Chapter 3

Hall Effect on Thermal Instability of Viscoelastic Dusty Fluid in Porous Medium
3.1 Introduction

The theoretical and experimental results of the onset of thermal instability (Bénard convection), under varying assumptions of hydrodynamics and hydromagnetics, have been discussed by Chandrasekhar [26] in his celebrated monograph. If an electric field is applied at right angles to the magnetic field, the whole current will not flow along the electric field. This tendency of the electric current is called the Hall current effect. The Hall effect is likely to be important in many geophysical and astrophysical situations as well as in flows of laboratory plasma, Singh and Gupta [37]. Sherman and Sutton [38] considered the effect of Hall currents on the efficiency of a magneto-fluid-dynamic generator. Gupta [39] studied the problem of thermal instability in the presence of Hall currents and found that Hall currents have a destabilizing effect on the thermal instability of a horizontal layer of a conducting fluid in the presence of a uniform vertical magnetic field. The use of the Boussinesq approximation has been made throughout, which states that the variations of density in the equations of motion can safely be ignored everywhere except in association with the external force. The approximation is well justified in the case of incompressible fluids.

When the fluids are compressible, the equations governing the system become quite complicated. To simplify them, Boussinesq tried to justify the approximation for compressible fluids when the density variations arise principally from thermal effects. Spiegel and Veronis [40] simplified the set of equations governing the flow of compressible fluids under the following assumptions:

- The depth of the fluid layer is much less than the scale height, as defined by them.
- The fluctuations in temperature, density and pressure, introduced due to motion, do not exceed their total static variations.

Under the above approximations, the flow equations are the same as those for incompressible fluids, except that the static temperature gradient is replaced by its excess over the adiabatic one and $C_v$ is replaced by $C_p$. In geophysical situations, the fluid is often not pure but contains suspended particles. Scanlon and Segel [23] considered the effects of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number is reduced because of the heat capacity of the particles. The suspended particles were thus found to destabilize the layer.
Palaniswamy and Purushotham [29] studied the stability of shear flow of stratified fluids with fine dust and found the fine dust to increase the region of instability. The fluids were considered to be Newtonian and the medium was considered to be non-porous in all the above studies.

There is growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry. Bhatia and Steiner [41] studied the problem of thermal instability of a Maxwellian viscoelastic fluid in the presence of rotation and found that rotation has a destabilizing influence in contrast to the stabilizing effect on an ordinary viscous (Newtonian) fluid. The thermal instability of an Oldroydian viscoelastic fluid acted on by a uniform rotation was studied by Sharma [42]. There are many elastico-viscous fluids that cannot be characterized by Maxwell’s or Oldroyd’s constitutive relations. The Rivlin-Ericksen elastico-viscous fluid is one such fluid. Rivlin and Ericksen [32] studied the stress, deformation, relaxations for isotropic materials. Thermal instability in viscoelastic Rivlin-Ericksen fluids in the presence of rotation and magnetic field, separately, was investigated by Sharma and Kumar [43] and [44]. Sharma and Kumar [45] studied the hydromagnetic stability of two Rivlin-Ericksen elasticoviscous superposed conducting fluids. Kumar and Singh [46] studied the stability of two superposed Rivlin-Ericksen viscoelastic fluids in the presence of suspended particles. In another study, Kumar et al. [47] studied the hydrodynamic and hydromagnetic stability of two stratified Rivlin-Ericksen elasticoviscous superposed fluids.

The flow through porous media is of considerable interest for petroleum engineers and geophysical fluid dynamicists. A great number of applications in geophysics may be found in the books by Phillips [48], Ingham and Pop [49], and Nield and Bejan [50]. When the fluid slowly percolates through the pores of a macroscopically homogeneous and isotropic porous medium, the gross effect is represented by Darcy’s law. As a result of this macroscopic law, the usual viscous term in the equations of fluid motion is replaced by the resistance term $-\frac{1}{k_1} (\mu + \mu' \frac{\partial}{\partial t} q)$, where $\mu$ and $\mu'$ are the viscosity and viscoelasticity of the Rivlin-Ericksen fluid, $k_1$ is the medium permeability and $q$ is the Darcian (filter) velocity of the fluid. Lapwood [27] studied the stability of a convective flow in hydromagnetics in a porous medium using Rayleigh’s procedure. The Rayleigh instability of a thermal boundary layer in flow through a porous medium
was considered by Wooding [28]. The stability of superposed Rivlin-Ericksen elastico-viscous fluids permeated with suspended particles in a porous medium was considered by Kumar [36]. Kumar et al. [51] studied the instability of two rotating viscoelastic (Rivlin-Ericksen) superposed fluids with suspended particles in a porous medium. In another study, Kumar et al. [52] considered the MHD instability of rotating superposed Rivlin-Ericksen viscoelastic fluids through a porous medium.

