CHAPTER 5
CALCULATION METHODOLOGY

In this chapter the procedure for the calculation of Nu, diffuser efficiency and loss coefficient are given. Sample of calculations also presented for the same.

5.1 Computational Procedure

In this chapter the procedure of the calculation of Nusselt number (Nu) and diffuser efficiency and loss coefficient are given. Sample of calculations are also presented for the same.

The variables of interest, Nusselt number (Nu) and efficiency of diffuser ($\eta_{\text{diff}}$) are calculated. Momentum and thermal energy balance is carried out across the diffuser. The stagnation pressure loss is also computed. This is used for obtaining the Loss Coefficient as well as dissipation (pumping power). The loss of stagnation pressure is obtained as

\[
\text{Loss of stagnation pressure} = P_{O_{in}} - P_{O_{out}}
\]  

(5.1)

Where $P_{O_{in}}$ and $P_{O_{out}}$ are,

\[
P_{O_{in}} = P_{in} + \frac{1}{2} \rho V_{in}^2
\]  

(5.1a)

\[
P_{O_{out}} = P_{out} + \frac{1}{2} \frac{\rho}{A_{out}} V_{out}^2
\]  

(5.1b)

$V_{in}$ and $V_{out}$ are average velocities at inlet and outlet. They are obtained as:

\[
V_{in} = \frac{m}{\rho A_{in}}
\]  

(5.2)

\[
V_{out} = \frac{m}{\rho A_{out}}
\]  

(5.3)

Dissipation power across the diffuser length obtained by,
\[ D = \text{Loss of stagnation pressure} \times \frac{m}{\rho} \quad (5.4) \]

Pressure rise along the diffuser obtain by \( P_3 - P_1 \).

Ideal pressure rise is obtained by Bernoulli equation along the diffuser length,

\[ P_3 - P_1 = \rho \frac{V_{in} - V_{out}}{2} \quad (5.5) \]

The efficiency of diffuser is given as,

\[ \eta_{\text{diff}} = \frac{\text{pressure rise}}{\text{Ideal pressure rise}} = \frac{P_3 - P_1}{\rho \frac{V_{in} - V_{out}}{2}} \quad (5.6) \]

The loss coefficient of diffuser is defined as,

\[ \zeta = \frac{\text{Loss of stagnation pressure}}{\frac{1}{2} \rho V_{in}^2} \quad (5.7) \]

Wall pressure coefficient \( C_p \) across the diffuser is given as,

\[ C_p = \frac{P_i - P_{\text{ref}}}{\frac{1}{2} \rho V_{in}^2} \quad i = 1, 2, 3 \quad (5.8) \]

The inlet pressure is taken as the reference pressure \( P_{\text{ref}} \).

### 5.2 Mass Flow Calculations

The velocity profile is determined at grid points at the outlet. This is in turn integrated to obtain mass flow rate and bulk temperature.

Mass flow rate obtain as,

\[ m = \rho \int \int u \, dy \, dz \]

The bulk temperature is computed as,

\[ \rho c_p \int \int V T_{\text{air}} \, dy \, dz = m c_p T_{\text{bulk}} \]

The average dynamic pressure is computed by integrating the discrete values at mesh points.
\[ \rho \dot{V}^2 = \frac{\rho}{A_{out}} \int V^2 dy dz \]

5.3 Heat Transfer Calculations

The heat transfer coefficient is calculated from the energy equation as:

\[ Q - Q_{loss} = hA(T_w - T_{mean}) \quad (5.9) \]

Where \( T_w \) is the average wall temperature computed as,

\[ T_w = \frac{\sum T_i}{n}. \quad (5.10) \]

Where \( T_i \) is correspond to the temperature at different location of the heated wall.

\( T_{mean} \) is the average of the inlet and outlet bulk air temperatures. The inlet temperature is the ambient, while the one at the outlet is obtained as

\[ \rho c_p \int T_{air} Vdydz = mc_p T_{bulk}. \quad (5.11) \]

\( Q \), the heat supplied, is obtained from electrical measurements as

\[ Q = V^* I^*. \quad (5.12) \]

Heat loss (\( Q_{loss} \)) is calculated using natural convection formulae for vertical and horizontal plates, using the measured outer wall and ambient temperatures.

Since the duct is laid horizontally, the heat loss is by free convection from the vertical sides and the horizontal top and bottom. For natural convection, the Rayleigh number is defined as,

\[ Ra = GrPr = \frac{g \beta L^3 (T_w - T_{\infty}) Pr}{\nu^2} \quad (5.13) \]

(i) For vertical sides, the average Nusselt number \([45,46]\) is given as

\[ \overline{Nu_L} = 0.59 Ra_L^{0.25} \quad \text{for} \quad 10^4 < Ra_L < 10^9 \quad (5.14) \]

(ii) For upper and lower surfaces, the average \( Nu \) is given as

\[ \overline{Nu_L} = 0.15 Ra_L^{\frac{1}{3}} \quad \text{for} \quad 10^7 < Ra_L < 10^{10} \quad (5.15) \]

The heat balance is verified by comparing \( Q - Q_{loss} \) with \( mc_p (T_{out} - T_{in}) \).
The heat loss from the lower surface is negligible.

