In this section, performance of the risk-return optimal portfolios formed using the simplified, single index model developed by Elton & Gruber is evaluated. Rate of returns on 250 shares traded on The Stock Exchange, Mumbai and the return on National Index are considered for the purpose of the study. For the computation of historical beta and unsystematic risk used by the model, 48 months' returns are taken. The excess return to beta sorting procedure and the cut-off formula are applied for constructing the portfolios. Risk free rate of return is taken as 10% per annum. The performance of the portfolios selected by the model is compared with geometric rate of return on the Natex.

Following five portfolios were formed and tested:

<table>
<thead>
<tr>
<th>PORTFOLIO</th>
<th>FORMATION PERIOD</th>
<th>GENERATION PERIOD</th>
<th>EVALUATION PERIODS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>APRIL 90 – MARCH 94</td>
<td>APRIL 1994</td>
<td>APRIL 94 – MAR 95</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>APRIL 94 – MAR 96</td>
</tr>
<tr>
<td>B</td>
<td>OCT 90 – SEP 94</td>
<td>OCT 1994</td>
<td>OCT 94 – SEP 95</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OCT 94 – SEP 96</td>
</tr>
<tr>
<td>C</td>
<td>APRIL 91 – MARCH 95</td>
<td>APRIL 1995</td>
<td>APRIL 95 – MAR 96</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>APRIL 95 – MAR 97</td>
</tr>
<tr>
<td>D</td>
<td>OCT 91 – SEP 95</td>
<td>OCT 1995</td>
<td>OCT 95 – SEP 96</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OCT 95 – SEP 97</td>
</tr>
<tr>
<td>E</td>
<td>APRIL 92 – MARCH 96</td>
<td>APRIL 1996</td>
<td>APRIL 96 – MAR 97</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>APRIL 96 – MAR 98</td>
</tr>
</tbody>
</table>
The portfolios selected by the model along with recommended proportion of investment made in various stocks and the historical betas of the shares are given in Annexure 2. Returns on stock and market index in the formation period have been the basis for calculating betas that are used for optimal portfolio generation. The performance of the portfolios constructed is monitored in subsequent periods of evaluation.

Analysis of the portfolios selected by the model reveals that the composition of the portfolios varies significantly over the period. The lower the beta of the security, the higher the proportion of investment suggested by the model. In other words, investment in the securities having high beta has been lower. Historical betas of the securities selected have generally been low, ranging between 0.3-0.7. Portfolios generated, though risk-return optimal, can not be said to be well-diversified for the model recommends higher proportion of investments in few securities. For example, in case of portfolio A, ranked by the proportion of investment, 53% of the total investments are in 3 shares and investments in 5 shares take away around 2/3 rd of the investible funds. Percentage-wise analysis shows that per share investment above 10% is 45% of the total investment and above 5% is 80% in case of Portfolio A. Following table shows the extent of concentration of scrips in the portfolio:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>IN TOP 3 SHARES</td>
<td>53.18</td>
<td>54.17</td>
<td>48.32</td>
<td>54.73</td>
<td>36.59</td>
</tr>
<tr>
<td>IN TOP 5 SHARES</td>
<td>67.27</td>
<td>68.02</td>
<td>60.75</td>
<td>68.5</td>
<td>46.71</td>
</tr>
<tr>
<td>ABOVE 10%</td>
<td>45.01</td>
<td>45.57</td>
<td>40.98</td>
<td>47.63</td>
<td>24.65</td>
</tr>
<tr>
<td>ABOVE 5%</td>
<td>80.24</td>
<td>84.7</td>
<td>60.75</td>
<td>73.61</td>
<td>41.99</td>
</tr>
</tbody>
</table>

160
Portfolio A:

**Top 5 shares and the % of Investment**

<table>
<thead>
<tr>
<th>Share Name</th>
<th>% of Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asian Paints (India) Ltd.</td>
<td>0.2746</td>
</tr>
<tr>
<td>Flex Industries Ltd.</td>
<td>0.1755</td>
</tr>
<tr>
<td>Oriental Hotels Ltd.</td>
<td>0.0817</td>
</tr>
<tr>
<td>Engine Valves Ltd.</td>
<td>0.0718</td>
</tr>
<tr>
<td>Automobile Corporation Of Goa Ltd.</td>
<td>0.0691</td>
</tr>
</tbody>
</table>

**Performance Evaluation:**

When money is invested in the portfolio in April 1994 as per the proportions indicated by the model, geometric return on the portfolio was 0.0148% per month at the end of 12 month period. During the same period geometric mean return on National Index has been minus 0.0104% per month. The portfolio selected by the model has thus outperformed Natex in the one year period.

Had an investor continued to hold on to the same portfolio for a period of two years (commencing from April 1994 till March 1996), he would have earned geometric monthly return of 0.0097% against market return of minus 0.0062%. This indicates that the portfolio held for long term without reshuffling also outperformed the market index in this evaluation period.
Portfolio B:

*Top 5 shares and the % of Investment*

<table>
<thead>
<tr>
<th>Shares</th>
<th>% of Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asian Paints (India) Ltd.</td>
<td>0.2916</td>
</tr>
<tr>
<td>Flex Industries Ltd.</td>
<td>0.1641</td>
</tr>
<tr>
<td>Gujarat State Fertilizer Company Ltd.</td>
<td>0.0860</td>
</tr>
<tr>
<td>Oriental Hotels Ltd.</td>
<td>0.0706</td>
</tr>
<tr>
<td>Engine Valves Ltd.</td>
<td>0.0679</td>
</tr>
</tbody>
</table>

*Performance Evaluation:*

The portfolio constructed in October 1994 as per the proportions indicated by the model when held for one year had earned geometric monthly return of minus 0.0036. During the same period geometric mean return on National Index has been negative 0.0204. It can be noted that the portfolio return was marginally higher than the market proxy.

Had the same portfolio been held for a period of two years, it would have earned geometric return of negative 0.0082% against market return of minus 0.0132, indicating that the portfolio held for long term could beat the market index during this period.
Portfolio C:

**Top 5 shares and the % of Investment**

<table>
<thead>
<tr>
<th>Shares</th>
<th>Investment %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asian Paints (India) Ltd.</td>
<td>0.3003</td>
</tr>
<tr>
<td>Flex Industries Ltd.</td>
<td>0.1095</td>
</tr>
<tr>
<td>Engine Valves Ltd.</td>
<td>0.0734</td>
</tr>
<tr>
<td>Oriental Hotels Ltd.</td>
<td>0.0679</td>
</tr>
<tr>
<td>Automobile Corporation Of Goa Ltd.</td>
<td>0.0564</td>
</tr>
</tbody>
</table>

**Performance Evaluation:**

Returns on market index as well as the portfolios were negative during the evaluation periods of one and two years.

<table>
<thead>
<tr>
<th></th>
<th>Monthly Rate of Return (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12 months</td>
</tr>
<tr>
<td>Portfolio</td>
<td>-0.0009</td>
</tr>
<tr>
<td>Natex</td>
<td>-0.0019</td>
</tr>
</tbody>
</table>

As can be seen from the Table, the portfolio had outperformed the market proxy in 12 month period whereas underperformed in the 24 month period.
Portfolio D:

Top 5 shares and the % of Investment

<table>
<thead>
<tr>
<th>Share</th>
<th>% of Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asian Paints (India) Ltd.</td>
<td>0.3504</td>
</tr>
<tr>
<td>Flex Industries Ltd.</td>
<td>0.1259</td>
</tr>
<tr>
<td>Oriental Hotels Ltd.</td>
<td>0.0710</td>
</tr>
<tr>
<td>Engine Valves Ltd.</td>
<td>0.0693</td>
</tr>
<tr>
<td>Parke Davis (India) Ltd.</td>
<td>0.0684</td>
</tr>
</tbody>
</table>

Performance Evaluation:

During both the evaluation periods the portfolio could not beat the performance of the market proxy as seen in the Table given below:

<table>
<thead>
<tr>
<th></th>
<th>12 months</th>
<th>24 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio</td>
<td>-0.0089</td>
<td>-0.0133</td>
</tr>
<tr>
<td>Natex</td>
<td>-0.0059</td>
<td>-0.0041</td>
</tr>
</tbody>
</table>

Portfolio E:

Top 5 shares and the % of Investment

<table>
<thead>
<tr>
<th>Share</th>
<th>% of Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asian Paints (India) Ltd.</td>
<td>0.2465</td>
</tr>
<tr>
<td>Jay Shree Tea &amp; Industries Ltd.</td>
<td>0.0630</td>
</tr>
<tr>
<td>Flex Industries Ltd.</td>
<td>0.0564</td>
</tr>
<tr>
<td>Oriental Hotels Ltd.</td>
<td>0.0540</td>
</tr>
<tr>
<td>Gujarat State Fertilizer Company Ltd.</td>
<td>0.0472</td>
</tr>
</tbody>
</table>
Performance Evaluation:

When money is invested in this portfolio in April 1996 in the proportions indicated by the model, geometric return on the portfolio at the end of 12 month period was minus 0.0286% per month. During the same period geometric mean return on National Index has been negative 0.0037% per month. The portfolio selected by the model had underperformed Natex.

