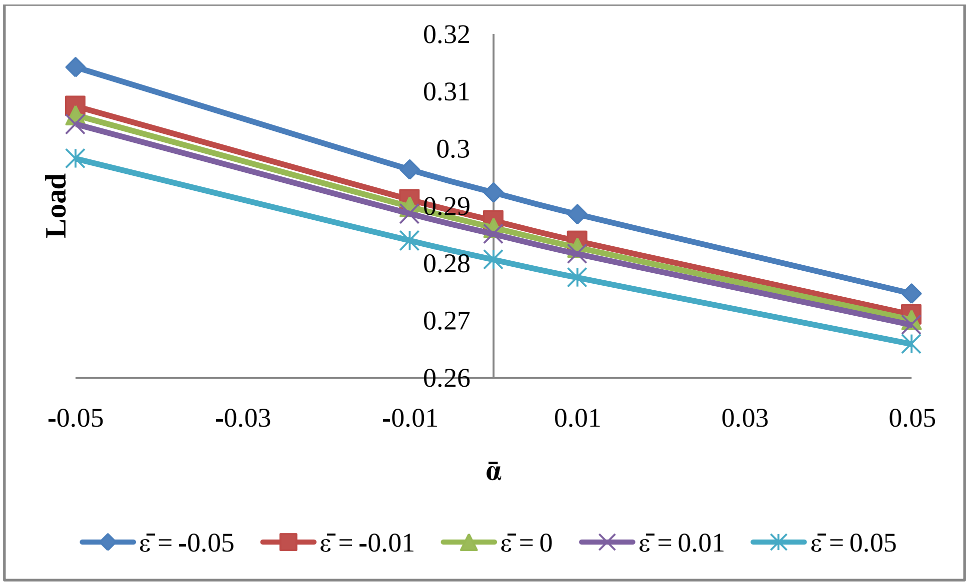

Fig.4.37 Variation of Load carrying capacity with respect to $\bar{\alpha}$ and $\bar{\varepsilon}$

The effect of variance presented in figures 4.32–4.37 indicates that negative variance results in increased load carrying capacity while, positive variance gives rise to decreased load carrying capacity. In addition, the effect of magnetization on the load carrying capacity remains almost negligible up to certain values of the magnetization parameter.