Here our interest is to bring out the suspended particles effect on thermal instability of a compressible viscoelastic (Rivlin-Ericksen) fluid in a porous medium including the effect of Hall currents.

3.2 Formulation of the Problem

In porous medium, an infinite horizontal layer of thickness $d$ confined between two planes $z = 0$ and $z = d$ of a compressible viscoelastic Rivlin-Ericksen fluid in the presence of uniform horizontal magnetic field $\vec{H}(0, 0, H)$ is considered. For the study thermal instability, layer is heated from underside and steady adverse temperature gradient $\beta$ is maintained, where $\beta = \left|\frac{dT}{dz}\right|$. The equations of motion and continuity for the fluid are:

$$\frac{\rho}{\epsilon} \left[ \frac{\partial \vec{v}}{\partial t} + \frac{1}{\epsilon} (\vec{v}.\nabla) \vec{v} \right] = -\nabla p - \rho g \vec{\lambda} - \frac{1}{k_1} \left( \mu + \mu' \frac{\partial}{\partial t} \right) \vec{v} + \frac{KN}{\epsilon} (\vec{u} - \vec{v}) + \frac{\mu_e}{4\pi} (\nabla \times \vec{H}) \times \vec{H}$$

(3.1)

and

$$\frac{\epsilon}{\epsilon} \frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{v}) = 0$$

(3.2)

where $\rho \rightarrow$ density, $\mu \rightarrow$ viscosity, $\mu' \rightarrow$ viscoelasticity, $p \rightarrow$ pressure and $\vec{v}(u, v, w) \rightarrow$ velocity of the pure fluid. Here $\vec{u}(l, r, s) \rightarrow$ velocity of the suspended particles, $N(\bar{x}, t) \rightarrow$ number density of the suspended particles, $\epsilon \rightarrow$ medium porosity, $k_1 \rightarrow$ medium permeability, $\mu_e \rightarrow$ magnetic permeability, $g \rightarrow$ acceleration due to gravity, $\bar{x} = (x, y, z)$, $\vec{\lambda}(0, 0, 1)$ and $K = 6\pi \mu \eta'$, $\eta'$ being the particle radius, is the Stokes’ drag coefficient.
In the above equations of conservation of momentum (3.1), some assumptions regarding the shape and velocity of the suspended particles are taken as:

- Shape of the suspended particles in the fluid is uniform spherical.
- Relative velocities between the fluid and particles is small.
- Large distance between the particles as compared to their diameter. So Interparticle reactions are ignored.
- Gravity, pressure, Darcian force and magnetic field effect on the suspended particles are negligibly small, so ignored.
- Extra force due to the presence of particles is proportional to velocity difference between the particles and the fluid.
- Force exerted by fluid on particles and force exerted by particles on fluid balance each other.

So there must be an extra force equal in magnitude but opposite in sign in the equations of conservation of momentum or motion for the particles. If \( mN \) is the mass of particles per unit volume, then under the above assumptions, equations of conservation of momentum and mass for the particles are

\[
mN \left[ \frac{\partial \vec{u}}{\partial t} + \frac{1}{\epsilon} (\vec{u} \cdot \nabla) \vec{u} \right] = KN(\vec{v} - \vec{u})
\]  

(3.3)

and

\[
\epsilon \frac{\partial N}{\partial t} + \nabla \cdot (N \vec{u}) = 0.
\]  

(3.4)

Let at constant volume, \( C_v \) is the heat capacity of the fluid, at constant pressure, \( C_p \) is the heat capacity of the fluid, \( C_{pt} \) denote the heat capacity of the particles \( T \) is the temperature and \( q \) is effective thermal conductivity of the pure fluid. Assuming, fluid particles are in thermal equilibrium, then equation of heat conduction is given by

\[
[\rho C_v \epsilon + \rho_s C_s (1 - \epsilon)] \frac{\partial T}{\partial t} + \rho C_v (\vec{v} \cdot \nabla) T + mN C_{pt} \left( \epsilon \frac{\partial}{\partial t} + \vec{u} \cdot \nabla \right) T = q \nabla^2 T
\]  