Had the same portfolio held for a period of two years (commencing from April 1996 till March 1998), geometric monthly return of minus 0.0442% against market return of 0.0051% would have been earned. This indicates that the portfolio held for long term without reshuffling also could not beat the market index in this period.

If the investor makes use of the model results to invest money in equities at given time intervals, the recommendations can be grouped in the category of partial disinvestments in few scrips, disposal of certain stocks, additional investments in case of some existing holdings and inclusion of new shares. In other words, by taking Portfolio A that was generated in April 1994 and comparing with the shares in Portfolio B, an investor is in a position to list out the new shares added in the portfolio and the shares that had been sold out. Similarly, comparing the proportions in two successive periods of the shares that are common in the portfolios, say between Portfolio A and Portfolio B, gives an idea as to additional investment / partial disinvestments recommended by the model. Changes in the portfolio through inclusion of new shares and complete disinvestments of scrips (additional investments and partial sale in the existing holdings are not considered) are given below:
<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Investment (New)</th>
<th>Disinvestment (Full)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Kirloskar Cummins Ltd.</td>
<td>Rane (Madras) Ltd.</td>
</tr>
<tr>
<td></td>
<td>Caprihans India Ltd.</td>
<td>I F B Industries Ltd.</td>
</tr>
<tr>
<td></td>
<td>Electrosteel Castings Ltd.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ingersoll-Rand (India) Ltd.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Morarjee Goculdas Spg. &amp; Wvg. Co. Ltd.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Parke Davis (India) Ltd.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shanthi Gears Ltd.</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Bannari Amman Sugars Ltd.</td>
<td>Ashok Leyland Ltd.</td>
</tr>
<tr>
<td></td>
<td>E I D Parry (India) Ltd.</td>
<td>Caprihans India Ltd.</td>
</tr>
<tr>
<td></td>
<td>Hindustan Ciba-Geigy Ltd.</td>
<td>Ingersoll-Rand (India) Ltd.</td>
</tr>
<tr>
<td></td>
<td>Indo-Asahi Glass Company Ltd.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sandvik Asia Ltd.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unititech Ltd.</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>Bimetal Bearings Ltd.</td>
<td>Bannari Amman Sugars Ltd.</td>
</tr>
<tr>
<td></td>
<td>Ferro Alloys Corporation Ltd.</td>
<td>E I D Parry (India) Ltd.</td>
</tr>
<tr>
<td></td>
<td>Godrej Foods Ltd.</td>
<td>G S L (India) Ltd.</td>
</tr>
<tr>
<td></td>
<td>G T C Industries Ltd.</td>
<td>Hindustan Ciba-Geigy Ltd.</td>
</tr>
<tr>
<td></td>
<td>I B P Company Ltd.</td>
<td>Indo-Asahi Glass Company Ltd.</td>
</tr>
<tr>
<td></td>
<td>I F B Industries Ltd.</td>
<td>Indo National Ltd.</td>
</tr>
<tr>
<td></td>
<td>Ingersoll-Rand (India) Ltd.</td>
<td>Morarjee Goculdas Spg. &amp; Wvg. Co. Ltd.</td>
</tr>
<tr>
<td></td>
<td>I V P Ltd.</td>
<td>Parke Davis (India) Ltd.</td>
</tr>
<tr>
<td></td>
<td>Jay Shree Tea &amp; Industries Ltd.</td>
<td>Shanthi Gears Ltd.</td>
</tr>
<tr>
<td></td>
<td>Motor Industries Company Ltd.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Parasrampuria Synthetics Ltd.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Parasrampuria Industries Ltd.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rollatainers Ltd.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Siv Industries Ltd.</td>
<td></td>
</tr>
</tbody>
</table>

Additional investment / partial disinvestments made in the portfolios are given in Annexure II.
Following shares, with low excess return to beta ratio, found berth in all five portfolios generated by the model:

a) Asian Paints Ltd  
b) Engine Valves Ltd  
c) Flex Industries Ltd  
d) Gabriel India Ltd  
e) Gammon India Ltd  
f) Oriental Hotels Ltd

Except Gabriel India Ltd and Gammon India Ltd, the other four shares from the above list have also figured in the top 5 investments in all five portfolios. Asian Paints shares continued to occupy the first slot with the model recommending majority portion of the funds (as high as 30% in Portfolio A) to be invested in this.

**Overview of Model Results:**

The model gives an idea about what shares to be bought in which proportion and the portfolio so constructed would be risk-return optimal. Five portfolios have been formed in three consecutive years and their performance has been evaluated by taking 'beating the market index on the basis of geometric return' as criterion. Portfolio A and B outperformed the market index during both the evaluation periods, return on Portfolio C was higher than index in 12 month period but underperformed the proxy in the long term of two years whereas Portfolio D and portfolio E could not beat the market, in one year as well as two year evaluation periods. As the model generally chose low beta shares (the historical beta of the shares considered for portfolio selection has been less than 1) for inclusion in the portfolio,
the return on the portfolio is expected to be lower than that of the market. However, in one year evaluation period, 3 out of 5 portfolios generated returns higher than the market proxy and in long term 2 out of 5 portfolios could beat the index on the basis of return.

The model though optimizes risk and return through diversification, a close look at the proportion of investment made indicates that there is concentration on few securities. For instance, in case of portfolio A & B, nearly 75% of total funds were invested in 6 shares and a lion’s share of investment is held in Asian Paints Ltd (approximately 30%)

Fund managers of large financial and investment institutions may find it difficult to implement the recommendation (of the model) for the simple reason that major portion of the fund invested is in only one/two shares. Moreover, acquiring shares of one particular company in large quantities attracts high impact cost. Similarly, when the portfolio is to be reshuffled, disinvesting the share in large quantities would also depress the market mood, pushing the share price down.

Another drawback of the model recommendation is that asset allocation does not vary with the size of the fund to be invested. Larger the fund size, the more would be the problems relating to acquisition and disinvestment.
SECTION 2

CAPITAL ASSET PRICING MODEL

Results of Capital Asset Pricing Model are discussed in this section. Testing of the CAPM involves estimation of portfolio betas and the risk-return trade-off. Portfolio betas are estimated by using the following combinations:

I. Portfolios based on non-overlapping betas computed during non-overlapping periods;
II. Portfolios based on overlapping betas computed during overlapping periods.
III. Portfolios based on non-overlapping betas computed during overlapping periods;
IV. Portfolios based on overlapping betas computed during non-overlapping period.

Betas estimated on return as well as excess return form are used to make the above combinations and the CAPM test is done as under:

- **Predictive Model**, where the associationship between portfolio betas in time period t and returns in time period t+1 is tested.
- **Non-Predictive Model**, where associationship between both risk and return at time period t is tested.

The results of the CAPM tests are presented in the following sequence:
- CAPM tests of all the four combinations of betas estimated in excess return form (using Natex as well as RBI Index).
• CAPM tests of all the four combinations of betas estimated in return form.

Two sets of cross sectional regressions were performed for each combination of portfolios considered:

- Multiple regressions with beta, squared beta and standard error estimations on portfolios as independent variables and portfolio return as dependent variable.
- Simple regression with portfolio beta as the sole explanatory variable and return on portfolios as the predicted variable.

Cross sectional regression results are reported in the following order:
- First, the regression equations.
- Secondly, the standard error of the regression coefficients (stderr).
- Thirdly, depending on the form of CAPM test undertaken, t values of regression coefficients at 5% confidence level. t values significant at this level is indicated by an asterisk. (t values)
- Finally, values of coefficient of determination. (R²)

Estimated betas, beta squared values, standard errors and expected returns for each of the combinations are given in Annexure 4 to this report.
A. EXCESS RETURN

CAPM results using excess return are given in two parts. The results using Natex as market proxy under all the four combinations are given in part 1 and RBI index under part 2.

1. Natex As Market Proxy

I. Non - Overlapping Portfolios during Non-Overlapping Period

A) Preective Model

Twenty non-overlapping portfolios were constructed by sorting out betas based on excess returns on 200 securities and Natex during the 32 months from April 90 to Nov. 92; betas were estimated using data during the next 32 months i.e. from Dec. 92 to July 95; portfolio returns were estimated from Aug. 95 to Mar. 98. Independent and dependent variables used for regression are given in Annexure 4(1) (1).

