CHAPTER II

RISK-RETURN OPTIMISATION MODELS

Fundamental Analysis is useful for the selection of individual stocks but when the stocks so selected were to be combined into a portfolio, the portfolio formed may not be diversified one. Ranking securities on the basis of margin of safety and selecting scrips with high ranks or allocating the investible funds by following some diversification norms say, industry or sector exposure may not result in a diversified portfolio. Modern Portfolio Theory was developed to overcome this problem of fundamental analysis. With the development of Modern Portfolio Theory, risk quantification and risk–return trade off are made possible. These models help investors in fund allocation and portfolio performance evaluation. Models developed as extensions of this theory try to explain the pricing of capital assets in the market.

This chapter is divided into two sections. First section includes literature survey of models based on portfolio theory and the second section deals with empirical testing of Capital Asset Pricing Model.
SECTION A

MODELS BASED ON PORTFOLIO THEORY

I. MARKOWITZ MODEL

Markowitz's Portfolio Theory (1952) was developed to overcome the major drawbacks of Fundamental Analysis viz., arbitrary fund allocation and ineffective diversification. This theory assumes that

a) Investors are risk averse that they prefer to assume lower risk to higher risk for a given level of return.

b) Asset returns are normally distributed and two parameters viz., mean and standard deviation of returns are sufficient to form expectations about future returns.

Given historical data, expected returns are the average returns during some past time period and the risk is the variability of return measured through dispersion of returns around its mean. Thus, definition of the term 'risk' under Modern Portfolio Theory is entirely different from what is connoted in Fundamental Analysis which considers factors or attributes (like high leverage, low interest coverage etc.) affecting stability of earnings and in turn dividend payments.

Combining scrips into a portfolio is relatively a simple task under fundamental analysis, as securities with higher expected returns are preferred to lower return scrips. Under Modern Portfolio Theory, when a portfolio is constructed, return is weighted sum of individual securities' returns.
and risk is weighted sum of variance of return on individual securities as well as covariance of return among the securities.

\[ \sum_{i=1}^{n} x_i R_i \]

\[ \sigma_p^2 = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \]

The essential problem in Markowitz Model is of the form:

Minimise \[ z = V_p - \lambda R_p \]

subject to

\[ \sum x_i = 1 \]

where

- \( V_p \) = variance of portfolio
- \( R_p \) = return on portfolio
- \( x_i \) = proportion to be invested in each of the securities included in the portfolio.

Using quadratic programming, when estimated means, standard deviation and correlation of return are given, a unique solution can be found out to this basic problem. The solution is the proportions of investible fund to be invested in each security. The set of optimal portfolios for all possible levels of risk is the mean-variance efficient frontier. The optimal portfolio for any particular investor is the portfolio on the efficient frontier that is tangent to the 'utility curve' (based on level of risk assumed by the investor). Thus, the focus is on optimisation of risk and return rather than.
maximisation of return. Portfolio theory has therefore explicitly considers both risk and return for portfolio selection. This optimisation is achieved by combining securities with negative or less than perfect correlation. Diversification through this method ensures selection of a portfolio with minimum risk without sacrificing return.

The model could not be put to widespread usage by investment managers due to:

a) Large number of data estimations to be made; and
b) Complicated algorithms to be used for optimal portfolio selection

To elaborate, the model requires 'n' estimates of returns, 'n' estimates of variances and n (n-1)/2 estimates of correlation. An institutional investor, with a portfolio of 100 - 150 stocks, would have to make estimates around 2650 - 11475 in total. Once these estimates are made, the problem of selecting an optimal portfolio poses another complexity. That is, finding out efficient frontier which consists of dominant portfolios at different risk levels (λ) and choosing a portfolio with desirable risk-return attributes, require sophisticated computational techniques.

In order to overcome these problems, Markowitz had suggested,

a) consideration of an index through which the securities are correlated; and
b) identification of corner portfolios using lagrangian multipliers for portfolio selection.
Michaud (1989) in his article has brought out clearly other limitations of Markowitz model.

a) Mean - Variance optimisation significantly overweights (underweights) those securities that have large (small) estimated returns, negative (positive) correlations and small (large) variances.

b) The technique ignores the impact of liquidity and when the same is imposed as constraint results in less return enhancement and/or less risk reduction. For very large capitalisation portfolios, the Mean - Variance frontier is close to the original 'unoptimised' portfolio.

c) Constructing a covariance matrix based on insufficient historical data gives unstable solution; i.e., small changes in the input assumptions lead to large changes in the solution.

d) The model produces a unique 'optimal' portfolio for given level of risk. Given any point on the true Mean - Variance efficient frontier, there are infinite numbers of 'statistically equivalent' portfolios that may have significantly different portfolio structure.