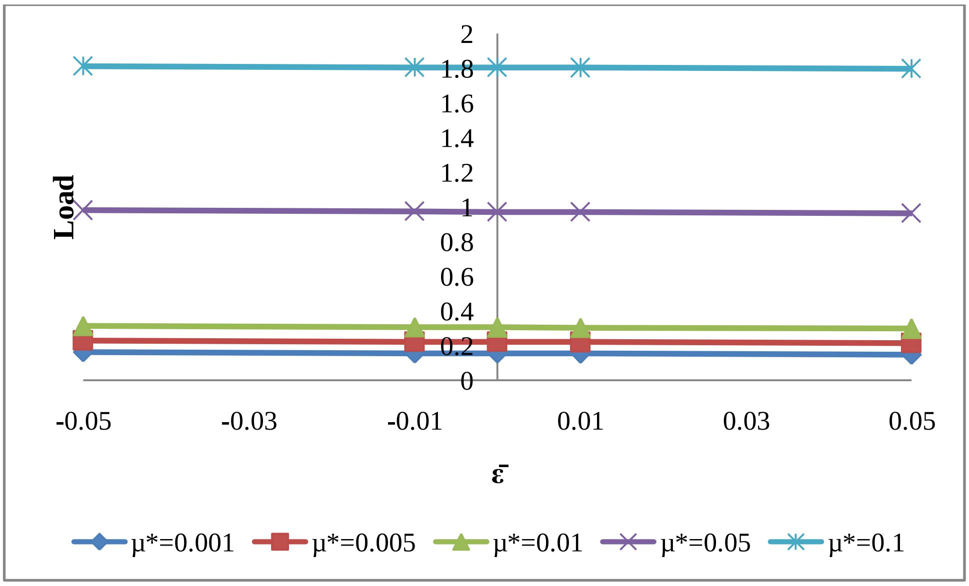

Fig.4.38 Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and μ^*

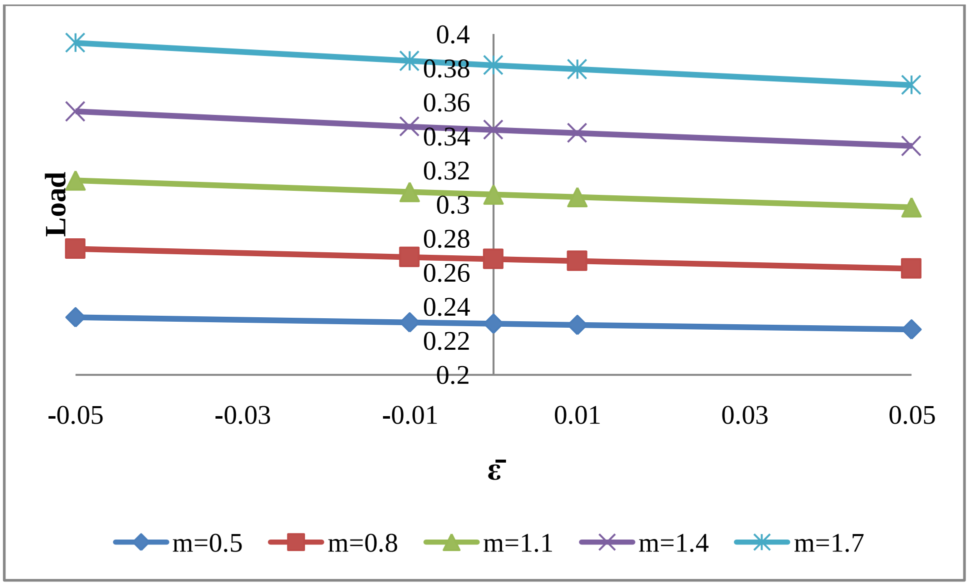

Fig.4.39 Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and m

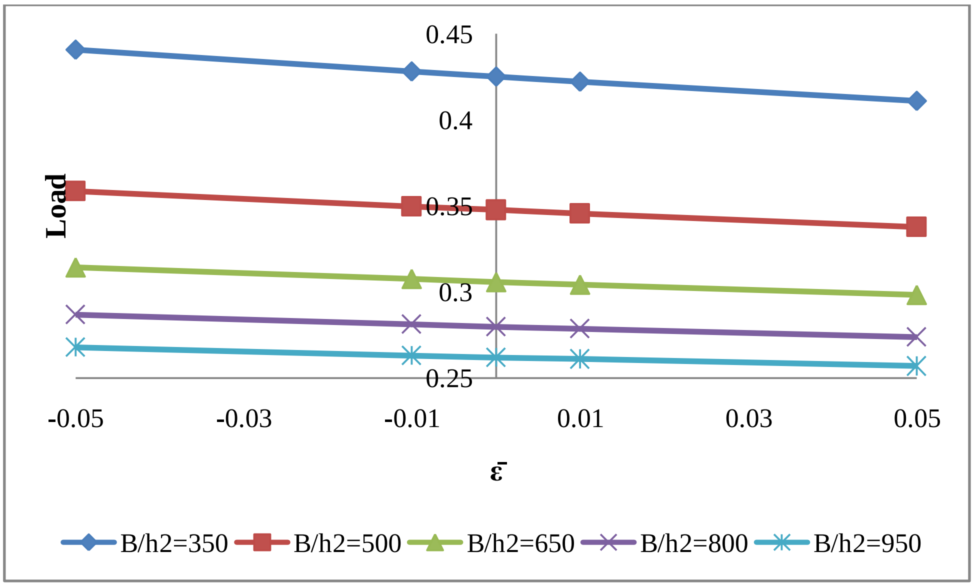

Fig.4.40 Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and B/h_2

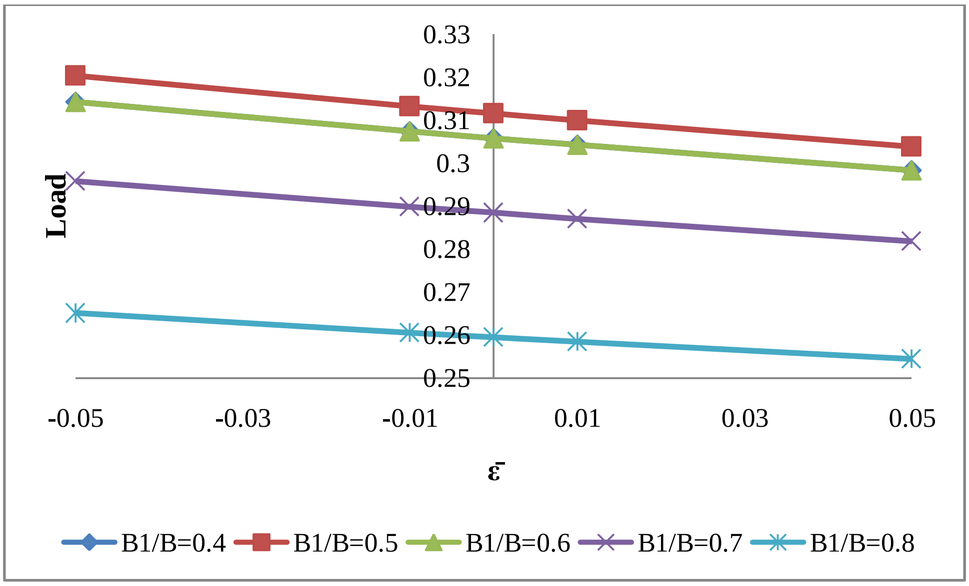

Fig.4.41 Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and B_1/B

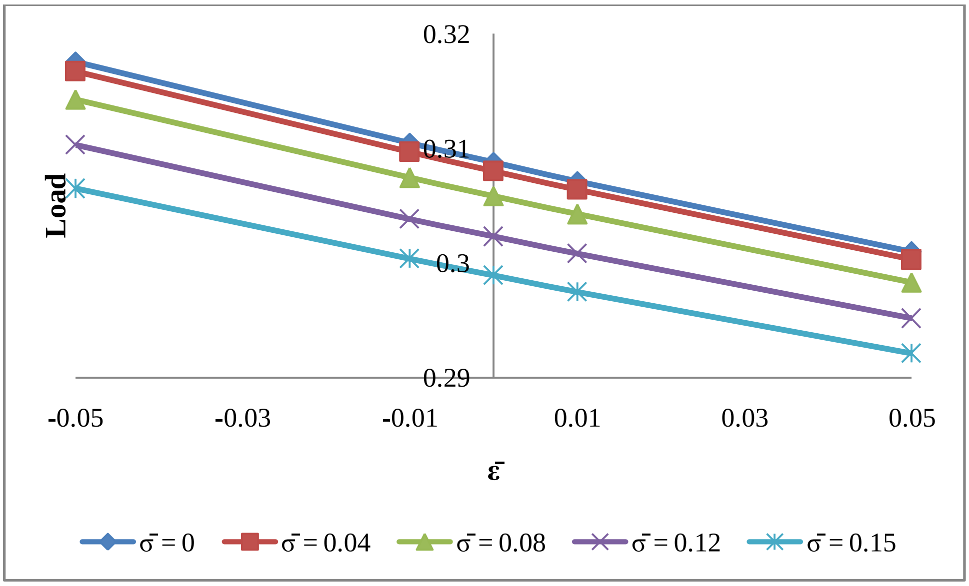

Fig.4.42 Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and $\bar{\sigma}$

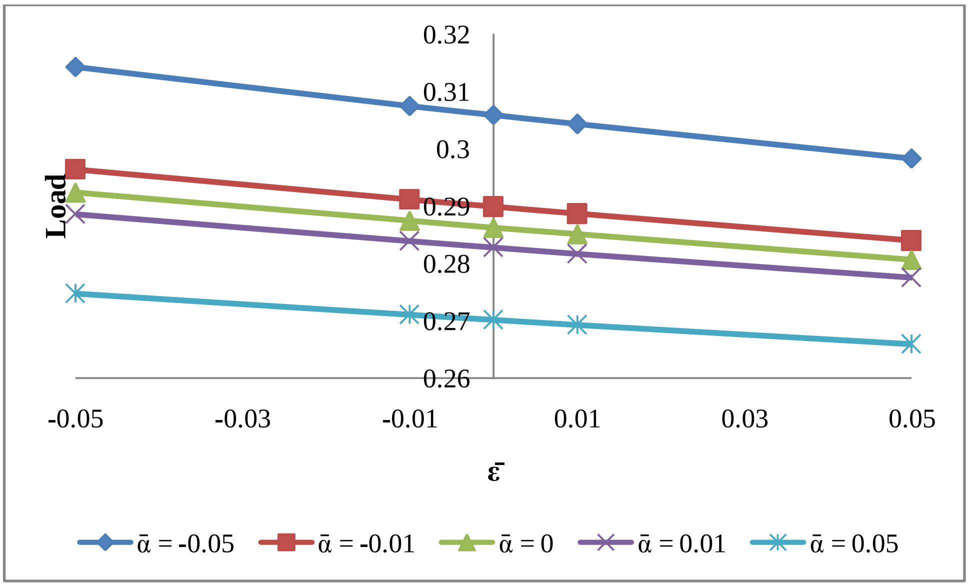

Fig.4.43 Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and $\bar{\alpha}$.

As can be seen from figures 4.38–4.43 the skewness follows the path of variance so far as the effect on the load distribution is concerned. This means the negatively skewed roughness increases the already increased load carrying capacity due to negative variance.

Further, the effect of magnetization on the load carrying capacity with respect to skewness can be disregarded up to some values of the magnetization.

This investigation offers the suggestion that the adverse effect of the roughness can be minimized by choosing suitably the magnetic strength and the step ratio. The effect of transverse surface roughness is adverse in general, which may be due to the fact that the roughness retards the motion of the lubricant resulting in decreased pressure and consequently decreased load carrying capacity.