(3.5)

where \( \rho_s \) is the density and \( C_s \) is the heat capacity of the solid matrix, R.C.Sharma and U.Gupta [53] had used the same parameters for their study.
Maxwell’s equations in the presence of hall currents give

\[ \nabla . \vec{H} = 0 \quad (3.6) \]

and

\[ \epsilon \frac{\partial H}{\partial t} = \nabla \times (\vec{v} \times \vec{H}) + \epsilon \eta \nabla^2 \vec{H} - \frac{\epsilon e}{4\pi N' e} \nabla \times \left[ (\nabla \times \vec{H}) \times \vec{H} \right] \quad (3.7) \]

where \( \eta \rightarrow \) resistivity, \( c \rightarrow \) speed of light, \( N' \rightarrow \) electron number density and \( e \) is charge of an electron. The initial state of the system is taken to be a quiescent layer (no settling) with a uniform particle distribution \( N_0 \) and is given by

\[ \vec{u} = (0, 0, 0), \quad \vec{v} = (0, 0, 0), \quad \vec{H} = (0, 0, H), \]

\[ T = T(z), \quad p = p(z), \quad \rho = \rho(z) \quad \text{and} \quad N = N_0 = \text{constant}. \quad (3.8) \]

Following the Spiegel and Veronis’ [40] we have

\[ T(z) = -\beta z + T_0, \quad p(z) = p_m - g \int_0^z (\rho_m + \rho_0) \, dz, \]

\[ \rho(z) = \rho_m \left[ 1 - \alpha_m (T - T_m) + K_m (p - p_m) \right], \]

\[ \alpha_m = -\left( \frac{1}{\rho} \frac{\partial \rho}{\partial T} \right)_m \quad \text{and} \quad K_m = \left( \frac{1}{\rho} \frac{\partial \rho}{\partial p} \right)_m. \quad (3.9) \]

Spiegel and Veronis’ [40] expressed any state variable say \( X \), in the form

\[ X = X_m + X_0(z) + X'(x, y, z, t) \quad (3.10) \]

where \( X_m \rightarrow \) constant space distribution of \( X \), \( X_0 \rightarrow \) variation of \( X \) in the absence of motion and \( X'(x, y, z, t) \rightarrow \) fluctuations in \( X \) due to motion of the fluid. Also, \( \rho_m \) is constant space distribution of \( \rho \) and \( p_m \rightarrow \) constant space distribution of \( p \) and \( \rho_0 \) is density at the lower boundary \( z = 0 \) and \( T_0 \rightarrow \) temperature of the fluid at \( z = 0 \). Again following Spiegel and Veronis[40] assumptions and results for compressible fluids, the flow equations are found to be the same as those of incompressible fluids except that the static temperature gradient \( \beta \) is replaced by its excess over the adiabatic \((\beta - g/C_p)\).
3.2.1 Perturbation of Equations

Let $\delta p$ denote the perturbation in pressure $p$, $\delta \rho$ denote the perturbation in density $\rho$, $\theta$ denote the perturbation in temperature $T$, $\vec{v}(u,v,w)$ denote the perturbation in fluid velocity (zero initially), $\vec{u}(l,r,s)$ denote the perturbation in particle velocity (zero initially), $N$ denote perturbations in suspended particles number density $N_0$ and $\vec{h}(h_x,h_y,h_z)$ denote perturbations in magnetic field $\vec{H}(0,0,H)$. Linearized perturbed equations of the viscoelastic fluid-particle layer are:

$$\frac{1}{\epsilon} \frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_m} \nabla \delta p - g \left( \frac{\delta \rho}{\rho_m} \right) \vec{x} - \frac{1}{k_1} \left( \nu + \nu' \frac{\partial}{\partial t} \right) \vec{v} + \frac{KN_0}{\epsilon \rho_m} (\vec{v} - \vec{u}) + \frac{\mu_e}{4\pi \rho_m} (\nabla \times \vec{h}) \times \vec{H}, \quad (3.11)$$

$$\nabla \cdot \vec{v} = 0, \quad (3.12)$$

$$mN_0 \frac{\partial u}{\partial t} = KN_0 (\vec{v} - \vec{u}), \quad (3.13)$$

$$\epsilon \frac{\partial N}{\partial t} + \nabla \cdot (N_0 \vec{u}) = 0, \quad (3.14)$$

$$(E + \epsilon) \frac{\partial \theta}{\partial t} = (\beta - g/C_p) (w + hs) + \kappa \nabla^2 \theta, \quad (3.15)$$

$$\nabla \cdot \vec{h} = 0 \quad \text{and} \quad (3.16)$$

$$\epsilon \frac{\partial \vec{h}}{\partial t} = \nabla \times (\vec{v} \times \vec{H}) + \epsilon \eta \nabla^2 \vec{h} - \frac{\epsilon \eta}{4\pi N' \epsilon} \nabla \times \left[ (\nabla \times \vec{h}) \times \vec{H} \right], \quad (3.17)$$

where $\alpha_m = \frac{1}{T_m} = \alpha$ (say), $\nu = \frac{\mu}{\rho_m}$, $\kappa = \frac{q}{\rho_m C_v}$ and $\frac{q}{C_p} \rightarrow$ adiabatic gradient, $\nu$ is kinematic viscosity and $\kappa$ is thermal diffusivity. Also,

$$h = \frac{f C_{pt}}{C_v}, \quad f = \frac{mN_0}{\rho_m} \quad \text{and} \quad E = \epsilon + \frac{(1 - \epsilon) \rho_s C_s}{\rho_m C_v}. \quad (3.18)$$