The models developed subsequently to overcome the difficulties of Markowitz model are covered below. These models are popularly known as the extensions of Markowitz model.
II. VARIATIONS OF MARKOWITZ MODEL

1. MODELS DEVELOPED TO OVERCOME DATA ESTIMATION PROBLEMS

a) Single Index Model

On the basis of the suggestion given by Markowitz in his monograph, Sharpe (1963) considered single index of general market performance that simplified data estimation problem under Markowitz model. The basic assumption of Sharpe's model is that various securities considered for inclusion in portfolio are related only through common relationship with the index. This eliminates the computation of pair-wise correlation among securities.

The return from a scrip is assumed to be determined by random factors and is also a linear relationship with the market index. Return on a security is,

\[ R_i = a_i + \beta_i R_m + \varepsilon \]

where,

\- \( R_i \) = Return on a security
\- \( a_i \) = Intercept
\- \( \beta_i \) = Slope, which indicates the change in security return corresponding to an unit change in the Index, referred to as beta factor.
\- \( R_m \) = Rate of return on market index
Thus, the return on a security has two components:

- A component which is insensitive to the return on the market as indicated by Intercept ($\alpha_i$):
- Another dependent on the market index movement as measured by beta factor ($\beta_i$).

The first component, unsystematic risk ($\alpha_i$) is expressed in two parts: $\alpha_i$ indicating the expected values of market independent return while $e_i$ denoting the random element. The expected value of $e_i$ is zero and $\alpha_i$ is constant. This unsystematic risk can be eliminated through diversification. The second component, systematic risk, $\beta_i$ is due to market and cannot be diversified away.

The special properties of index structure further simplified the process of combining different securities into portfolio. Return of the portfolio is given by,

$$R_p = \alpha_p + \beta_p R_m$$

where both alpha and beta are the weighted average of individual security values. The risk of the portfolio is,

$$\sigma_p^2 = \sum_{i=1}^{n} \beta_i^2 \sigma_m^2 + \sum_{i=1}^{n} \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_i \beta_j \sigma_{ij}^2$$

If $i = j$,

$$\sigma_p^2 = \beta^2 \sigma_m^2 + \sum_{i=1}^{n} \sigma_i^2$$

where,

- $\beta^2 \sigma_m^2$ is Portfolio systematic risk
- $\sigma_i^2$ is Portfolio residual variance which can be diversified.
If an institution considers 'n' securities for portfolio selection, using single index model only $3n + 2$ estimates are needed. That is 'n' estimate of $\alpha, \beta, \text{and } \sigma^2_n$ for each stock and expected return ($R_m$) only 152 to 302 estimates need to be made against 2650 to 11475 estimates under Markowitz framework.

b) Multi-Index Models

Sharpe's model replaced the portfolio volatility computation with an approximation that all stocks responded differentially to a single factor viz., market index. Apart from market, other common factors influencing security prices were shown by many researchers. These factors vary from industry related factors (King, 1966) to macro economic factors (Chen, Roll, and Ross, 1986). In order to capture the effects or influences of other factors, multi-index models were used. These models assume that security returns respond to more than one factor, which are represented by some indices. Assuming that the return on a security is influenced by market index and a couple of industry related factors, the same can be expressed as a linear function as follows

$$R_i = \alpha_i + \beta_i I_i + \beta_j I_j + \ldots \ldots + \beta_n I_n + c_i$$

where,

$\alpha_i$ = Intercept term of regression

$\beta_i I_i$ = Response of the stock to changes in Index 1, say market index.

$\beta_j I_j$ = Response of the stock to changes in Index 2.

$C_i$ = random variation as captured by standard error.
Comparison of Markowitz Model and Its Extensions:

Factor models were developed with a view to simplifying the complexities involved in estimating correlation factors. Depending on the level of approximation achieved, correlation matrix of Index Models will be closer to Markowitz correlation matrix. Studies have been carried out to find out which of these models are useful and these studies fall in either of the following two categories:

a) Keeping Markowitz variance and covariance of portfolio as the basis, variance - covariance matrix arrived at by using other index models are compared in order to test the ability of the models to reproduce the correlation matrix during a historical period. It has been found that with the addition of number of indices, the closer the correlation matrix is with the historical one. With only one index, such as market index, covariance of portfolio is lower than that of Markowitz's model. Thus, in the historical time period, when ranked, portfolio covariance computed using the multi-index models are in the middle, with Markowitz model calculation on the up and single index model at the bottom in the rankings.

b) The ability of these models to produce better performance in the future period. Statistical testing of the same involved examining how well the future correlation matrix of security returns are estimated by comparing forecasted results with actual results. Economic substance of the same is judged by comparing the return on the portfolios.