**Multiple Regression:**

\[
R_p = -0.35966 + 0.25119 - 0.08424^2 + 0.85825
\]

\[
\text{std error} \quad 0.160158 \quad 0.18491 \quad 0.05917 \quad 0.33661
\]

\[
\text{t values} \quad -2.24568 \quad 1.35848 \quad .42384 \quad 2.54972
\]

\[R^2 = 0.3\]

**Simple Regression:**

\[
R_p = 0.01388 + 0.0222(R_m - R_f)
\]

\[
\text{std error} \quad 0.07009 \quad 0.0721
\]

\[
\text{t values} \quad 0.1979 \quad 0.3086
\]

\[R^2 = 0.005000\]
The following could be inferred from these results:

- There was positive relationship between portfolio beta and returns;
- Null hypothesis (viz., both intercept and beta coefficients are equal to zero) tested using 't' test could not be rejected as observed 't' values under simple regression was lower than expected values at 5% significant level for 19 degrees of freedom.
- Unsystematic risk, with significant t value seems to have a positive impact on return rather than systematic risk under multiple regression analysis.

II. Overlapping Periods With Non-Overlapping Portfolios:

A. Non-Predictive Model

The testing (on the lines of Black, Jensen and Scholes methodology) is done in the following sequence:

1. Historical betas on 200 equity shares were calculated by considering monthly excess returns from April 1990 to March 1993 (36 months).
2. These shares were ranked in descending order of their betas.
3. Twenty equally weighted portfolios during each time period.
4. Monthly excess returns on National Index, which has been taken as market proxy, have also been estimated for these 20 time periods.
5. By taking return 1 for all 20 periods on portfolio 1 as dependent variable series and return on market proxy for the respective periods as independent variable series, regression analysis has been performed. This is done to estimate the portfolio betas.
6. Correlation analysis is done to find out the relationship between estimated betas on portfolios and average monthly excess return on all twenty portfolios.

7. Cross sectional regression analysis is done to test the properties of Security Market Line.

8. The following form of CAPM is tested.

\[ R_p - R_f = \alpha_p + \beta_p (R_m - R_f) \]

Cross sectional regression analysis is used with estimated excess return on portfolios as dependent variable and estimated betas as independent variable to test the properties of SML and the hypothesis that market portfolio is efficient.

Following hypotheses were tested:

a) High beta portfolios have high level of returns i.e., positive relationship between beta and return.

b) Regression coefficients take the values as expected theoretically.

c) Linear relationship exists between beta and return.

d) No factor other than beta is positively priced in the market.

By regressing portfolio returns on market returns, it is observed [Annexure IV (1)] that correlation between excess return on market index and portfolios were not very high as most of the portfolios have correlation coefficient around 0.5. Estimated intercept in all but two portfolios is positive. Despite negative returns on market index, estimated excess return in case of all but four portfolios were positive. Market factor accounts for around one-half of the risk of a portfolio. Moreover, on all 20 portfolios, beta estimated using monthly returns
during the 20 periods were very low, ranging narrowly between 0.22 and 0.48. Thus, identifying clearly the variations in return with different beta levels was not possible. Portfolio betas do not follow any particular pattern; high beta portfolio in period t-1 does not seem to have high betas in period t.

The results of second pass regression are shown below:

**Multiple Regression:**

\[
R_p = 0.02624 + 0.086955 - 0.1249^2 + 0.297822 \\
\text{Std. error} = 0.0382 \\
\text{t values} = 0.68691 \\
R^2 = 0.048
\]

**Simple Regression:**

\[
R_p = 0.00040 + 0.005084(R_m - R_f) \\
t values = 0.06436 \\
R^2 = 0.004692
\]

There is a positive relationship between beta and returns. Student's 't' test is used to find out whether regression coefficients are significantly different from zero. As can be seen from the table above, the t values for both intercept and slope coefficients were lower than the table values, indicating that these were not significantly different from zero. Testing for positive pricing of unsystematic risk and existence of non-linearity between risk and return confirmed the results were in accordance with the CAPM specifications i.e., the regression coefficients were not different from zero.
Risk-Return Framework Of Portfolios With 20 Scrips

Less variation in estimated beta factors (when portfolios with 10 scrips were formed earlier) induced an attempt to construct more diversified portfolios and observe the risk-return relationship. Thus, following the same procedure described earlier, ten equally weighted portfolios each with 20 scrips were formed and the results of the test are shown below.

Multiple regression:

\[
R_p = 0.0592 + 0.1952 \beta_1 + 0.292 \beta_2 + 0.671 \beta_3
\]

| Std. error | 0.039 | 0.192 | 0.295 | 0.286 |
| t values | -1.51 | 1.014 | 0.98 | 2.34 |
| \( R^2 \) | 0.52 |

Simple regression:

\[
R_p = 0.0156 + 0.07145(R_m - R_f)
\]

| t values | 0.4591 | 0.75631 |
| \( R^2 \) | 0.06673 |

Even in case of a well diversified portfolio as well, betas range in a narrow band of 0.42 to 0.22; [[Annexure 4 (II) (1)]] and risk-return characteristics, as depicted by second pass regression, was also not very different from the results obtained earlier. That is, both intercept and slope coefficients were close to zero.
B. Predictive Model

Testing under this is based on Fama & MacBeth methodology where portfolios are formed based on betas in period t-2, estimations on portfolio betas, and other variables are made based on returns in period t-1 and the CAPM testing is carried out in period t.

As mentioned earlier, this methodology enables testing not only for positive relationship between betas and returns on portfolios but also testing for non-linearity and for the influence of other unsystematic factors on returns. Apart from betas on twenty equally weighted portfolios, b_pmi, following additional variables were also calculated:

- Average of the squared values of betas of individual securities in each of the portfolio; (b^2_pmt-1)
- Average of standard deviations of market model residuals for individual securities in the portfolio. (S_pmt-1)
- All the three of the independent variables were recomputed from monthly returns and updated yearly.

a) Portfolio Formation Period: Monthly returns on all 200 scrips included in the sample and Natex during 30 months from April 1990 to September 1992 (t-2) were the data used for beta computation needed for portfolio formation;

b) Estimation Period: Next 30 months' portfolio as well as market returns from Oct. 92 to March 95 (t-1) were the basis for estimating the independent variables;

c) Testing Period: dependent variable (R_p) predicted every month from April 1995 through March 1998 (t) and the CAPM is tested.
Relationship between return and some combination of independent variables is tested using the least square regression method. Following are the equations used for establishing relationship:

\[
R_{pt} = \gamma_{1t} + \gamma_{2t} b_{pm,t-1} + \gamma_{3t} b^2_{pm,t-1} + \gamma_{4t} s_{p,t-1}(e_t) + \gamma_{pt} 
\]

\[
R_{pt} = \gamma_{1t} + \gamma_{2t} b_{pm,t-1} + \gamma_{3t} b^2_{pm,t-1} + \gamma_{pt} 
\]

\[
R_{pt} = \gamma_{1t} + \gamma_{2t} b_{pm,t-1} + \gamma_{4t} \bar{s}_{p,t-1}(e_t) + \gamma_{pt} 
\]

\[
R_{pt} = \gamma_{1r} + \gamma_{2r} b_{pm,t-1} + \gamma_{pt}, p = 1,2,\ldots,20. 
\]

The mean values and standard deviations of the regression coefficients for the full testing period (36 months) and two sub-periods of 18 months each are computed. For testing regression coefficients 't' statistics of following form is used between dependent variable, Return in period t and above mentioned independent variables at t-1.

\[
t(\gamma) = \frac{\hat{\gamma}_j}{S(\gamma_j)/\sqrt{t-1}}
\]

Durbin- Watson's d test

\[
d = \frac{\sum_{t=2}^{n}(\mu_t-\mu_{t-1})^2}{\sum_{t=2}^{n}(\mu_t)^2}
\]

has been used to test for the existence of autocorrelation of variables over the time.

In other words, equation (1) establishes relationship between return and all three independent variables, equation (2) and (3) considered in
Results Of Hypothesis Testing:

A) Intercept:

Hypothesis : \( H_0 : (\bar{R} - \bar{R}_f) = 0 \)

Average risk free return is deducted from the mean values of intercept computed from each of the equations. The average values of intercept were low. The null hypothesis tested using 't' value indicated that except in second sub-period in Panel A, the values were not significantly different from zero.

B) Non-Linearity:

Hypothesis \( H_0 : \gamma_3 = 0 \)

Average of squared values of individual security betas has been taken as an additional variable along with beta (in panel B) and standard error (in panel D) to test for non-linearity.