The linearized dimensionless perturbation equations relevant to the problem are

$$N^{-1}_{p1} \frac{\partial u}{\partial t} = -\frac{\partial}{\partial x} \delta p - \frac{1}{p} \left( 1 + A \frac{\partial}{\partial t} \right) u + \omega (l - u) + N_Q \left( \frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x} \right), \quad (3.18)$$
\[ N_{p1}^{-1} \frac{\partial v}{\partial t} = -\frac{\partial}{\partial y} \delta p - \frac{1}{p} \left( 1 + A \frac{\partial}{\partial t} \right) v + \omega (r - v) + N_Q \left( \frac{\partial h_y}{\partial z} - \frac{\partial h_z}{\partial y} \right), \quad (3.19) \]

\[ N_{p1}^{-1} \frac{\partial w}{\partial t} = -\frac{\partial}{\partial z} \delta p - \frac{1}{p} \left( 1 + A \frac{\partial}{\partial t} \right) w + \omega (s - w) + N_R \theta, \quad (3.20) \]

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (3.21) \]

\[ \left( \tau \frac{\partial}{\partial t} + 1 \right) l = u, \quad \left( \tau \frac{\partial}{\partial t} + 1 \right) r = v, \quad \left( \tau \frac{\partial}{\partial t} + 1 \right) s = w, \quad (3.22) \]

\[ \frac{\partial M}{\partial t} + \frac{\partial l}{\partial x} + \frac{\partial r}{\partial y} + \frac{\partial s}{\partial z} = 0, \quad (3.23) \]

\[ \frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + \frac{\partial h_z}{\partial z} = 0, \quad (3.24) \]

\[ N_{p2} N_{p1}^{-1} \frac{\partial h_x}{\partial t} = \epsilon^{-1} \frac{\partial u}{\partial z} + \nabla^2 h_x - M_1 \frac{\partial}{\partial z} \left( \frac{\partial h_x}{\partial y} - \frac{\partial h_y}{\partial z} \right), \quad (3.25) \]

\[ N_{p2} N_{p1}^{-1} \frac{\partial h_y}{\partial t} = \epsilon^{-1} \frac{\partial v}{\partial z} + \nabla^2 h_y - M_1 \frac{\partial}{\partial z} \left( \frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x} \right), \quad (3.26) \]

\[ N_{p2} N_{p1}^{-1} \frac{\partial h_z}{\partial t} = \epsilon^{-1} \frac{\partial w}{\partial z} + \nabla^2 h_z - M_1 \frac{\partial}{\partial z} \left( \frac{\partial h_y}{\partial x} - \frac{\partial h_z}{\partial y} \right) \quad \text{and} \quad (3.27) \]

\[ (E + h\epsilon) \frac{\partial \theta}{\partial t} = \left( \frac{G - 1}{G} \right) (w + h s) + \nabla^2 \theta \quad (3.28) \]

where

- \( N_{p1} = \frac{\nu}{\kappa} \) is modified Prandtl number, \( N_{p2} = \frac{\nu}{\eta} \) is modified magnetic Prandtl number,
- \( N_R = \frac{g \alpha}{\nu \kappa} \) is Rayleigh number, \( N_Q = \frac{\mu e H^2 d^2}{4 \pi \rho_m \nu \eta} \) is Chandrasekhar number, \( M = \frac{\epsilon N}{N_0} \).
- \( M_1 = \frac{m H}{4 \pi N \nu \eta} \) is Hall parameter, \( \omega = \frac{KN_0 d^2}{\rho_m \nu e}, \tau = \frac{m e}{K d^2}, A = \left( \frac{\nu'}{\nu} \right)^2 \frac{\nu}{\mu}, f = \frac{m N_0}{\rho_m} = \tau \omega, N_{p1} \) is mass fraction, \( G = \frac{C_p \rho}{\gamma} \) and \( P = \frac{k}{d^2} \).
Here physical variables have been scaled using \( d, \frac{d^2}{\kappa}, \frac{\rho \nu \kappa}{d^2}, \beta d \) and \( \frac{H \kappa}{\eta} \) as the length, time, velocity, pressure, temperature and magnetic field scale factors, respectively. The boundary conditions suitable to the problem, two free boundaries and the medium adjoining the fluid as non conducting, are considered as

\[
w = \frac{\partial^2 w}{\partial z^2} = \theta = 0, \quad \xi = \frac{\partial \zeta}{\partial z} = 0, \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1.
\] (3.29)

and \( h_x, h_y, h_z \) are continuous with an external vacuum field.