Comparison of the performance of portfolios selected using these models has been a subject of interest to many researchers. Cohen and Pogue (1967) evaluated ex-ante (1947-50) and ex-post (1950-64) performances of a number of single period portfolio selection
models. The results of the study indicated that ex-post performance of the index models is not dominated by the Markowitz formulation. For common stock portfolios, the performance of the multi-index models is not superior to that of the single index formulation. Ex-post performance of the efficient sets when compared to performance of portfolios of naive selection and 78 common stock mutual funds, indicated dominance of efficient sets. This study considered 543 common stocks and both dividend and capital gains were considered for measuring the yield. An unweighted arithmetic average of yields of all 543 securities was computed and used as the factor for single index model. The universe of securities was divided into ten industry subgroupings and industry indices were constructed for using in multi-index models.

Elton and Gruber (1971)'s study found that though adding more indices led to a better explanation of the historical correlation matrix but resulted in poorer prediction of future correlation matrix and in the selection of portfolio that had lower returns at each level of risk. These studies recorded evidence for poor predictive ability of multi index models as compared to single index model.

Thus, simplification of the Markowitz Model comes with a cost. This is because of the violation of the basic assumption on which factor models are built. That is, if the residual variance of the securities is positively related to one another, true variance of portfolio (under Markowitz formulation) will be higher than the variance of portfolios selected using factor models. The difference between these variances is referred as tracking error. With the advancement of computing techniques and reduction in costs it is now possible to employ Markowitz model even with large sample. Thus, the focus of the recent studies is to find out the extent of
tracking error. Portfolio managers, instead of considering difference in the volatility try to optimise tracking error arising out of return differences.

2. MODELS DEVELOPED TO OVERCOME COMPUTATIONAL COMPLEXITIES:

High computing costs and more time requirement for optimal portfolio construction under Markowitz Model made researchers to focus on the task of simple algorithms. Elton and Gruber (1987) have developed algorithms to simplify the computation required to find optimal portfolios using both Markowitz Model and Single Index Model. These models assume the existence of a risk free security.

a). Selecting an Optimal Portfolio on Efficient Frontier

Assumption regarding the existence of riskless security allows investors to identify a risky portfolio (on the efficient frontier). Maximising the ratio of portfolio return in excess of risk free rate to risk, through quadratic programming gives this optimal solution. The basic problem under this model takes the form of,

$$ \text{maximize } \theta = \frac{R_p - R_f}{\sigma_p} $$

Subject to the constraint,

$$ \sum_{i=1}^{n} x_i = 1 $$
By including constraint in the objective function, the problem is solved as an unconstrained one.

\[ \theta = \frac{\sum_{i=1}^{n} (R_i - R_f)}{\left( \sum_{i=1}^{n} x_i \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \right)^{1/2}} \]

Taking partial derivative with respect to \( x_i \) to \( x_n \) and by multiplying each \( X_k \) with constant \( \lambda \). By definition, \( Z_k = \lambda x_k \) and by rearranging terms

\[ R_i - R_f = Z_1 \sigma_1 + Z_2 \sigma_2 + \ldots + Z_{n-1} \sigma_{n-1} + Z_n \sigma_n \]

Using simultaneous equations, the values of \( x \), which are proportion to the optimum amount of investment in each security, can be found out. Depending on the level of risk premium chosen, the proportion of money invested in securities included in optimum portfolio will vary. It is possible to modify the problem for scenarios like short sale of risky assets, restrictions on short sale etc. by suitably incorporating conditions in the constraint function.

b) Models Based on Rankings

Ranking of stocks based on a unique attribute helps not only to shortlist the securities from the universe but also to form judgement of relative desirability. After ranking, if a stock is included in a portfolio, any other higher ranked stocks will also automatically be considered for inclusion. This model developed by Elton and Gruber is based on 'risk adjusted excess return' ranking. Under this, the securities are, in the first place, ranked by their excess return to beta ratio. That is,

\[ \frac{R_i - R_f}{\beta_i} \]
where
\[ R = \text{Expected Return on Security} \]
\[ R_f = \text{Risk Free Rate} \]
\[ \beta_i = \text{Systematic risk associated with stock } I. \]

Once ranking is done, the next step in selecting optimal portfolio is to decide the cut-off beta (c) which provides basis for the inclusion/exclusion of scrips in the portfolio. The cut-off beta is calculated by applying the following formula:

\[ C_i = \frac{1 + \sum_{j=1}^{l} \frac{(R_j - R_f) \beta_j}{\sigma_{\beta_j}}} {1 + \sum_{j=1}^{l} \frac{\beta_j^2}{\sigma_{\beta_j}^2}} \]

where
\[ \sigma_m^2 = \text{Variance of Market returns.} \]
\[ \sigma_{\beta_j}^2 = \text{Unsystematic risk of the securities.} \]