A comparison of this study with the discussion carried out in earlier investigations suggests that in spite of the roughness the load carrying capacity gets increased by $\frac{\mu^*}{6}$, as compared to the case of conventional lubricant.

4.4 Validation

To verify the effect of megnetization in the bearing system the following comparision is made with the results done by Patel V. (2010) without considering magnetization. From the following Table 4.1 to 4.3, we can see that load carrying capacity increases more than 100% in the present study :

Table 4.1

	Load Carrying Capacity (calculated for $\mu* = 0.01$, $m = 1.1$, $\bar{\sigma} = 0.08$, $\bar{\varepsilon} = -0.05$)		
$\bar{\alpha} =$	**with consideration of Magnetic fluid as in this chapter**	**without consideration of Magnetic fluid by Patel, V. (2010)***	**increase in %**
-0.05	0.314248958	0.147582292	112.93
-0.01	0.296385868	0.129719201	128.48
0	0.292384943	0.125718277	132.57
0.01	0.288550239	0.121883572	136.74
0.05	0.274706899	0.108040233	154.26

Table 4.2

	Load Carrying Capacity (calculated for $\mu^* = 0.01$, $m= 1.1$, $\overline{\alpha} = -0.005$, $\overline{\sigma} = 0.08$)		
$\overline{\varepsilon} =$	**with consideration of Magnetic fluid as in this chapter**	**without consideration of Magnetic fluid by Patel, V. (2010)***	**increase in %**