(3.30)

Here \( \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \) and \( \xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y} \) are the z-components of vorticity and current density, respectively. Equations (3.18)-(3.28), after eliminating \( u, v \) and \( \delta p \) can be expressed as

\[
\left[ L_1 + \frac{L_2}{P} \left( 1 + A \frac{\partial}{\partial t} \right) \right] \nabla^2 w = L_2 N_Q \nabla^2 \frac{\partial h_z}{\partial z} + L_2 N_R \nabla_1^2 \theta,
\] (3.31)

\[
\left[ L_1 + \frac{L_2}{P} \left( 1 + A \frac{\partial}{\partial t} \right) \right] \zeta = L_2 N_Q \frac{\partial \xi}{\partial z},
\] (3.32)

\[
\left[ N p_2 N^{-1} p_1 \frac{\partial}{\partial t} - \nabla^2 \right] \xi = \epsilon^{-1} \frac{\partial \zeta}{\partial z} + M_1 \frac{\partial}{\partial z} \left( \nabla^2 h_z \right),
\] (3.33)

\[
\left[ N p_2 N^{-1} p_1 \frac{\partial}{\partial t} - \nabla^2 \right] h_z = \epsilon^{-1} \frac{\partial w}{\partial z} - M_1 \frac{\partial \xi}{\partial z} \quad \text{and}
\] (3.34)

\[
L_2 \left[ (E + h \epsilon) \frac{\partial}{\partial t} - \nabla^2 \right] \theta = \left( \frac{G - 1}{G} \right) \left( \tau \frac{\partial}{\partial t} + \overline{H} \right) w
\] (3.35)

where

\[
L_1 = N p_1^{-1} \left( \tau \frac{\partial^2}{\partial t^2} + F \frac{\partial}{\partial t} \right), \quad F = f + 1, \quad L_2 = \tau \frac{\partial}{\partial t} + 1, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},
\]

\[
\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad \overline{H} = h + 1.
\]
3.3 The Dispersion Relation

Perturbed quantities are assumed to be of the following form and for the analysis of disturbances into normal modes by seeking solutions whose dependence on \( x, y \) and \( t \) is given by

\[
[w, \theta, h_z, \zeta, \xi] = [W(z), \Theta(z), K(z), Z(z), X(z)] \exp(ik_xx + ik_yy + nt) \tag{3.36}
\]

where \( k_x \) is the wave number along \( x \)-direction, and \( k_y \) is wave number along \( y \)-direction. \( k = \sqrt{k_x^2 + k_y^2} \) = resultant wave number and \( n \) = growth rate. Equations (3.31)-(3.35), with the help of expression (3.36), become

\[
\begin{align*}
L_1 + \frac{L_2}{P} (1 + An) & \left( D^2 - \alpha^2 \right) W = L_2 N_Q \left( D^2 - \alpha^2 \right) DK - L_2 N_R \alpha^2 \Theta, \tag{3.37} \\
L_1 + \frac{L_2}{P} (1 + An) & \left[ N_{p2} N_{p1}^{-1} n - \left( D^2 - \alpha^2 \right) \right] X = \epsilon^{-1} DZ + M_1 \left( D^2 - \alpha^2 \right) DK, \tag{3.38} \\
L_2 \left[ (E + h\epsilon) n - \left( D^2 - \alpha^2 \right) \right] & \Theta = \left( \frac{G - 1}{G} \right) \left( \tau n + \overline{H} \right) W \tag{3.39}
\end{align*}
\]

where \( D = \frac{d}{dz} \), \( L_1 = N_{p1}^{-1}(\tau n^2 + Fn) \) and \( L_2 = \tau n + 1 \).

By eliminating \( X, Z, K, \) and \( \Theta \) from the equations (3.37)-(3.41), we obtain

\[
\begin{align*}
L_1 + \frac{L_2}{P} (1 + An) & \left[ (D^2 - \alpha^2) - (E + h\epsilon)n \right] \left( D^2 - \alpha^2 \right) W \\
+ \frac{L_2 N_Q \left[ (D^2 - \alpha^2) - (E + h\epsilon)n \right]}{M_1} & \left( \frac{(D^2 - \alpha^2) - N_{p2} N_{p1}^{-1} n}{M_1} \right) \left( \frac{L_2 N_Q D^2}{M_1^{1/2} (1 + An)} \right) = \left( \frac{G - 1}{G} \right) N_R \alpha^2 (\tau n + \overline{H}) W. \tag{3.42}
\end{align*}
\]
Using the boundary conditions and equations (3.29) and (3.30), obviously that the even order derivatives of $W$ vanish on the boundaries and hence the proper solution of equation (3.42) characterizing the lowest mode is

$$W = W_0 \sin(\pi z), \quad \text{where} \quad W_0 = \text{Constant}. \quad (3.43)$$