The percentage invested in each security is

\[ x_i = \frac{Z_i - \sum_{j=1}^{l} z_j}{x_i} \text{ where } Z_i = \frac{\beta_i}{\sigma_{\beta_i}} \left( \frac{R_i - R_f}{\beta_i} - C_i \right) \]

In short, this model involves ranking of securities by their excess return to beta, calculation of cut-off beta and finding out the proportion to be invested in each of the securities whose beta are equal to or more than the cut off beta. This model is very simple to use and portfolio selection is less time consuming.
SECTION B

MODELS BASED ON CAPITAL MARKET THEORY

Extension of modern portfolio theory explains how an asset is priced in the market place in relation to the risk(s) associated with that asset. These extensions are normative approach to Modern Portfolio Theory and are based on the assumption of homogeneous expectations among all market participants. Capital Market Theory explains how there are two equilibrium models that explain relation between risk and return. They are:

a) Capital Asset Pricing Model (CAPM); and
b) Arbitrage Pricing Theory (APT)

The CAPM is based on the principle of utility maximisation at a given risk and return level, whereas APT is based on the law of one price. CAPM assumes that there is only one factor viz., market movement influencing prices of all assets and the same cannot be diversified; in case of APT, a return generating process is identified and through the use of multi-index models, expected level of return is arrived at. Thus, CAPM is a special case of APT with an assumption of only one factor influencing asset prices. Of the two asset pricing models, the CAPM was developed in the early sixties and has been extensively tested, especially in developed markets. In view of the complexities involved in identifying the factors influencing asset prices, only few tests of APT have been carried out so far.
CAPITAL ASSET PRICING MODEL

A simple assumption on including a risk free asset in the risky portfolio has led to the development of Capital Asset Pricing Model (CAPM). This model explains the asset pricing mechanism in the market place when expectations of all the investors / players in terms of return and systematic risk are in complete agreement for a given investment horizon. As Capital Market Theory is an extension of Modern Portfolio Theory, it would be appropriate to list out the assumptions of the CAPM in the following manner:

Assumptions of Portfolio Theory

- Parameters considered for portfolio selection are expected mean and standard deviation of return on capital assets included in the portfolio; this is based on the condition that the probability distributions of portfolio returns are normally distributed.
- Portfolio with highest return at a given level of risk or lowest risk at a given level of return is preferred. This is on the premise that all investors are risk averse and prefer to derive maximum utility from the wealth deployed. This assumption limits the choice of investors to portfolios on efficient frontier.
- Holding period of the portfolio is single period.

Assumption of Capital Market Theory

- Expectations regarding the parameters viz. return and risk as well as time horizon are homogeneous. That is, in perfect market condition where information about securities is freely available to all market players, it is assumed that the investors' expectations regarding risk and return would be uniform for a single time horizon.
Assumptions of Standard CAPM

- Unlimited lending and borrowing are possible at risk free rate of interest. The terms ‘lending’ and ‘borrowing’ are invariably associated with buying and selling of risk free securities (like Treasury Bills and / or Government Bonds) respectively.
- Unlimited short sale of risky securities is possible. This assumption leads to leveraging of funds i.e., the investors could sell the securities which are expected to be inefficient and utilise the proceeds to buy securities with are expected to be efficient.

Additional Assumptions of CAPM

- All assets are marketable and any quantity of these assets can be bought and sold in perfect market conditions.
- No transaction cost is payable for buying and selling of assets and no income tax on dividend, interest and capital gains realised from holding /selling the assets is payable.

Of the assumptions listed above, the one on the homogeneous expectations regarding expected return and risk leads to the concept of market portfolio \(^1\) and the one on unlimited lending and borrowing at risk free rate leads to separation theorem \(^2\). Both market portfolio and the selection of efficient portfolio in some combination of risky as well as risk free asset using separation theorem is the centrepieces of the CAPM. The additional assumptions are basically considered to avoid arbitraging in securities due to differences in transaction costs or taxation rates.

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\(^1\) Market Portfolio is the one, which contains all risky assets, and each asset is held in proportion to the total market value of that asset relative to the total value of all risky assets.
or impediments in the flow or processing of information; in other words, only possible differences in risk-return level are considered for arriving at an equilibrium price.