-0.05	0.314248958	0.147582292	112.93
-0.01	0.307429193	0.140762526	118.40
0	0.305821608	0.139154941	119.77
0.01	0.304250328	0.137583661	121.13
0.05	0.298304717	0.131638050	126.60

Table 4.3

	Load Carrying Capacity (calculated for $\mu^* = 0.01$, $m= 1.1$, $\overline{\alpha} = -0.005$, $\overline{\varepsilon} = -0.05$)		
$\overline{\sigma} =$	**with consideration of Magnetic fluid as in this chapter**	**without consideration of Magnetic fluid by Patel, V. (2010) ***	**increase in %**
0	0.317583098	0.150916431	110.43
0.04	0.316735519	0.150068853	111.06
0.08	0.314248958	0.147582292	112.93
0.12	0.310282885	0.143616218	116.05
0.15	0.306478662	0.139811995	119.20

4.5 Conclusion

The performance of the bearing system improves significantly owing to the presence of the magnetic fluid lubricant. The adverse effect of roughness can be minimized by the positive effect of magnetization at least in the case of negatively skewed roughness. Even if, the magnetic field is suitably chosen, this chapter offers the suggestion that the roughness deserves to be accounted for while designing the bearing system. This investigation holds out some measures to mitigate the adverse effect of roughness by the magnetization suitably choosing the aspect ratio and the step location. Further, it is observed that this type of bearing system can support a load even in the absence of flow, unlike, the case of conventional lubricant.