On substituting the solution (3.43) in equation (3.42), we get the dispersion relation as

$$N_R = \left( \frac{G}{G-1} \right) \left( \frac{\pi^2 + \alpha^2}{\alpha^2 (\tau n + H)} \right) \left[ \left\{ L_1 + \frac{L_2}{P} (1 + An) \right\} + L_2 N_Q \pi \left[ \frac{(\pi^2 + \alpha^2) + N_p N - 1}{M_1 \epsilon} \right] + \frac{L_2 N_Q \pi}{\left\{ L_1 + \frac{L_2}{P} (1 + An) \right\} M_1 \epsilon^2} \right]. \quad (3.44)$$

### 3.4 Stationary Convection

When the instability sets in as stationary convection, the marginal state will be characterized by $n = 0$ and the dispersion relation equation (3.44) reduces to

$$N_R = \left( \frac{G}{G-1} \right) \left( \frac{\pi^2 + \alpha^2}{\alpha^2 H} \right) \left[ 1 + \frac{N_Q \epsilon^{-1} \pi^2 \left\{ (\pi^2 + \alpha^2) + N_Q P \epsilon^{-1} \pi^2 \right\}}{P \left( \pi^2 + \alpha^2 \right) \left\{ M_1^2 \pi^2 + (\pi^2 + \alpha^2) + N_Q P \epsilon^{-1} \pi^2 \right\}} \right]. \quad (3.45)$$

Thus for stationary convection, the viscoelastic parameter vanishes with $n$, and stress and strain rate showed linear realtion for Rivlin- Ericksen viscoelastic fluid. Also, for fixed values of $P$, $N_Q$, $M_1$ and $H$, let the non-dimensional number $G$ accounting for the compressibility effects be also kept as fixed, then we have

$$N_R^C = \left( \frac{G}{G-1} \right) N_R^C \quad (3.46)$$

where $N_R^C$ is critical Rayleigh number in the absence compressibility and $N_R^G$ is critical Rayleigh number in the presence of compressibility. Since the critical Rayleigh number $> 0$ and finite which implies $G > 1$, which means stabilizing effect due to compressibility.
Now we study, the effect of suspended particles which depends upon the nature of \( \frac{dN_R}{dH} \), the effect of medium permeability which depends upon the nature of \( \frac{dN_R}{dP} \), the effect of magnetic field which depends upon the nature of \( \frac{dN_R}{dN_Q} \), the effect of hall current which depends upon the nature of \( \frac{dN_R}{dM_1} \). From equation (3.45) we have

\[
\frac{dN_R}{dH} = - \left( \frac{G}{G-1} \right) \frac{(\pi^2 + \alpha^2)^2}{\alpha^2 H^2} \left[ \frac{1}{P} + \frac{N_Q \epsilon^{-1} \pi^2 \{ (\pi^2 + \alpha^2) + N_Q P \epsilon^{-1} \pi^2 \}}{(\pi^2 + \alpha^2) \{ M_1^2 \pi^2 + (\pi^2 + \alpha^2) + N_Q P \epsilon^{-1} \pi^2 \}} \right].
\]

(3.47)

which is \(< 0 \Rightarrow \) destabilizing effect of suspended particles on the thermal instability of the compressible fluid-particle layer in the presence of and hall currents through a porous medium. It is obvious from equation (3.45) that

\[
\frac{dN_R}{dP} = \left( \frac{G}{G-1} \right) \frac{(\pi^2 + \alpha^2)^2}{\alpha^2 H} \left[ - \frac{1}{P^2} + \frac{(N_Q \pi^2 \epsilon^{-1}) M_1^2 \pi^2}{(\pi^2 + \alpha^2) \{ M_1^2 \pi^2 + (\pi^2 + \alpha^2) + N_Q P \epsilon^{-1} \pi^2 \}} \right].
\]

(3.48)

which is \(> 0 \) if \( P \left[ M_1 \pi - \sqrt{\pi^2 + \alpha^2} \right] > \frac{\sqrt{\pi^2 + \alpha^2} \{ M_1^2 \pi^2 + (\pi^2 + \alpha^2) \}}{N_Q \epsilon^{-1} \pi^2} \).

which is \(< 0 \) if \( P \left[ M_1 \pi - \sqrt{\pi^2 + \alpha^2} \right] < \frac{\sqrt{\pi^2 + \alpha^2} \{ M_1^2 \pi^2 + (\pi^2 + \alpha^2) \}}{N_Q \epsilon^{-1} \pi^2} \).