The Capital Asset Pricing Model developed simultaneously by Sharpe, Lintner and Mossin (1964, 1965 & 1966) is based on stringent, not-so-practical assumptions and is referred as the standard form whereas in the non-standard form of the model, most of the restrictive assumptions have been relaxed.

a. Standard CAPM:

The process of optimal portfolio selection has been simplified to a great extent with the assumptions regarding the existence of a risk free bond and unrestricted short-selling of the same. Since market portfolio is the one with the highest risk adjusted rate of return, every investor would be holding this portfolio. The risk averse investors would lend more money whereas risk seekers would borrow to leverage their returns. By investing all the funds in government securities whose maturity term matches exactly with investment horizon, investors could earn a rate of return, which is relatively free of default and reinvestment risks. Any portion of investible funds deployed in risky securities would increase the portfolio risk, thus forcing investors to expect a return higher than risk free rate. The level of risk premium would be the deciding factor for choosing an investment opportunity. Higher the risk premium better would be the desirability.

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2 The ability to determine the optimum portfolio of risky assets without having to know anything about the risk preferences of the investor.
Depending on whether the portfolio is efficient or inefficient, the measure of risk considered for reckoning the risk premium varies. If the portfolio is minimum variance - efficient portfolio, the appropriate risk measure is the total risk of the portfolio measured in terms of variance or standard deviation of returns. The relevant risk measure for individual securities and inefficient portfolios would be the systematic or non-diversifiable risk associated with it, as quantified in terms of beta.

Risk - Return Framework of Efficient Portfolios:

Portfolio risk varies with the level of 'risky' securities included in the portfolio, ranging from less-than-market-portfolio risk (in case of lending) to more-than-market-portfolio risk (in case of borrowing). On the bottom of the spectrum, portfolio with riskless securities having zero risk would offer risk-free return. In terms of return, portfolio with lending would have lower return than a geared or leveraged portfolio. Thus, return is linear function of risk, in contrast to the curvilinear function in case of Markowitz. Capital Market Line (CML) is the line that passes through the market portfolio on the efficient frontier and has the maximum excess return. This explains that return is a linear function of risk, with intercept equals to risk free rate and the premium per unit of risk is the gradient. Thus, return on the portfolio is

\[
E(R_p) = R_f + \frac{E(R_m - R_f)}{\sigma_m} \sigma_p
\]
Risk - Return Framework of Inefficient Portfolios:

Security Market Line (SML) explains risk-return trade off of other-than-efficient portfolios and individual securities, which plot below and to the right of efficient frontier and are dominated by efficient portfolios. Sharpe (1964) has exhibited that the appropriate measure of risk for both efficient and inefficient portfolios is the systematic risk (as measured by Beta), which cannot be diversified. Higher the beta, greater would be the return.

As in the case of the CML, risk-return relationship for inefficient portfolios is linear. Portfolios consisting of only risk free assets would have zero risk associated with the returns; hence the return would be equal to $R_f$. Beta on market portfolio is equal to one. Aggressive portfolio with beta more than one should have return, which is higher than market portfolio. Returns on defensive portfolios (with beta less than one) will be lower than return on market portfolio. Thus, the expected return on inefficient portfolios is equal to

$$E(R_i) = R_f + \beta_i[E(R_m - R_f)]$$

Both CML and SML show that expected return on portfolios (whether efficient or not) is the linear function of risk with different risk measures. Thus, given a risk level, it is possible to select portfolio with higher expected return and / or evaluate level of return against the benchmark return.
b. Non-Standard CAPM:

The equilibrium model discussed above was made practicable by relaxing some of its restrictive assumptions. The main assumption viz., unrestricted lending and borrowing at risk free rate is relaxed and the CAPM is modified on the following lines:

a) Riskless lending and borrowing is not allowed
b) Riskless lending is allowed but not borrowing
c) Lending and borrowing at two different rates

Models were also developed under conditions of short selling of risky securities, heterogeneous expectations among investors, personal taxation on investment income, etc. Two of these models, which have significance in terms of CAPM testing, are briefly explained in the following paragraphs.

i) CAPM with No Risk Free Bond (Zero Beta Model)

Taking into consideration that there is no security which is free from all types of risk the basic assumption of standard model has been relaxed which led to the development of zero beta model. Black (1972) suggested consideration of return on a zero beta portfolio as a proxy to the riskfree security. This zero beta portfolio can be constructed by combining securities in such a way that the weighted sum of the betas add to zero and the mean of the unsystematic risk of the securities considered also equal to zero. Zero beta portfolio does not imply that the securities in the portfolio have zero variance of return but it simply means that these securities collectively contribute nothing to the variance of return on market portfolio.
With zero beta portfolios as well, the linear relationship between risk and return holds. This is because all investors prefer to hold efficient portfolios, with the assumption of homogeneous expectations, this leads to every investor holding securities in proportion to market portfolio which itself would be on the efficient set. The risk-return relationship of the same will be linear. In Black's model,

\[ \hat{R}_{it} = \alpha_i + \beta_{i\alpha} R_m + \beta_{i\mu} R_{m t} + \mu_i \]

\[ a_i = R_{it} - b_{iz} R_{zt} - b_{im} R_{mt} \]

\[ \beta_{i\alpha} = \frac{\text{Cov}(R_i, R_z)}{\sigma^2(R_z)} \]

\[ \beta_{i\mu} = \frac{\text{Cov}(R_i, R_m)}{\sigma^2(R_m)} \]