For the full testing period in both the panels, t values of the coefficient
was quite low and significantly not different from zero. For the sub-
periods, low values in case of panel A and fairly high values in Panel
B when compared against expected t values at 17 degrees of freedom,
showed that the above stated hypothesis can not be rejected.

C) Pricing Of Unsystematic Risk:

Hypothesis: $H_0 : \gamma_4 = 0$

This is a test aimed at confirming / rejecting the proposition that
systematic risk is the only factor which is priced by the market. The
results obtained were indicative of the hypothesis as can be seen from
low values of $\gamma$ in panel C & D and t values are also statistically
smaller and not significantly different from zero.

D) Relationship Between Beta And Returns:

Hypothesis: $H_0 : \gamma_2 = 0$

If the equilibrium model holds, positive relationship is hypothesised
between systematic risk of the portfolio and the expected return on
the portfolio. Testing the hypothesis with t test should confirm the
observed values are significantly greater than zero.

For full testing period, the sign of the coefficient was negative in case
of all panels except in panel A; For the sub-period 1, the sign was
negative in all panels; in case of sub-period 2, negative signs are
observed in panels B & D and in other two panels it is positive. Since
the observed coefficient values are small and comparison of the same
against expected t values offers confirmation of that they are close to
zero. Thus, the results obtained under the predictive model of CAPM
are not different from the results obtained in non-predictive model.
Testing For Autocorrelation:

The result of Durbin-Watson 'd' tests \((r_1-r_f), r_3 \text{ and } r_4\) computed about means that are assumed to be zero, to test the proposition that there is no information in the time series of past values that could be used as a tool for making excess returns in future, confirmed that there was no autocorrelation in these parameter estimates. \(d\) values of these coefficients were closer to 2 and were statistically insignificant.

If market is efficient and pricing of asset takes place in a fair manner, deviations from \(\mu\) from its mean \(R_m\) \(R_f\) would be random, regardless of what happened in time period \(t-1\). Autocorrelations were computed by assuming sample means to test whether some information in the time series of past values help in estimating future values. Negative autocorrelation has been found in case of betas for all test periods of all other panels except for full period and sub-period 2 in panel B.

III Overlapping Portfolios During Overlapping Periods

B. Non - Predictive Model

The methodology described under non-predictive model under II above is followed and betas and returns were estimated for 20 periods. However, while forming portfolios, instead of dividing the sample into 20 parts, first 5 highest beta scrips and 5 lowest beta scrips were excluded from the sample; remaining 190 scrips were then grouped into portfolios of 10 scrips each and returns were estimated. Overlapping period portfolios depicted low beta range as under combination II and except in one period, estimated intercept values were positive. [Annexure 4 (III) (1)]
**Multiple Regression:**

\[
R_p = 0.007157 - 0.08187 + 0.132008^2 + 0.325991
\]

\[
\text{Std. error} = 0.028643 \quad 0.132489 \quad 0.194648 \quad 0.297285
\]

\[
t \text{values} = 0.249872 \quad -0.61792 \quad 0.67819 \quad 1.096561
\]

\[
R^2 = 0.16
\]

**Simple Regression:**

\[
R_p = 0.006120 + 0.013236(R_m - R_f)
\]

\[
\text{Std. error} = 0.00703 \quad 0.01908
\]

\[
t \text{value} = 0.87018 \quad 0.69365
\]

In this case, positive relationship is seen between risk and return and regression coefficients for intercept and slope are close to zero as in the case of other combinations.

**IV Overlapping Portfolios During Non-Overlapping Period**

**A. Predictive Model**

Betas were estimated using monthly returns during 32 month period, i.e. from December 92 to July 95 and overlapping portfolios with 10 scrips each were formed. For the purpose of making portfolios, initially scrips were ordered by betas in descending order and top ten securities were included in portfolio 1. Leaving 5 scrips i.e. from scrip 6 to scrip 15, (10 scrips) were considered as portfolio 2 and so on. In all, 39 portfolios were created and returns during the next 32 months i.e. from August 95 to March 98, on all these 39 portfolios were estimated.
Unlike portfolio betas under combinations II and III, estimated betas on portfolios in non-overlapping period were wider in range and estimated returns on portfolios in many cases were negative. [Annexure 4 (IV)(1)] Results of the cross sectional regression are reproduced below:

**Multiple Regression:**

\[
R_p = 0.2826 + 0.292393 \cdot \beta \\
\text{Std. error} = 0.2571, 0.391947 \\
t \text{values} = 0.099, 0.746002 \\
R^2 = 0.086
\]

**Simple Regression:**

\[
R_p = 0.0139 - 0.005(R_m - R_f) \\
t \text{values} = 0.27399, 0.0928 \\
R^2 = 0.0002
\]

It can be noted that results obtained here are not different from the results of other three options. The hypothesis that intercept is equal to zero is not rejected as observed t value is close to zero while the relationship between beta and return is negative and the slope is not significantly different from zero. Coefficients for squared beta values and standard error were also close to zero.
Results of CAPM testing are summarised below:

<table>
<thead>
<tr>
<th>Combination</th>
<th>Model</th>
<th>Intercept #</th>
<th>Slope*</th>
<th>Sign@</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Predictive</td>
<td>$\gamma_1 = 0$</td>
<td>$\gamma_1 = 0$</td>
<td>$\gamma_2 = 0$</td>
</tr>
<tr>
<td>II</td>
<td>Non-Predictive</td>
<td>$\gamma_1 = 0$</td>
<td>$\gamma_1 = 0$</td>
<td>$\gamma_2 = 0$</td>
</tr>
<tr>
<td>II</td>
<td>Predictive</td>
<td>$\gamma_1 = 0$</td>
<td>$\gamma_1 = 0$</td>
<td>$\gamma_2 = 0$</td>
</tr>
<tr>
<td>III</td>
<td>Non-Predictive</td>
<td>$\gamma_1 = 0$</td>
<td>$\gamma_1 = 0$</td>
<td>$\gamma_2 = 0$</td>
</tr>
<tr>
<td>IV</td>
<td>Predictive</td>
<td>$\gamma_1 = 0$</td>
<td>$\gamma_1 = 0$</td>
<td>$\gamma_2 = 0$</td>
</tr>
</tbody>
</table>

# Null hypothesis tested is intercept term is equal to zero.
* Null hypothesis tested is slope term is equal to zero.
@ Sign indicates the relationship between beta and return on portfolios.

M.R. Multiple Regression, S.R. Simple Regression

2. RBI Index As Market Proxy

I. Non-Overlapping Portfolios During Non-Overlapping Period

A. Predictive Model

Multiple Regression:

\[
R_p = 0.027022 \quad 0.14217 \quad 0.04237^2 \quad 0.012626
\]

| Std. error | 0.02621 | 0.050678 | 0.01599 | 0.044173 |
| t values   | 1.0308  | 0.80532  | 0.64873 | 0.2858  |
| R^2        | 0.71    |
Simple Regression:
\[ R_p = 0.03895 + 0.0276(R_m - R_d) \]
Std. error 0.0138 0.0139
\( t \) values 0.8201 0.9756
\( R^2 \) 0.32

There was negative association between portfolio betas and portfolios return under the predictive model of the CAPM. Under simple regression analysis the hypothesis that intercept coefficient is zero was rejected with significant \( t \) value, beta coefficient is close to zero. Whereas, multiple regression analysis showed that coefficients of beta and beta \(^2\) were significantly different from zero and intercept was equal to zero. [Annexure 4 (I) (2)]

II. Non-Overlapping Portfolios During Overlapping Period

B. Non-predictive Model

Multiple Regression:
\[ R_p = .05 + 0.107 + 0.047^2 + 0.174 \]
Std. error 0.055 0.124 0.066 0.356
\( t \) values .897 0.865 0.715 0.489
\( R^2 \) 0.469

Simple Regression:
\[ R_p = .0156 + 0.008(R_m - R_d) \]
Std. error 0.007 0.008
\( t \) values .068 2.38
\( R^2 \) 0.41
Both multiple and simple regression analysis showed a positive relationship between beta and returns and $R^2$ values were higher when compared to values under Group 1. ‘t’ values were significant in case of simple regression thereby indicating the hypothesis that both intercept and slope coefficients are equal to zero was rejected. In case of multiple regression, where ‘t’ values were insignificant confirming the null hypothesis tested in case of all coefficients. [Annexure 4 (II) (2)]