### 5.4 Sample Calculation

The calculations below show for Reynolds number equal $2.3E5$ for smooth diffuser with half angle $\beta=5.7^\circ$

\[
\frac{m}{\rho} = 0.033746 \text{ kg/m}^3
\]

\[
V_{in} = \frac{m}{\rho A_{in}} = 29.8 \text{ m/s}
\]

\[
\text{Re}_L = \frac{V_{in} L}{v} = 2.3E5
\]

\[
V_{out} = \frac{m}{\rho A_{out}} = 12.81 \text{ m/s}
\]

\[
P_{\text{in}} = P_{\text{in}} + \frac{1}{2} \rho V_{in}^2 = 251.845 \text{ Pa}
\]

\[
P_{\text{out}} = P_{\text{out}} + \frac{1}{2} \rho V_{out}^2 = 100.919 \text{ Pa}
\]

Loss of stagnation pressure is obtained:

\[
\text{Loss} = P_{\text{in}} - P_{\text{out}} = 150.926 \text{ Pa}
\]

Dissipation power across the diffuser length obtained by:

Ideal pressure rise is obtained by Bernoulli equation along the diffuser length,

\[
P_3 - P_1 = \rho \frac{V_{in}^2 - V_{out}^2}{2} = 445.23 \text{ Pa}
\]

Pressure rise along the diffuser obtain by:

\[
P_3 - P_1 = 294.3 \text{ Pa}
\]

Efficiency of diffuser is given as,

\[
\eta_{\text{diff}} = \frac{\text{pressure rise}}{\text{Ideal pressure rise}} = \frac{P_3 - P_1}{\rho \frac{V_{in}^2 - V_{out}^2}{2}} = 0.661
\]
Loss coefficient of diffuser is defined as,

\[ \zeta = \frac{\text{Loss of stagnation pressure}}{\frac{1}{2} \rho V_{in}^2} = 0.2764 \]

5.4.1 Heat loss

The heat loss from the vertical sides and upper surface is given as

a) For the vertical right side

\[ R_a = G_r \cdot Pr = 1285000 \]

\[ Nu = 19.86 \]

\[ h_v = \frac{Nu \cdot K}{L} = 4.5 \text{ W/m}^2\text{K} \]

\[ Q_v = h_v A (T_w - T_\infty) = 0.7241 W \]

The heat losses from the two vertical sides are equal.

b) For the upper surface

\[ Gr_L \cdot Pr = 358733 \]

\[ Nu = 13.2156 \]

\[ h_{\text{top}} = \frac{Nu \cdot K}{L} = 5.5791 \text{ W/m}^2\text{K} \]

\[ Q_{\text{top}} = h_{\text{top}} A (T_w - T_\infty) = 1.7918 W \]

5.4.2 Total heat loss

The heat loss at bottom surface is negligible. The total heat loss is thus equal to,

\[ Q_{\text{loss}} = 2Q_v + Q_{\text{top}} = 3.24 W \]

5.4.3 Calculation of heat transfer coefficient

The heat transfer coefficient is calculated from the energy equation as

\[ Q - Q_{\text{loss}} = h A (T_w - T_{\text{mean}}) \]

\[ Q_{\text{net}} = Q - Q_{\text{loss}} = VT - Q_{\text{loss}} = 131 \times 1.315 - 3.24 = 169.04 W \]
\[ h = 163.65 \ \frac{W}{m^2 K} \]

\[ T_f = \frac{T_{wall} + T_{air}}{2} = 354.31 K \]

The thermal conductivity \( K \) of air at \( T_f \) is

\[ K = 0.030342 \ \frac{W}{mK} \]

\[ Nu = \frac{hL}{K} = 647.23 \]

5.5 Uncertainty Estimates

The numerical value of uncertainty is an estimate of the error. The uncertainty quantifies the expected accuracy. It provides an estimate of the accuracy measured data and calculated parameter. Most of the physical parameters are measured against a suitable reference point directly like temperature measurement, speed measurement etc. A few physical parameters such as heat transfer rate, heat transfer coefficient cannot be measured directly. Therefore, uncertainty analysis is carried out to estimate the uncertainty in calculated parameters, as outlined by Robert and Moffat [44].