The standard model has only single beta factor which arises out of covariance the risky assets would have with market portfolio; investments in risk free securities do not have any variance at all. Black's model is a 'two factor version' of the standard CAPM. Securities included in portfolio z have two beta factors, one being the covariance with zero beta portfolio, z and the other is the covariance with market portfolio, m. Return on zero beta portfolio is higher than that of risk free return because this portfolio has some variance associated with it. The zero beta model takes the form of,

\[ E(R_i) = E(R_z) + [E(R_m - R_z)]\beta_i \]

where,

Rz is the return on the zero beta portfolio.
If zero beta version of the CAPM holds good, the intercept of the regression of estimated beta on expected return will be equal to the return on zero beta portfolio which is higher than risk free rate and the slope is less steep than the one under the standard model.

ii) **CAPM with Two Interest Rates**

Considering the fact that there is a spread between rates of interest on borrowing and lending and only lending but not borrowing at riskless rate is possible, suitable modifications were made in the Standard Model by Brenann (1971). Under this, the market portfolio (M), which is efficient, would lie between the portfolio formed by borrowing money (B) and the portfolio created by lending money (L) on the efficient frontier. This is because of the characteristics of low risk - low return of portfolio L and high risk-high return of portfolio B. The market portfolio which consists of all risky assets would have a risk-return profile which is higher than portfolio L but lower than portfolio B, thus lying in the middle of the two portfolios. Return on the portfolio would still be linear function of risk.

The CAPM is widely used by fund managers in the areas of:

- **Security selection/valuation** by identifying mispriced securities, i.e., securities with low returns compared to the level of systematic risk associated with it (overpriced scrips) or securities which have low risk but giving high returns (underpriced scrips).
• **Portfolio Performance Evaluation** based on expected level of return given a risk level under equilibrium conditions vis-à-vis the actual performance of a fund / portfolio or ranking of funds on this basis.

• **Portfolio Construction** to suit the needs of the investors (say, aggressive or defensive portfolios) based on the relevant measure of risk for efficient or inefficient portfolios and individual securities and the expected return for that level of risk.

Major criticisms of the model were by Roll (1977). They are:

1. **CAPM TESTABILITY**

• Since market model is the centrepiece on which equilibrium prices are arrived at, the one and only testable hypothesis of the CAPM is to find out whether the market portfolio is 'mean-variance efficient'. Only when testing of this hypothesis confirms the efficiency of market portfolio, other implications of general equilibrium model can be tested. The empirical studies that have so far been carried out are not direct tests of CAPM but were indirect tests of properties of SML. Though the results of the empirical tests were in support of the SML properties, Roll showed that the same could not simply be taken to imply that market portfolio is efficient. This is because it is possible to observe positive, linear relationship between portfolio betas and average returns in case of large, well-diversified portfolios tested against equally weighted index taken as proxy for market portfolio.
• Moreover, the theory is testable only with the 'true' market portfolio that should contain every asset in the international economic system. However, construction of such a market portfolio is very difficult and determining whether such a portfolio is efficient in expected return, risk framework is impossible. Thus, CAPM can never be tested!!

2. PORTFOLIO PERFORMANCE MEASURABILITY

• The portfolio performance measures that are widely used are based on the CAPM. While Jensen's and Treynor's measures are based on the SML, Sharpe's measure is based on the CML. Roll showed that if the CAPM can not be tested, measures based on the model can not be used for performance evaluation. If the index chosen as proxy for true market portfolio is mean-variance efficient even though the true market portfolio was not, then no security would have abnormal performance when measured as a departure from the security market line.
EMPIRICAL TESTS OF THE CAPM

Capital Asset Pricing Model is based on investors' homogeneous expectations regarding return and risk leading to a market portfolio which is efficient. Any direct testing of the model would be for testing the efficiency of the market portfolio. However, due to the difficulty involved in constructing an index which is a proxy for market model and testing the same for its efficiency, most of the tests were designed to test the properties of Security Market Line (SML) and conclusions were drawn about the market efficiency indirectly. Capital Asset Pricing Model explains that difference in stock/portfolio returns are mainly due to differences in the systematic risks that can not be diversified. Though the CAPM is an ex-ante model, because of difficulties involved in forming expectations about future returns and betas, the model is generally tested using historical data. Basic assumptions on which this ex-post testing is done are:

- beta is stable over time
- market model holds in every period
- Probability distribution for return on securities does not change their shape over time, thus estimates needed can be predicted through the sampling of past returns.