III. Overlapping Portfolios under Overlapping Periods

B. Non-Predictive Model

Multiple Regression:

\[
\begin{align*}
R_p &= 0.078 & 0.138 & 0.064^2 & 0.220 \\
\text{Std. error} &= 0.090 & 0.197 & 0.107 & 0.247 \\
t \text{ values} &= 0.863 & 0.705 & 0.594 & 0.890 \\
R^2 &= 0.30
\end{align*}
\]

Simple Regression:

\[
\begin{align*}
R_p &= 0.019 & 0.024(R_m - R_d) \\
\text{Std. error} &= 0.009 & 0.010 \\
t \text{ values} &= 0.13 & 2.39 \\
R^2 &= 0.25
\end{align*}
\]

Results under Black, Jenson and Scholes Model showed that null hypothesis (i.e. regression coefficients are zero) was rejected, as the ‘t’ values were more than two under simple regression analysis. Multiple regression confirm that coefficients were equal to zero as expected under excess return form of the CAPM. [Annexure 4 (III)(2)]
IV. Overlapping Portfolios During Non-Overlapping Periods

A. Predictive Model

Multiple Regression:
\[
R_p = 0.076 - 0.036 + 0.016^2 + 0.067
\]
Std. error 0.017 0.023 0.008 0.027
\[t\text{ values}\] 0.57 0.57 1.98 0.48
\[R^2\] 0.17

Simple Regression:
\[
R_p = 0.114 - 0.0009(R_m - R_d)
\]
Std. error 0.006 0.007
\[t\text{ values}\] 0.73 0.13
\[R^2\] 0.0004

Using 32 months returns for the estimation, formation, and testing of the model showed that there is negative associationship between beta and returns under both multiple and simple regression; interecept coefficients was significantly greater than zero and beta coefficient was close to zero. [Annexure 4 (IV)(2)]. Results of the CAPM testing using RBI Index as market proxy are summarised below:

<table>
<thead>
<tr>
<th>Combination Model</th>
<th>Intercept#</th>
<th>Slope*</th>
<th>Sign@</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M.R.</td>
<td>S.R.</td>
<td>M.R.</td>
</tr>
<tr>
<td>I Predictive</td>
<td>(\gamma_{1=0})</td>
<td>(\gamma_{1#0})</td>
<td>(\gamma_{2=0})</td>
</tr>
<tr>
<td>II Non-Predictive</td>
<td>(\gamma_{1=0})</td>
<td>(\gamma_{1#0})</td>
<td>(\gamma_{2=0})</td>
</tr>
<tr>
<td>III Non-Predictive</td>
<td>(\gamma_{1=0})</td>
<td>(\gamma_{1#0})</td>
<td>(\gamma_{2=0})</td>
</tr>
<tr>
<td>IV Predictive</td>
<td>(\gamma_{1#0})</td>
<td>(\gamma_{1#0})</td>
<td>(\gamma_{2=0})</td>
</tr>
</tbody>
</table>
Null hypothesis tested is intercept term is equal to zero.
* Null hypothesis tested is slope term is equal to zero.
@ Sign indicates the relationship between beta and return on portfolios.
M.R. Multiple Regression, S.R. Simple Regression

The CAPM Tests (under different combinations of portfolios made in excess return form using Natex and RBI Index) show that intercept was not significantly different from zero as expected in case of standard CAPM, and slope co-efficient was not significantly higher than zero, and relationship between beta and return was not always positive. There was no evidence of unsystematic risk getting priced in market place (except in case of Combination II using Natex and combination IV using RBI Index). Regression coefficient of beta squared values and return confirmed non-linearity relationship between systematic risk and return.
B. RETURN

Betas estimated using return on securities and market index for different combinations were generally higher than the beta estimates made using excess returns. Correlation coefficients between security and market returns were also generally high and the standard error of the estimates was low. Testing of CAPM in this form involves finding out whether the intercept coefficient is equal to risk free rate while the slope is closer to \( R_m - R_f \). The results of cross sectional regression in this form are reported below:

I. Non-Overlapping Portfolios During Non-Overlapping Period

A. Predictive model

**Multiple Regression:**

\[
R_p = 0.13663 + 0.03286(0) + 0.00659^2 + 0.023256
\]

<table>
<thead>
<tr>
<th>Std error</th>
<th>t value</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03740</td>
<td>0.86532*</td>
<td>0.03740</td>
</tr>
<tr>
<td>0.04387</td>
<td>0.82375</td>
<td>0.01339</td>
</tr>
<tr>
<td>0.07111</td>
<td>0.32704</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Since t value of intercept coefficient is greater than expected values, the null hypothesis that \( \beta \) is equal to risk free rate is rejected. If zero beta model holds, \( \beta \) is expected to be greater than \( R_f \); testing of the same indicated that the intercept was in fact less than \( R_f \). Testing for non-linearity (2) and pricing of unsystematic risk (3) showed that these factors were not getting priced in the market place as t values were closer to zero as expected. The relationship between beta and return was on expected lines with a positive sign and the coefficient value was not different from \( R_m - R_f \).
Simple Regression:

\[ R_p = 0.11913 + 0.01266(R_m - R_f) \]

Std. error 0.01327 0.01376

t values .5779* 1.15815

\[ R^2 = 0.04 \]

Positive relationship between beta and return and slope coefficient was close to estimated risk premium. However, intercept term was significantly lower than risk free rate. [Annexure 4 (I) (3)]

II. Overlapping Periods With Non-Overlapping Portfolios

B. Nonpredictive model

Multiple Regression:

\[ R_p = 0.01039 - 0.01327 + 0.01902 - 0.20784 \]

Std. error 0.05035 0.10152 0.05147 0.22907

t values 0.02696 -0.15913 0.3690 -0.90732

\[ R^2 = 0.33 \]

The results were in conformity with standard model with low t values indicating that hypotheses used for testing regression coefficients were not to be rejected. However, standard errors of these estimations are much greater than the coefficients indicating low level of confidence in accepting the results.

Simple Regression:

\[ R_p = 0.01433 + 0.024656(R_m - R_f) \]

Std. error 0.0094 0.00904

t values .4785* 2.4084*

\[ R^2 = 0.29 \]
The results were much different from those obtained using multiple variables. The hypotheses relating to intercept and slope coefficients were rejected with significantly higher t values. That is, intercept was neither equal to nor higher than time value, but was found to be lower than $R_f$; slope was greater than excess return on market portfolio. [Annexure 4 (II)(3)]

III Overlapping Portfolios During Overlapping Periods

B. Nonpredictive Model

Multiple Regression:

\[
R_p = 0.06279 - 0.12805 + 0.07112^2 + 0.0959
\]

\[
\text{Std error} = 0.0791 \quad 0.1575 \quad 0.0767 \quad 0.2999
\]

\[
\text{t values} = 0.6798 \quad -0.8315 \quad 0.9273 \quad 0.3198
\]

\[
R^2 = 0.22
\]

Except that the relationship between return and beta was negative, the other parameters of coefficient testing using t statistics showed conformity with the standard CAPM properties. Indirect testing of the hypothesis that market model is on the efficient frontier seems to have been not rejected by this combination.

Simple Regression:

\[
R_p = 0.00094 + 0.00901(R_m - R_d)
\]

\[
\text{Std. error} = 0.0053 \quad 0.0046
\]

\[
\text{t values} = 0.5294 \quad 1.9674
\]

\[
R^2 = 0.185
\]
Testing associationship between return and beta also recorded evidence for the conformity of standard model parameters. [Annexure 4 (III)(3)]

IV Overlapping Portfolios During Non Overlapping Period.

A. Predictive Model

Multiple Regression:
\[
R_p = .12036 + 0.01732 -0.0023^2 .00081 \\
\text{Std. error} = 0.0241 0.02871 0.0092 0.0455 \\
t \text{values} = .3284* 0.7174 -0.2504 -0.0178 \\
R^2 = 0.035
\]

Simple Regression:
\[
R_p = .11651 + 0.0101(R_m - R_d) \\
\text{Std. error} = 0.0088 0.0091 \\
t \text{values} = .1917* 1.4762 \\
R^2 = 0.032
\]

Results under this combination of betas also confirmed that factors other than systematic risk did not get priced in the market place and the relation between return and beta was positive with the slope term equivalent to risk premium. However, significantly high t values in case of intercept provided evidence for \( i \) is much lower than \( R_f \). [Annexure 4 (IV)(3)]
Results of the CAPM testing are summarised below:

<table>
<thead>
<tr>
<th>Combination Model</th>
<th>Intercept #</th>
<th>Slope *</th>
<th>Sign@</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M.R</td>
<td>S.R</td>
<td>M.R</td>
</tr>
<tr>
<td>I</td>
<td>Predictive</td>
<td>Lower</td>
<td>Equal</td>
</tr>
<tr>
<td>II</td>
<td>Non-Predictive</td>
<td>Equal</td>
<td>Lower</td>
</tr>
<tr>
<td>III</td>
<td>Non-Predictive</td>
<td>Equal</td>
<td>Equal</td>
</tr>
<tr>
<td>IV</td>
<td>Predictive</td>
<td>Lower</td>
<td>Lower</td>
</tr>
</tbody>
</table>

# Null hypothesis tested is intercept term is equal to Rf.
* Null hypothesis tested is slope term is equal to Rm - Rf.
@ Relation between return and beta

M.R. Multiple Regression. S.R. Simple Regression

The tests under non-predictive models show that intercept was not significantly different from risk free rate as expected in case of standard CAPM, but under predictive models, the same seems to be significantly lower than Rf. Except in one case, slope co-efficient was equal to RmRf. Relation between beta and return was not always positive.