Consider a physical variable \( y \), a function of several variables as

\[ y = f(x_1, x_2, x_3, ..., x_n) \]

The error associated with \( y \) is given as

\[ dy = \left[ \left( \frac{\partial y}{\partial x_1} \right)^2 \left( dx_1 \right)^2 + \left( \frac{\partial y}{\partial x_2} \right)^2 \left( dx_2 \right)^2 + \ldots + \left( \frac{\partial y}{\partial x_n} \right)^2 \left( dx_n \right)^2 \right]^{1/2} \]

Where each of the \( dx \) terms is a combination of bias and precision uncertainties. The most basic forms of the \( Nu \), Reynolds number are used so that the errors associated with the directly measured quantities could be incorporated.

The variables measured are pressure and temperature. Pressure is measured by a digital manometer with an uncertainty of \( \pm 0.5 \) mm of water. Temperature is
measured with a digital sensor with an uncertainty of ±0.5°C. The velocity profile is measured using a pitot tube.

5.5.1 Uncertainty in mass flow rate

The mass flow rate is calculated as

\[ \dot{m} = \rho AV \]

As the uncertainty in the measurement (estimation) of \( \rho \) and \( A \) are negligible,

\[ \frac{\Delta m}{\dot{m}} = \frac{\Delta V}{V} \]

The uncertainty in velocity can be written as

\[ \frac{\Delta V}{V} = \frac{1}{2} \frac{\Delta h^*}{h^*} \]

Where \( h^* \) is the dynamic head, obtained from the manometer as mm water column.

The uncertainty in \( h \) is given as

\( \Delta h^* = 0.5 \text{ mm (water)} \)

\( \bar{h}^* \) the average head is computed as

\[ \bar{h}^* = \frac{1}{2} \frac{\rho_{\text{air}} V}{\rho_{\text{water}} g} \]

Thus for \( V = 12.4 \text{ m/s} \), one obtains \( \bar{h} = 10 \text{ mm (water)} \). Thus

\[ \frac{\Delta m}{\dot{m}} = \frac{1}{2} \frac{\Delta h^*}{\bar{h}^*} = 2.5\% \]

Thus, the uncertainty in mass flow rate (measurement) is 2.5 %.

5.5.2 Uncertainty in heat flux

The power supplied to the heating filament is measured by voltmeter and ammeter as

\[ Q_h = VI = 131 \times 1.3105 = 171.68 \text{ W} \]

Taking instrumental error as ±1% of the full scale
V = V ± 2 Volt
I = I ± 0.05 Amp

\[ \frac{\partial Q_h}{\partial I} = V = 131 \]
\[ \frac{\partial Q_h}{\partial V} = I = 1.3105 \]

\[ \Delta Q_h = [(\frac{\partial Q_h}{\partial I})^2 (\Delta I)^2 + (\frac{\partial Q_h}{\partial V})^2 (\Delta V)^2]^{1/2} = [(131)^2 (0.05)^2 + (1.3105)^2 (2)^2]^{1/2} = 7.05 \, W \]

The relative uncertainty in \( Q_h \) is

\[ \frac{\Delta Q_h}{Q_h} = \frac{7.05}{171} = 4.1\% \]

Thus the uncertainty in heat flux is 4%.

\[ \Delta Q_h = 171 \times 0.04 = 6.8 \, W \]

**5.5.3 Uncertainty in heat transfer coefficient**

The uncertainty estimation in the rate of heat transfer coefficient is given as follows

\[ h = \frac{Q_{\text{real}}}{A(T_w - T_{in})} \]

\[ \frac{\partial h}{\partial Q} = \frac{1}{A(T_w - T_{in})} = \frac{1}{0.151 \times 0.062 (119.6 - 26.53)} = 1.15 \]

\[ \frac{\partial h}{\partial T_w} = \frac{-Q_{\text{real}}}{A(T_w - T_{in})^2} = \frac{-199.8}{0.151 \times 0.062 \times (119.6 - 26.53)^2} = -2.464 \]

\[ \frac{\partial h}{\partial T_{air}} = \frac{Q_{\text{real}}}{A(T_w - T_{in})} = 2.464 \]

\[ \Delta h = [(\frac{\partial h}{\partial Q})^2 (\Delta Q_h)^2 + (\frac{\partial h}{\partial T_w})^2 (\Delta T_w)^2 + (\frac{\partial h}{\partial T_{air}})^2 (\Delta T_{air})^2]^{1/2} \]

The uncertainty in heat transfer coefficient is thus obtained as

\[ \Delta h = [(1.15)^2 (6.8)^2 + (-2.464)^2 (0.5)^2 + (2.464)^2 (0.5)^2]^{1/2} = 7.9 \, \frac{W}{m^2 \, K} \]

The relative uncertainty in \( h \) is thus

\[ \Delta h/h = 7.9/180 = 4.4\% \]
5.5.4 Uncertainty in Nusselt number

\[ \text{Nu}_L = \frac{hL}{k} \]

Nu is proportional to h other two being constant. Thus the uncertainty in Nu can be taken as 4.4%. 