Generally, two-pass regression technique is adopted to test SML.

a) First, time series regression is used to calculate historical betas on individual securities. This methodology of calculating beta ensures that the residuals are independent. Since it is assumed that the betas are stable over time, and historical betas are expected to have high correlation with future (true) beta, the
same is used as the basis to form portfolios in the future period for testing the model. This avoids any bias in selection. Expected returns on these portfolios are then regressed on expected market returns to have an estimate about future betas.

b) Second pass regression on cross-sectional data is used to find out how portfolio betas are related to average portfolio returns in future period. The line of best fit thus arrived at is the SML.

c) 't' test is used to test the hypothesis regarding expected values of regression coefficients. This is to find out whether properties of the SML estimates are in accordance with the CAPM predictions.

There have been numerous studies testing the efficacy of the CAPM using data from Indian and other developed markets. In the following paragraphs, some of the studies have been briefly reviewed.

1. **EMPIRICAL TESTS USING DATA FROM DEVELOPED CAPITAL MARKETS**:

   a) One of the earliest tests of the model was by Sharpe and Cooper (1972). They studied whether higher return has been associated with higher risk over a long period of time. The methodology used was to calculate betas using 60 months' stock returns for the period 1931-1967 and to make ten equally weighted portfolios based on betas. This study confirmed that in general, stocks with higher betas gave higher returns. The co-efficient of determination (R squared) from second pass regression was 95% implying that beta explained a significant portion of the difference in return between portfolio deciles. This study did not verify whether beta was the only measure of risk priced in the market.
b) The studies done by Douglas (1970) Miller and Scholes (1972) Blume and Friend (1973) showed that the intercept of second pass regression was consistently higher than the risk free rate, thus violating the Sharpe -Lintner hypothesis that price of time was equivalent to Rf. These studies also showed that the price of risk is lower than Rm - Rf which is expected in case of zero beta model.

CAPM tests by Black, Jensen and Scholes (1972) & by Fama and MacBeth (1974) were instrumental in recording evidence of positive association between return and systematic risk in the long run.

c) Black, Jensen and Scholes formed portfolios based on betas computed on 60 monthly returns for 35 years (1935-1969). Portfolio returns and betas were estimated on the basis of average return during the following twelve months and tested cross sectionally for the properties of the CAPM.

d) The CAPM test by Fama and MacBeth considered monthly returns of all equity shares traded on the NYSE for the period 1926-1968 and an equally weighted average index was considered as market model. The efficacy of the CAPM was tested over three major periods and six sub-periods under the predictive mode by following a three-step procedure in arriving at the estimates of security market line. That is, in formation period, betas were estimated by using first five years monthly returns and equally weighted portfolios were formed; in estimation period, monthly returns on all securities and market index during next five years were used to estimate betas; and in testing period, the returns on every month is predicted using least square
method for the last five years. Thus, the period for which beta estimations were made and the returns predicted for testing the relationship were of two different time periods. Risk return framework was tested in the following form:

\[ R_{pt} = \gamma_1 + \gamma_2 b_{pm,t-1} + \gamma_3 b^2_{pm,t-1} + \gamma_4 s_{p,t-1(ei)} + u_{pt} \]

In addition to the hypotheses used for testing the properties of the SML, this study considered -

- Impact of residual risk on return;
- Linearity of security market line; and
- Existence of positive price of risk in the capital market.

This study concluded that the pricing of securities was in line with the implications of two factor model (zero beta model) and the behaviour of return through time was consistent with an efficient capital market. These tests recorded evidence for the prevalence of zero beta CAPM during pre-1970 period. These results have been criticised by Tinic & West (1986) on the ground that relation between return and systematic risk was positive only in the month of January (January effect) and risk premiums in other months were not significantly different from zero.

e) A study by Friend, Westerfield and Granito (1978) has used *ex-ante* expectations rather than *ex-post* realised returns. Another test by the same authors constructed a portfolio of marketable risky assets by incorporating return of bonds as well as stocks in order to test CAPM. In this study Beta was calculated by using quarterly rate of returns from the fourth quarter of 1968 through second quarter of 1973. As against
grouping done on the basis of beta, this study adopted a
grouping technique based on residual standard deviation.
This study confirmed that there was a significant influence of
residual standard deviation on the return on assets and
concluded that standard deviation of individual asset could
be an appropriate additional measure of the risk apart from
beta.