SUMMARY

Results of various tests under different combinations indicate that Capital Asset Pricing Model does not hold good in India. The intercept and slope terms were not as per the properties of SML indicating that market portfolio is not an efficient portfolio. Highlights of the results are summarised below.

Under the excess return form of the beta estimation and CAPM testing using Natex, the slope coefficient was not different from zero, with intercept term is on expected lines of standard CAPM. Whereas under
return form, slope coefficients were in consonance with expected values, intercept term, under many of the combinations, were less than time value. Results under excess return form using RBI Index were also not indicative of SML's properties.

Changes in the risk-free rate of return seem to be affecting not only the beta estimations but also time value that an investor expects to earn from capital market assets. Portfolios formed on the basis of beta estimations in the return form and tested for equilibrium showed that when risk free rates rise, the intercept of the regression is much lower than expected time value. This is in contrast to the zero beta model wherein time value is slightly higher than riskless rate to account for variations among securities included in the zero beta portfolio.

A look at the alpha values of regression on estimated returns and estimated portfolio betas showed that their values did not follow any particular pattern or sign. Values of squared betas used for testing, under all combinations of periods and portfolios, showed that there was no non-linearity between portfolio returns and betas. However, in some cases of portfolio combinations, residual variance or unsystematic risk factor seems to have positive impact on the portfolio returns.

An interesting observation of this study is that portfolios formed during overlapping time period in excess return form had estimated betas varying in a narrow range (less than 0.5) than the portfolios formed during non-overlapping time period. Similarly, estimated returns on portfolios during non-overlapping periods, in both return as well as excess return forms, were generally negative whereas
during overlapping periods, returns estimated in majority of portfolios were positive. CAPM test carried out in the excess return form using RBI index as market proxy generally explained the dependence of portfolio returns on beta, beta^2 and unsystematic risk well than the tests carried using the Natex. The correlation coefficient in most of the cases were above 40%.

Most of the CAPM tests carried as part of study, confirm that slope coefficient is closer to zero indicating that the relation between return and risk is flat; but in view of unsystematic risk as measured by standard error is not being priced in the market it can be concluded that beta is the only explanatory variable, though not an effective variable, in these cases.

In many tests slope coefficients obtained were negative indicating that portfolio betas and returns move in opposite directions; this is in contrast to what is postulated by capital market theory. Following table shows the CAPM tests that clearly brought out the negative relation.

<table>
<thead>
<tr>
<th>Type</th>
<th>Combination</th>
<th>Model</th>
<th>Regression Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess Return</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Using Natex</td>
<td>II</td>
<td>Predictive</td>
<td>S.R.</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>Non-predictive</td>
<td>M.R.</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>Predictive</td>
<td>S.R. &amp; M.R.</td>
</tr>
<tr>
<td>Using RBI index</td>
<td>I</td>
<td>Predictive</td>
<td>S.R. &amp; M.R.</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>Predictive</td>
<td>S.R. &amp; M.R.</td>
</tr>
<tr>
<td>Return</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Using Natex</td>
<td>II</td>
<td>Non-predictive</td>
<td>S.R.</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>Non-predictive</td>
<td>M.R.</td>
</tr>
</tbody>
</table>

S.R. - Simple Regression, M.R. - Multiple Regression.
SECTION 3

LIQUIDITY CONSIDERATION

Elton & Gruber Model which is the Single Index Model based on Modern Portfolio Theory (see section 1 of this chapter) has following drawbacks:

a) Concentration of fund allocation in few shares
b) The problems faced by fund managers, especially one having sizable investible funds.

Optimal portfolios generated using Markowitz model also have similar problems pertaining to fund allocation. (Fisher & Statman (1992), Frost & Savario (1988) and Green & Holkfield (1992)). The fund managers find that the Markowitz portfolios suffer from following drawbacks, viz.,

a) the portfolios are insufficiently diversified — some stocks get awfully large positive or negative weightages,

b) the composition of the portfolios in terms of the number of stocks included and the allocations to them is independent of the fund size and

c) the portfolios include several stocks that are so thinly traded that they endanger the portfolios’ liquidity.

Fund managers don’t like to get these drawbacks remedied by imposing asset allocation constraints, say in the form of minimum or maximum exposures, because
a) such imposition amounts to sacrificing return and/or taking risk which is unacceptable to them;
b) features like portfolio liquidity and diversity are objectives of the fund, not extrinsic impositions and
c) the use of external asset allocation constraints is dicey because it is well-nigh impossible to determine how sensitive are the portfolio return and risk to changes in these constraints — fund managers feel that they lose control on the key parameters of their funds.

In short, fund managers find the portfolios too risky and too illiquid. This suggests that there is a large gap between the perceptions of the investment theorists and the fund manager. An attempt has been made to fill this gap by introducing liquidity considerations. As time series data on the liquidity measure used in the study is not available, illustrations based on Markowitz model are given.

**Liquidity and Impact Cost**

Liquidity of stocks is normally measured in terms of (a) market capitalisation (b) turnover (c) trading frequencies and (d) bid-ask spreads. All these measures are of course fairly well correlated. Impact cost considered as a liquidity measure and the portfolios are constructed with risk, return and liquidity parameters. The impact cost of a stock is the change in price that occurs due to the arrival of an order in the market. A buy order causes the price to rise. A sell order causes it to fall. The magnitude of the change in price varies and is different for different stocks. Liquid stocks are those whose price is not much affected even though large orders hit the market. These are stocks for which large numbers of large orders, buy and
sell, market and limit, are waiting to be executed in the close vicinity of the going price so that even large additional orders are absorbed without a substantial change in the market price. On the other hand, stocks that are sparsely traded would have large impact costs.

The impact cost is measured from the electronic limit order book of the stock exchange which contains the cheapest sell order (ask) and the costliest buy order (bid). The limit orders in the order book will be for a certain number of shares. If the buyer’s order size is lesser than the order size at the best asking price, the order is absorbed without any effect on the price. But if it exceeds that the limit order size the computer moves to the next best quote. A buyer would therefore face a schedule of rising asking prices and the seller would face a schedule of falling bid prices. This is illustrated in the diagram (1) below. As the buy order size increases it will result in larger increases in the price. Likewise as the sell order size increases.

Diagram 1

![Diagram of price realisation](Image)
it will result in larger decreases in the price realised. To get an idea of
the computation consider the following example\(^1\). Suppose the order
book at a particular instant stands as follows. The best bid price is 100
and the best

<table>
<thead>
<tr>
<th>BUY</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>Quantity</td>
</tr>
<tr>
<td>98</td>
<td>1000</td>
</tr>
<tr>
<td>99</td>
<td>800</td>
</tr>
<tr>
<td>100</td>
<td>600</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SELL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>Quantity</td>
</tr>
<tr>
<td>101</td>
<td>500</td>
</tr>
<tr>
<td>102</td>
<td>600</td>
</tr>
<tr>
<td>103</td>
<td>800</td>
</tr>
</tbody>
</table>

asking price is 101. The middle price is 100.50. Now consider a buyer
who places an order to buy 1500 shares. He will be able to buy 500
shares at 101, 600 shares at 102 and the remaining 400 shares at 103.
His average buying price will be,

\[
\left( \frac{(500)(101) + (600)(102) + (400)(103)}{1500} \right) = 101.93
\]

A seller who wishes to sell say 2000 shares will realise 100 for the first
600 shares, 99 for the next 800 shares and 98 for the remaining 600.
His average selling price realised is

\[
\left( \frac{(600)(100) + (800)(99) + (600)(98)}{2000} \right) = 99
\]

The impact cost of the buy order of 1500 shares is

\[
\left( \frac{101.93 - 100.50}{100.50} \right)(100) = 1.42\%
\]

The impact cost of the sell order of 2000 shares is

\(^1\) See Shah and Thomas(97) for an analysis of impact cost of market portfolios in
India.
\[
\left( \frac{100.50 - 99}{100.50} \right) (100) = 1.49\% 
\]

The impact cost will rise steeply as the order size increases both for the buyer as well as the seller - the seller would have to move down the buyers' bid price schedule and the buyer would have to move up the sellers' ask price schedule. We shall characterise the impact cost function of a stock on the following intuitive grounds,

\[ p_i = f(Q_i) \]

where \( p_i \) is the change in the market price and \( Q_i \) is the size of the order and the function \( f \) is such that,

\[ \frac{dp_i}{dQ_i} > 0 \quad \text{and} \quad \frac{d^2 p_i}{dQ_i^2} > 0 \]

A function that meets this requirement is,

\[
p_i = a_i Q_i^{1+\delta_i} \quad \delta_i > 0 \tag{1}
\]

where \( a_i \) and \( \delta_i \) are determined by the factors affecting liquidity, being lower for liquid stocks and greater for illiquid stocks.

