Chapter 1

Introduction
The last century has witnessed an upsurge growth in theory of graphs and its related concepts. The unprecedented rise of computer science and its applications is one of the reasons for recognition of graph theory as a subject of larger interest. It is believed that the well celebrated Königsberg bridge problem has been the origin of graph theory and the representation of the problem by means of vertices and edges initiated by Swiss Mathematician Leonhard Euler(1707-1783) was the first graph drawn ever in the history of graph theory.

The subject, which remained almost untouched for more than hundred years after Euler’s death, suddenly came into limelight at the turn of 19th century due to four color problem. The problem was first posed by A. F. Möbius (1790-1868) in one of his lecture in 1840. About 10 years later Augustus De Morgan (1806-1871) discussed this problem with Sir William Hamilton in London. The problem was posed by his student Francis Guthrie who noticed that it was possible to color the counties(states) of England using only four colors so that no two neighbouring counties receive the same color in the Atlas. The question was then generalized with the feeling that whether four colors would be always sufficient to color any possible decompositions of the plane into regions? The problem became well known after Arthur Cayley published it in the first volume of the Proceedings of The Royal Geographic Society in 1879. The four color problem remained unsolved for almost 100 years till it was settled by Wolfgang Haken and Kenneth Appel in 1976.

In mathematical terminology the ‘Conjecture’ is an observation with the characteristic that it is supposed to be true but its proof is not known. Whereas, to disprove it, the counter example is also not available.

The problem stated above is known as four color conjecture because the proof given by Haken and Appel is not widely accepted as it is highly specialized, intricate and long. They first reduced the seemingly infinite problem of considering every planar graph to checking a finite and unavoidable set of (over 1900) reducible configurations. To reach at the conclusion more than 1200 hours of computer time were used which
includes 1010 units of operations on a high speed computers. The countless efforts to settle the conjecture and strong belief in it stimulated the research activity in the field of graph theory.

Besides of graph coloring problem the theory of domination has also triggered the research work in graph theory. The problem of determining the minimum number of queens to be placed on $8 \times 8$ chessboard so that all the squares are either attacked by a queen or occupied by a queen. In 1862 de Jaenisch determined that the minimum number of such queens is five. The problem is ultimately to find a dominating set of five queens which can be precisely stated as follows.

A vertex subset of graph $G$ is called a dominating set if every vertex is an element of vertex subset or is adjacent to at least one element of that subset.

Domination as a theoretical area in graph theory was formalized by Berge [6] in 1958 and Ore [35] in 1962. A detailed survey on theory of domination presented by Cockayne and Hedetniemi [13] has received considerable attention. The first comprehensive title “Fundamentals of Domination in Graphs” by Haynes et al. [23] contains the elegant discussion on domination and subset related problems such as independence, covering and matching. Theory of domination has close interaction with many branches of mathematics, engineering and social sciences.

The relation between graph coloring and dominating set as well as various vertex subsets is immediate. A vertex subset is called independent if no two vertices of that subset are adjacent. The independence number $\beta_0(G)$ is the maximum cardinality of an independent set of a graph. The minimum number of an independent set required to partition vertex subset of a graph is called the chromatic number of that graph and the process is known as the chromatic partitioning of graph.

A property $P$ with reference to a set $S$ is said to be minimal if no proper subset of $S$ have that property while $P$ is called maximal if no superset of $S$ have that property.

A minimal dominating set may or may not be independent whereas an independent set has the dominance property only if it is a maximal independent set. An independent
dominating set is same as a maximal independent set. Thus it is interesting to discuss coloring problems with reference of dominating set.

The present thesis is aimed to report some investigations on b-coloring of graphs and dominator coloring of graphs.

The first chapter is of introductory nature which provides a quick look of the remaining chapters while all the basic definitions, terminology and notations are introduced in Chapter-2.

There are many variants of graph colorings. The a-coloring, b-coloring, dominator coloring, fall coloring, equitable coloring, edge coloring, acyclic coloring are really noteworthy. The concept of b-coloring was introduced by Irving and Manlove [24]. The third chapter is focused on b-coloring of some cycle and path related graphs. The exact value of b-chromatic number is investigated for the graphs obtained from the given graph by means of some graph operations like switching of a vertex, duplication of a vertex and edge splitting of a graph. We have also discussed b-continuity and b-spectrum of the graphs.

The penultimate Chapter-4 is dedicated to the discussion of b-coloring of some wheel related graphs. The b-chromatic number is investigated for the graphs obtained by duplication of a vertex and splitting of a graph. The b-continuity and b-spectrum are also discussed for the graphs under consideration.

A variant of proper coloring with a blend of domination is introduced by R.Gera et al. [18] which is termed as dominator coloring. The last Chapter-5 is focused on the dominator coloring of graphs. We contribute several results on dominator chromatic number of graphs.

Throughout the thesis some open problems are posed and further scope of research is indicated. A list of symbols is provided just after the table of contents. The bibliography is provided alphabetically at the end in MLA (Modern Language Association) format.
The work reported in the thesis give rise to 12 research papers out of which 9 papers are already published in scholarly, indexed and peer reviewed journals. The reprints of the published papers are provided as an Annexure.
Chapter 1. Introduction

List of Publications Arising From the Thesis

   
   ISSN: 2319-5215(Online) ISSN: 2249-3328(Print)
   
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9. Dominator Coloring of Some Cycle Related Graphs, *Journal of Graph Labeling*, (Accepted for publication).

10. Dominator Coloring of Some Graphs, (To be communicated for publication).

11. Dominator Coloring of Total Graph of Some Graphs, (To be communicated for publication).

12. Dominator Coloring of Path and Bistar Related Graphs, (To be communicated for publication).
Details of the Work Presented in Conferences

1. The paper entitled as “b-chromatic Number of Some Cycle Related Graphs” was presented in The Tenth Annual Conference of Academy of Discrete Mathematics and Applications (ADMA) held at Reva Institute of technology and Management, Bangalore (Karnatak) during 10-13 June, 2014.

2. The paper entitled as “b-chromatic Number of Some Cycle Related Graphs” was presented in One Day National Conference on Recent Trends in Mathematics and its Applications held at C.U.Shah University, Wadhwan City (Gujarat) on 19th December, 2014.

3. The paper entitled as “Some New Results on b-coloring of Graphs” was presented in The Eleventh Annual Conference of Academy of Discrete Mathematics and Applications (ADMA) held at B. S. Abdur Rahman University, Chennai (Tamil Nadu) during 10-12 June, 2015.

4. The paper entitled as “Dominator Coloring of Some Cycle Related Graphs” was presented as poster in The XXX Gujarat Science Congress-2016 on Challenges For Science and Technology Education During Coming Decades: Preparing For a Sustainable Gujarat, held at K.S.K.V. Kachchh University, Bhuj (Gujarat) during 6-7 February, 2016 and awarded as the best research paper.

5. The paper entitled as “Dominator Coloring of Some Wheel Related Graphs” was presented in The 9th National Level Science Symposium on Recent Trends in Science and Technology held at Christ College, Rajkot (Gujarat) on 14th February, 2016.

6. The paper entitled as “Dominator Coloring of Some Degree Splitting Graphs” was presented in The 7th National Conference On Emerging Vistas of Technology in 21st Century held at Parul University, Vadodara (Gujarat) on 8-9 April, 2016.
7. The paper entitled as “Dominator Coloring of Some Graphs” was presented as poster in The International conference on Discrete Mathematics (ICDM-2016) held at Siddaganga Institute of Technology, Tumkur (Karnatak) during 9-11 June, 2016.