f) Stambaugh (1982) created a market portfolio by considering
a market portfolio of common stocks, real estate and
consumer durables and tested the efficacy of the CAPM. The
outcome of this study indicated that CAPM inferences varied
according to the composition of the portfolios considered.

g) In a more recent study, Fama and French (1992) shown that
two easily measured variables viz., size and book to market
equity seem to describe the cross section of average stock
returns and concluded that beta is insufficient to explain
variation in portfolio returns. The failure of the CAPM is
attributed to the usage of bad proxies to market portfolio.
The study also inferred that if the market proxies used were
inefficient, then applications of the model in the areas of cost
of capital estimation and portfolio performance evaluation
would not be correct.

2. TESTS OF EQUILIBRIUM IN INDIAN MARKET:

In the recent years quite a few studies have been undertaken to
test if the CAPM holds good in the Indian Market. While many of
the studies indicate that the returns generated are not
commensurate with the level of risks undertaken by investors, few
research papers on this subject do confirm the positive relation
between systematic risk and return. An interesting point that
emerges out of almost all these studies are that the intercepts of the second pass regression is nowhere near the risk free rate.

**a) Tests Confirming CAPM Hypothesis**

i) One of the first tests of CAPM using Indian data was by Srinivasan (1988). This study used 85 companies' data from July 1982 to Oct. 1985 and Economic Times Index as market proxy for beta estimation, had concluded that the CAPM holds good only in the long run. This also confirmed that betas and returns were linearly related and beta explained the variation in stock returns.

ii) Two other studies conducted during the same period by Varma (1988) and Yalawar (1988) provide evidence in favour of CAPM.

**b. Tests Rejecting CAPM**

i) Two studies of M. Obaidullah (1991) observed the existence of abnormal returns and these are attributed to price-earnings ratios rather than betas. Small portfolio size and inefficient market structure were the reasons attributed for the price disequilibrium.

ii) Ray (1994) tested the model by considering monthly returns of 170 actively traded scrips on the BSE for the data period 1980-91. Using Fama and MacBeth methodology, he concluded that Betas computed using monthly returns and three different market indices viz., RBI Index, Economic Time Index and BSE Sensex did not confirm the applicability of the CAPM in Indian context.
iii) Gupta & Sehgal's (1993) study used average monthly prices of thirty Sensex scrips for the period April 1979 to March 1989. Sensex was used as market proxy and Fama & MacBeth methodology was adopted to test the CAPM. Betas were estimated for the entire study period as well as for four sub-periods, each of 30 months' duration. In order to avoid having an upward bias in the slope coefficient of cross section regression, the odd-even procedure suggested by Ball, Brown & Officer was used. For the full study period and for the first three sub-periods, slope coefficient was found to be positive. Negative slope observed in the fourth sub-period was explained by the slow down in economic activities during that period. Similarly, except for the fourth sub-period, the intercept term was not significantly different from zero. Tests of linearity pointed out that both $B_p$ and $B_p^2$ together explained about 96% of return on risky assets.

iv) The methodology used by Dhankar (1996) was to test CAPM based on estimated returns and betas, rather than ex-post realised returns. Fifty frequently traded scrips from the specified group on Bombay Stock Exchange during the period April 1989 to March 1993 were selected for the study. National Index was considered as market proxy. Estimations regarding returns were made based on the assumptions that there are not much of fluctuations in the last five years' average. Considering the estimates provided by market players regarding the next year's market price and dividend on a stock, the estimates were refined further. Future betas have been computed by adjusting the historical betas using Blume's method. Beta during the second period is estimated as $\beta_2 = 0.75 \beta_1 + 0.25 \beta_p$ where.
\( \beta_1 \) = Beta during 1st period
\( \beta_2 \) = Beta during 2nd period
\( \beta_p \) = Average Beta of the portfolio.

Of the ten portfolios constructed from the sample, except in three cases, there existed a linear relationship among the future betas and expected returns. The second pass regression results confirmed that Beta is a significant explanatory variable. However, like in many other studies, the intercept is not equal to risk free rate.

v) A recent CAPM test by G.Ramchandran (1998) using Fama-MacBeth framework (sample of fifty scrips listed on BSE during the period 1987-97 and BSE Sensex as benchmark) estimated beta over three sub-periods and found that risk classes of scrips change across time. This study also tested CAPM under predictive and non-predictive framework. Under the former only Beta is found statistically significant while the intercept term is found to be much higher than the risk free rate. Under the non-predictive framework, the CAPM is rejected.