Suppose a fund manager decides to allocate an amount of money \( X_t \) to stock \( i \). Then at the going price \( P_t \) he will place an order of size

\[
Q_t = \frac{X_t}{P_t}
\]

\[
\left[ \frac{X_t}{P_t} \right] P_t = a_i \left[ \frac{X_t}{P_t} \right]
\]
whose impact cost will be,

\[ p_i = a_i \left[ \frac{X_i}{P_i} \right]^{1+\delta_i} \]

Summing this over all the stocks that he buys/sells, the total impact cost of the portfolio will be,

\[ C_p = \sum p_i \left[ \frac{X_i}{P_i} \right] = \sum a_i \left[ \frac{X_i}{P_i} \right]^{2+\alpha_i} \]

(2)

where \( \alpha_i = 2\delta_i \). Dividing by the fund size gives the decrease in the rate of return due to the presence of impact costs,

\[ \frac{C_p}{F} = \frac{1}{F} \sum a_i \left[ \frac{X_i}{P_i} \right]^{2+\alpha_i} \]

(3)

It may noted at this point that the impact cost functions of stocks exhibit a very steep rise with every increase in the order size. It follows that a given outlay, if it is concentrated on any one stock, will result in a much greater impact cost than if it is spread across stocks. And the greater the spread of the outlay across stocks, the slower would be the rise in the impact cost.

**The Potable Frontier**

For determining the frontier of efficient portfolios in the presence illiquid stocks, the fund manager should minimise

\[ Z = -\lambda R_p + V_p \quad \text{subject to} \]

\[ \Sigma x_i = 1 \]

(4)
where

\[ R_p = \sum x_i R_i - \frac{C_p}{F} \]

is the portfolio return net of portfolio impact cost, \( x_i = X_i/F \) is the proportion of the fund size \( F \) allocated to stock \( i \), \( V_p = \sum \sum x_i x_j C_{ij} \) is the portfolio variance and \( R_i \) and \( C_{ij} \) are respectively the expected rate of return of the stock \( i \) and the covariances between rates of return. The multiplier \( \lambda \) measures the risk tolerance of the investor showing the additional risk he is willing to take for a unit increase in portfolio return.

To solve the problem we set up the Lagrangean,

\[
W = -\lambda R_p + V_p + \lambda_f (1 - \sum x_i)
\]  \hspace{1cm} (5)

and set its partial derivatives with respect to \( x_i \) and \( \lambda_f \) equal to zero.

\[
\frac{\partial W}{\partial x_i} = -\lambda R_i + \frac{\lambda a_i(2 + \alpha_i) R^{2 + \alpha_i} x^{(1 + \alpha_i)}}{R^{2 + \alpha_i}} + 2C_{i1}x_1 + 2C_{i2}x_2 + \ldots + 2C_{in}x_n - \lambda_f = 0
\]

\[
\frac{\partial W}{\partial \lambda_f} = 1 - \sum x_i = 0
\]  \hspace{1cm} (6)

The system of equations (6) contain \( n+1 \) equations in \( n+1 \) unknowns, viz. \( x_1 \ldots x_n \) and \( \lambda_f \). Arranging the equations in vector matrix notation,

\[
\begin{bmatrix}
2C_{11} + \lambda_1 B_1 x_1^{\alpha_1} & 2C_{12} & \ldots & 2C_{1n} & -1 \\
2C_{21} & 2C_{22} + \lambda_2 B_2 x_2^{\alpha_2} & \ldots & 2C_{2n} & -1 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
2C_{n1} & 2C_{n2} & \ldots & 2C_{nn} + \lambda_n B_n x_n^{\alpha_n} & -1 \\
1 & 1 & \ldots & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n \\
\lambda_f
\end{bmatrix}
= 
\begin{bmatrix}
\lambda R_1 \\
\lambda R_2 \\
\vdots \\
\lambda R_n \\
1
\end{bmatrix}
\]  \hspace{1cm} (7)
where

\[ B_i = \frac{a_i(2 + \alpha_i)R^{2+\alpha_i}}{R^{2+\alpha_i}} \]

The system (7) can only be solved by a process of trial and error. The process of solving it is as follows. Select a value of \( \lambda \). Start by plugging in hypothetical values of \( x_{1h}, \ldots, x_{nh} \) into the matrix on the left hand side. Invert the matrix and multiply the right hand side vector to obtain the solution for \( x_{0h}, \ldots, x_{n0} \). Use the bisection method and plug in the averages of the \( x_{0h} \) and \( x_{n0} \) into the matrix on the left hand side and find the next solution. Repeat the process till the \( x_i \) solution converges. This is an efficient portfolio. Vary the value of \( \lambda \) from 0 to \( \infty \) to find the entire locus of efficient portfolios. This resulting locus may be called the *potably efficient frontier* or simply the *potable frontier*.

Some properties of potable portfolios in relation to efficient portfolios should be noted. Firstly, in the system of equations (7), the impact cost differentials appear as add-ons to the own risks (variances) of the stocks, i.e. they are added to the principal diagonal elements. The effect, speaking purely in mathematical terms, is to make the diagonal more dominant. Its result is to make the solution of \( x_i \) more positive and/or less negative as compared to the Markowitz portfolio which corresponds to a less dominant diagonal. Thus for any given fund size, the proportions allocated to the various stocks are more uniform on the potable portfolio. And greater uniformity in allocation implies greater diversity of holdings. Portfolio diversity is measured by

\[
D = \frac{1}{\sum x_i^2}
\]
where
\[ x_i \] is the proportion allocated to stock \( i \).

Secondly, the sizes of the add-on elements to the principal diagonal vary directly with the size of the fund. The larger the fund size the more dominant diagonal is the matrix and the more the proportions allocated will tend towards positivity and uniformity. In other words, the greater the size of the fund the greater would its diversity be. To interpret, the fund manager diversifies not only to spread out the risk of fluctuation in asset prices but also to prevent the impact costs (which increase with the degree of concentration) from reducing the return on the portfolio. This secondary diversification is to preserve return just as the primary diversification is to reduce risk. The larger the size of the fund he manages the greater will his outlay be on every stock and the greater will be the impact cost if he concentrates his outlays. He will therefore diversify over a greater number of stocks more uniformly than the manager of a small fund.

The upshot of all this is that the potable frontier is a locus of diversified portfolios each of which affords the maximum possible return adjusted for liquidity at every given level of risk. Because the liquidity, or impact cost, varies with the size of the fund it follows that the position of the potable frontier will differ for different fund sizes, being lower at greater fund sizes.

These results are illustrated by means of a numerical example. The data are as follows. The expected returns from three securities are assumed to be \( R_1 = 0.2 \) and \( R_2 = 0.3 \) and \( R_3 = 0.15 \), their variances \( C_{11} = 0.0225 \), \( C_{22} = 0.01 \) and \( C_{33} = 0.04 \) and the matrix of correlations is
The impact cost coefficients are $a_1$, $a_2$, $a_3 = 1$ and $\alpha_1 = 0.2$, $\alpha_2 = 0.5$ and $\alpha_3 = 0.3$. The stock prices are $P_1 = 50$, $P_2 = 70$, $P_3 = 40$. Under the assumption of zero impact cost we obtain the following standard portfolios.