Thus, for the different values of parameter, medium permeability has both destabilizing and stabilizing effect. Presence and absence of magnetic field plays an important role in stabilizing effect of permeability. Its absence destabilize the effect. Since for the case

\[
\frac{dN_R}{dP} = - \left( \frac{G}{G-1} \right) \frac{(\pi^2 + \alpha^2)^2}{\alpha^2 H P^2}.
\]

(3.49)

which is always \(< 0 \). Thus, in the presence of magnetic field, medium permeability succeeds in stabilizing the thermal instability of the compressible fluid-particle layer for certain wave numbers. Now from equation (3.45), we get

\[
\frac{dN_R}{dN_Q} = \left( \frac{G}{G-1} \right) \frac{(\pi^2 + \alpha^2) \pi^2 \epsilon^{-1}}{\alpha^2 H \{ M_1^2 \pi^2 + (\pi^2 + \alpha^2) + N_Q P \epsilon^{-1} \pi^2 \}} \left[ \{ (\pi^2 + \alpha^2) + N_Q P \epsilon^{-1} \pi^2 \} \right.
\]

\[+ \frac{M_1^2 \pi^4 N_Q P \epsilon^{-1}}{M_1^2 \pi^2 + (\pi^2 + \alpha^2) + N_Q P \epsilon^{-1} \pi^2} \].

(3.50)
which is always \( > 0 \) which implies that magnetic field has a stabilizing effect. To find the effect of hall currents, from equation (3.45), we have

\[
\frac{dN_R}{dM_1} = -2 \left( \frac{G}{G-1} \right) \frac{\alpha^2}{\alpha^2 H} \left[ N_Q \epsilon^{-1} M_1 \pi^4 \left\{ (\pi^2 + \alpha^2) + N_Q P \epsilon^{-1} \pi^2 \right\} \right] \quad \text{(3.51)}
\]

which is always \( < 0 \) which means that, in porous medium, hall current destabilize the thermal convection in the compressible fluid-particle layer. We analyze graphically all the four effects as

![Figure 3.1: Variation of \( N_R \) with \( \alpha \) for a fixed \( H = 1000, G = 9.8, \pi = 3.14, N_Q = 20, M_1 = 10, \epsilon = 0.5 \) and for different values of \( P(2, 4, 6) \).](image_url)

![Figure 3.2: Variation of \( N_R \) with \( \alpha \) for a fixed \( H = 1000, G = 9.8, \pi = 3.14, P = 4, M_1 = 10, \epsilon = 0.5 \) and for different values of \( N_Q = (10, 20, 30) \).](image_url)
Figure 3.3: Variation of $N_R$ with $\alpha$ for a different value of $H = (500, 1000, 1500)$ for fixed values of $G = 9.8, \pi = 3.14, P = 2, M_1 = 10, \epsilon = 0.5$.

Figure 3.4: Variation of $N_R$ with $\alpha$ for a fixed values $H = 1000, G = 9.8, \pi = 3.14, P = 2, N_Q = 20, \epsilon = 0.5$ for different values of $M_1 = (10, 20, 30)$.

We find from Figure 3.1 (refer table 4), when the value of the medium permeability($P$), increased then the value of $N_R$ is increased which shows the stabilizing effect. Similarly from Figure 3.2 (refer table 5), when the value of magnetic field $N_Q$ is increased, and the value of $N_R$ is increased which again shows the case of stabilizing effect. In Figure 3.3 (refer table 6), as the value of suspended particle $H$ increased, the value of $N_R$ decreased, which shows the destabilizing effect. Also Figure 3.4 (refer table 7) shows as the value of hall currents $M_1$ through the porous medium increased, the value of $N_R$ decreased, which is again the case of destabilizing effect on the system.
3.5 Oscillatory Modes

Multiplying equation (3.37) by the complex conjugate of \( W \) i.e. \( W^* \), integrating over the range of \( z \) from \( z = 0 \) to \( z = d \) and using equations (3.38)-(3.47) together with the boundary conditions (3.29) and (3.30)

\[
\left[ L_1 + \frac{L_2}{P} (1 + An) \right] I_1 + A_1 (nI_2 + n^* I_5) + L_2 N Q \epsilon (I_3 + I_6) + \frac{L_2}{L_2} \left[ L_1^* + \frac{L_2^*}{P} (1 + An^*) \right] I_4
\]

\[
= L_2 L_2^* N R \alpha^2 \left( \frac{G - 1}{G} \right) \left( \frac{1}{\tau n^* + \bar{H}} \right) [I_7 + (E + h\epsilon) n^* I_8].
\]