If impact costs are taken into account the optimum portfolios corresponding to different fund sizes are as follows:

**Fund Size = 50**

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$D$</th>
<th>$R_P$</th>
<th>$\sqrt{V_P} = \sigma_P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>0.154</td>
<td>0.9370</td>
<td>-0.092</td>
<td>1.097</td>
<td>0.298</td>
<td>0.099</td>
</tr>
<tr>
<td>0.05</td>
<td>0.060</td>
<td>1.145</td>
<td>-0.025</td>
<td>0.736</td>
<td>0.324</td>
<td>0.115</td>
</tr>
<tr>
<td>0.08</td>
<td>-0.053</td>
<td>1.394</td>
<td>-0.341</td>
<td>0.484</td>
<td>0.356</td>
<td>0.142</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.128</td>
<td>1.560</td>
<td>-0.432</td>
<td>0.378</td>
<td>0.377</td>
<td>0.162</td>
</tr>
</tbody>
</table>

**Fund Size = 75**

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$D$</th>
<th>$R_P$</th>
<th>$\sqrt{V_P} = \sigma_P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>0.160</td>
<td>0.926</td>
<td>-0.086</td>
<td>1.12</td>
<td>0.289</td>
<td>0.098</td>
</tr>
<tr>
<td>0.05</td>
<td>0.097</td>
<td>1.089</td>
<td>-0.186</td>
<td>0.811</td>
<td>0.306</td>
<td>0.111</td>
</tr>
<tr>
<td>0.075</td>
<td>0.070</td>
<td>1.216</td>
<td>-0.287</td>
<td>0.637</td>
<td>0.320</td>
<td>0.125</td>
</tr>
<tr>
<td>0.1</td>
<td>0.084</td>
<td>1.328</td>
<td>-0.412</td>
<td>0.515</td>
<td>0.331</td>
<td>0.143</td>
</tr>
</tbody>
</table>

204
From the three tables above it is at once apparent that as the fund size increases so does the diversity, although the effect is prominently visible at higher values of $\lambda$. As regards the expected portfolio return, observe that the standard method will trace out the usual efficient frontier, increasing $\lambda$ will be associated with increasing return and increasing risk. However, once impact costs are brought into the story there is a value of $\lambda$ at which expected net portfolio return (net of impact costs) is maximum and then starts falling. The efficient frontier rises upto a point and then declines - the rewards for increased risk taking become negative! Obviously only the rising portions of the locus should be deemed to be
efficient. The investor is confronted with a much smaller risk-return opportunity set than the standard theory would lead him to believe. As the diagrams 2, 3, and 4 clearly show the efficient frontier drops down with every increase in the fund size. Diagram 5 shows how the diversity increases with the fund size.

To sum up, the presence of liquidity problems structurally alters the decision making environment of the fund managers in such a way that the problem cannot be solved by finding the optimum portfolio first and then solving for the net-of-impact cost return which can be presumed optimal. If impact costs are indeed significant in the real world, standard models of portfolio theory may be rendered inapplicable.

The Potable CAPM
The Markowitz procedure leads to the Sharpe-Mossin-Lintner Capital Asset Pricing Model. Empirical tests of the CAPM show that the market factor does not explain the variation in stock return. The potable procedure should lead to a general version. To derive this we follow Sharpe (1990) and define the kth individual’s portfolio utility as its risk-adjusted rate of return,

$$U_k = R_k - C_k - \frac{V_k}{T_k}$$

(8)

where $R_k = \sum x_{ik}R_i$, $C_k = \sum a_i(X_i/P_i)^{2*\alpha_1}$, $V_k = \sum \sum x_{ik}x_{jk}C_y$ and $T_k$ is the marginal rate of substitution of risk for expected return. The individual $k$ seeks to maximise $U_k$ subject to the funds constraint.
Setting up the Lagrangean,

$$\mathcal{Z}_k = R_k - C_k - \frac{V_k}{T_k} + \lambda_{jk} (1 - \sum x_k)$$  \hspace{1cm} (9)

and differentiating $\mathcal{Z}_k$ partially with respect to $x_{ik}$ gives,

$$\frac{\partial \mathcal{Z}_k}{\partial x_{ik}} = R_t - L_t - \frac{2 \sum x_{jk} C_y}{T_k} - \lambda_{jk} = 0$$  \hspace{1cm} (10)

where $L_t = \partial C_k / \partial x_{ik}$ is the additional impact cost of stock $i$ due to an additional allocation towards stock $i$. Now $\sum x_{jk} C_y = C_{ik}$, where $x_{jk}$ represents the $k^{th}$ individual's portfolio denoted $k$. Multiplying (10) by $T_k$ gives,

$$T_k R_t - T_k L_t - 2C_{ik} = \lambda_{jk} T_k$$

Multiplying both sides by $F_k$ (the fund size of investor $k$) and summing over $k$ gives,

$$\sum_k F_k T_k R_t - \sum_k F_k T_k L_t - 2\sum_k F_k C_{ik} = \sum_k F_k T_k \lambda_{jk}$$

Letting $\sum F_k T_k = T_m$ measure the market's risk tolerance and remembering that $C_{ik} = \text{Cov}(R_t, R_k)$ so that

$$\sum F_k C_{ik} = \text{Cov}(R_t, \sum F_k R_k) = \text{Cov}(R_t, R_m) = C_{m\cdot m},$$

we get

$$T_m (R_t - L_t) - 2C_{m\cdot m} = T_m \lambda_{m\cdot m}$$

or

$$R_t = L_t + \lambda_{m\cdot m} + \frac{2C_{m\cdot m}}{T_m}$$

$$= L_t + \lambda_{m\cdot m} + \frac{2\beta_{m\cdot m} V_m}{T_m}$$  \hspace{1cm} (11)
where $\beta_i$ is Sharpe’s beta-coefficient obtained by regressing $R_i$ on $R_m$ and $V_m$ is the variance of the market portfolio. If $i$ were the market portfolio, $\beta_i = 1$ so that

$$R_m = L_m + \lambda_{jm} + \frac{2V_m}{T_m}$$

or

$$\frac{2}{T_m} = \frac{R_m - L_m - \lambda_{jm}}{V_m}$$

(12)

where $L_m$ denotes the impact cost of the market portfolio.

Substituting for $2/T_m$ from (12) into (11) gives

$$R_i = \lambda_{jm} + L_i + \beta_i (R_m - L_m - \lambda_{jm})$$

Finally if we let $\lambda_{fm}$, the market marginal utility of wealth, equal to the risk free interest rate $R_F$ we obtain,

$$R_i = R_F + L_i + \beta_i (R_m - L_m - R_F)$$

(13)

or in another way,

$$R_i = R_F + \beta_i (R_m - R_F) + (L_i - \beta_i L_m)$$

(14)

In other words the required rate of return of stock $i$ must contain compensation for the risk free rate foregone plus the systematic risk premium as per the usual CAPM. In addition it must contain an illiquidity premium for the illiquidity the stock in relation to the systematic portion of the illiquidity of the market portfolio. (Equation 14).

If the potable CAPM were to be true of real life it would predict the following about attempts to empirically test the regular CAPM,

$$R_i = a + b\beta_i$$
The value of \( a \) obtained would be greater than \( R_F \) and closer to \( R_F + L_i \) in equation\((13)\) i.e. the risk-free rate plus the impact cost of security \( i \), and the value of \( b \) could either be less or more than the market excess return \( R_m - R_F \) depending on whether security \( i \) was less or more liquid as compared to the market portfolio. Of course, for reasons recorded earlier, a security is always likely to be more illiquid than the market portfolio so that the observed \( b \) would be less than the market excess return. It is perhaps interesting to note that several empirical studies of the CAPM have found \( a > R_F \) and \( b < R_m - R_F \). (Douglas (1968), Miller and Scholes (1972), Jensen (1972), Black, Jensen and Scholes (1972) and Fama and Macbeth (1974)). Miller and Scholes (1972) in particular find that neither the negative correlation between \( R_F \) and \( R_m \) nor even the presence of heteroscedasticity accounted for the phenomenon of an intercept greater than \( R_F \) and a slope lower than \( R_m - R_F \). The literature seems to have concluded that all these observations support a two-factor CAPM instead of a one factor (beta) CAPM. The analysis underlying equations \((13)\) and \((14)\) would seem to suggest that the deviations of the observed \( a_i \) from \( R_F \) and \( b_i \) from \( R_m - R_F \) may be due to the neglect of liquidity premiums.

To sum up it can be said that the search for liquidity on the part of fund managers may lead them to invest in portfolios that dominate the standard Markowitz portfolios in terms of return and risk. When the potable portfolio analysis is extended towards the capital market equilibrium we obtain a generalised version of the Capital Asset Pricing Model which is consistent with empirical findings that otherwise diverge from the original model. These findings suggest that portfolio liquidity is a strategic objective that ought to be given the same attention as portfolio return and risk. Incidentally the quest for liquidity on the part
of fund managers also helps to explain why, as fund sizes grow bigger, the chosen portfolios come to resemble more and more the important index portfolios. After all the securities are themselves chosen to constitute the index on account of their greater liquidity.