(3.52)

where \( A_1 = L_2 N Q N P_2 N P_1^{-1} \) and

\[
I_1 = \int_0^1 (|DW|^2 + a^2 |w|^2) \, dz, \quad I_2 = \int_0^1 |X|^2 \, dz,
\]

\[
I_3 = \int_0^1 (|DX|^2 + a^2 |X|^2) \, dz, \quad I_4 = \int_0^1 |Z|^2 \, dz
\]

\[
I_5 = \int_0^1 (|DK|^2 + a^2 |K|^2) \, dz, \quad I_6 = \int_0^1 (|D^2 K|^2 + 2a^2 |DK|^2 + a^4 |K|^2) \, dz,
\]

\[
I_7 = \int_0^1 (|D\Theta|^2 + a^2 |\Theta|^2) \, dz, \quad I_8 = \int_0^1 |\Theta|^2 \, dz.
\]

(3.53)

all \( I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8 \) all are positive definite, take \( n = i n_0 \) in equation (3.52), where \( n_0 \) is real, and equate imaginary parts on both sides, we get

\[
n_0 = 0 \quad \text{or} \quad n_0^2 = -\tau^{-2} \frac{A - B}{C - D + E}
\]

(3.54)

where

\[
A = \left( N_p^{-1} \bar{H} F - \frac{\tau}{P} - \frac{\tau}{P} A \right) I_1 + N Q \epsilon N P_2 N P_1^{-1} \bar{H} (I_2 - I_3) - N Q \epsilon \tau (I_3 + I_6),
\]

\[
B = \left( N_p^{-1} \bar{H} F + \frac{\tau}{P} + \frac{\tau}{P} A \right) I_4 - N R \alpha^2 \left( \frac{G}{G - 1} \right) \{\tau I_7 + (E + h\epsilon) I_8\},
\]

\[
C = \left( N_p^{-1} (\bar{H} + 1 - F) - \frac{\tau}{P} - \frac{\tau}{P} A \right) I_1 + N Q \epsilon N P_2 N P_1^{-1} \bar{H} (I_2 - I_5),
\]

\[
D = N Q \epsilon \tau (I_3 + I_6) - \left( N_p^{-1} (1 - \bar{H} - F) - \frac{\tau}{P} - \frac{\tau}{P} A \right) I_4 \quad \text{and}
\]

\[
E = N R \alpha^2 \left( \frac{G}{G - 1} \right) \{\tau I_7 + (E + h\epsilon) I_8\}. \quad \text{Whereas in the absence of magnetic field,}
\]

\[
n_0^2 = -\tau^{-2} \left[ \left( N_p^{-1} \bar{H} F - \frac{\tau}{P} - \frac{\tau}{P} A \right) I_1 + N R \alpha^2 \left( \frac{G}{G - 1} \right) \{\tau I_7 + (E + h\epsilon) I_8\} \right]
\]

\[
\left( N_p^{-1} (\bar{H} + 1 - F) - \frac{\tau}{P} - \frac{\tau}{P} A \right) I_1 + N R \alpha^2 \left( \frac{G}{G - 1} \right) \{\tau I_7 + (E + h\epsilon) I_8\}.
\]

(3.55)
3.6 Conclusion

Problem was formulated to discuss the combined effect of compressibility, hall current, magnetic field, medium permeability and suspended particles on thermal instability of a Rivlin-Ericksen fluid and the results obtained as:

(I) Constitutive relation of Rivlin-Ericksen fluid becomes linear i.e. the relation between stress and strain becomes linear for stationary convection due to the vanishing of the viscoelastic parameter.

(II) Magnetic field, suspended particles and medium permeability introduce oscillatory modes in the system otherwise effects the principle of exchange of stabilities is hold good.

(III) When magnetic field is not present, \( n_0^2 < 0 \) if

\[
C_{pt} > C_v \left[ 1 + \frac{\epsilon m}{f k_1 K d^2} \left\{ \nu d^2 + \nu' \right\} \right]
\]

For all \( N_R > 0 \), since \( n_0 \) is real and \( n_0^2 < 0 \) which implies \( n_0 = 0 \). This shows that \( n \) is real when \( N_R > 0 \) in the absence of the magnetic field. If equation (3.55) holds true and that the principle of exchange of stabilities is valid for this case, however, if equation (3.55) is violated, then the oscillatory modes may come into play even in the absence of the magnetic field, Singh and Gupta [37].

(IV) Equation (3.46) indicates compressibility effect is to postpone the onset of instability.

(V) To study the various effects of suspended particles, medium permeability, magnetic filed and Hall currents in a compressible Rivlin-Ericksen viscoelastic fluid, we examined the expressions \( \frac{dN_R}{dH} \), \( \frac{dN_R}{dP} \), \( \frac{dN_R}{dQ} \) and \( \frac{dN_R}{dM_1} \) analytically. The magnetic field postpones the onset of instability, suspended particles and Hall currents both hasten the onset of convection, which is in contrast with the result of Gupta et al. [54].