Chapter 2: Review of Interpolation Methods

This Chapter presents the literature review related to interpolation of images using: traditional interpolation methods, edge directed interpolation methods and wavelet transform methods. Traditional interpolation methods includes review of nearest neighbor, linear bicubic and spline functions based interpolation techniques. Edge directed interpolation methods include review of EDI, NEDI, INEDI and ICBI whereas wavelet transforms methods includes review of DWT-SR, DT-CWT, DWT-SWT-SR methods.

2.1 Introducing image interpolation

In digital image processing, a process which transforms a discrete image defined at one set of coordinate locations to a new set of coordinate points is termed as resampling (13-14). The process of resampling uses the interpolation functions to find the value(s) of pixel(s) at new coordinates as shown in Figure 2.1. Resampling process is used in many image processing operations such as rotation and resizing of images. This resampling process is accomplished in two steps. In the first step digital image is interpolated into a continuous image and then this interpolated continuous image is again sampled to convert it into a digital image. For example, convolution is done between a discrete function \( x(n) \) and an interpolating function \( h(t) \) to produce a continuous function \( y(t) \) as shown in Figure 2.1. In the sampling process, this continuous function is multiplied by a sampling function to produce a discrete function resampled at a new set of points. The performance of the resampling process significantly depends on the interpolation functions used (15-17) and their frequency domain characteristics in the pass band and the stop band. The magnitude of Fourier transformation is used to examine the performance in the passband and the logarithmic plot of the magnitude is used to examine the performance in the stopband.
There are many interpolation techniques developed to date which are divided into three main categories defined as:

- Traditional interpolation techniques
- Edge directed interpolation techniques
- Wavelet Based interpolation techniques

A discussion on all these techniques is presented in the following sections.

### 2.2 Traditional interpolation functions

#### 2.2.1 Ideal filter or sinc function

This interpolating function have constant value in the passband and zero value within the stop band as shown in Figure 2.2 (18-19). Mathematically, it is defined as:

\[ h(x) = sinc(x) \]  

(2.1)

It behaves like positive from 0 to 1, negative from 1 to 2, positive from 2 to 3 cycle, and so on which makes the interpolation impractical. So the sinc function is truncated to a
shorter length (20-21) to overcome this problem but this causes wiggling in the stopband as can be seen in Figure 2.2.

2.2.2 Nearest-neighbor
This is the simplest interpolation method which has a rectangular shape in space domain as shown in Figure 2.2 (22-23). Mathematically, it is defined as:

\[ h(x) = \begin{cases} 
1 & |x| < 0.5 \\
0 & \text{elsewhere} 
\end{cases} \]  

(2.2)

The advantage of this method is that it has very less processing time but the major disadvantage is the poor visual quality of interpolated image (22-25).

2.2.3 Linear
The mathematical representation of linear function can be expressed as (26-28):

\[ h(x) = \begin{cases} 
1 - |x| & |x| < 1 \\
0 & \text{elsewhere} 
\end{cases} \]  

(2.3)

This type of interpolation function has a triangle shape in space domain as shown in Figure 2.2. The comparison between the time domain, frequency domain and logarithmic domain response of Ideal, Nearest neighbor and linear interpolation functions is shown in Figure 2.2-2.4.

Figure 2.2: Time domain response of Nearest Neighbor, Ideal and Linear interpolation functions.
Figure 2.3: Frequency domain response of Nearest Neighbor, Ideal and Linear interpolation functions.

Figure 2.4: Logarithmic frequency domain response of Nearest Neighbor, Ideal and Linear interpolation functions.
2.2.4 B-splines
Splines can be referred as piecewise polynomials with pieces smoothly connected together (29-30). The most commonly used spline function is the B-splines which is expressed mathematically as:

\[ b^n(x) = \sum_{k=0}^{n+1} (-1)^k \frac{(n + 1) (n + 1 - k)}{(n + 1 - k)!} \left( \frac{n + 1}{2} + x - k \right)^n \quad (2.4) \]

\[ \forall x \in \mathbb{R}, \forall n \in \mathbb{N}_+ \]

And, \( (X)^n_+ = (\text{max} (0, x))^n \) for \( n > 0 \) \quad (2.5)

The B-spline functions for different values of \( n \) i.e. for \( n=0, n=1, n=2 \) & \( n=3 \) are mentioned in Table 2.1 and their respected time and frequency domain representation are shown in Figure 2.5, 2.6 & 2.7.

<table>
<thead>
<tr>
<th>( n=0 )</th>
<th>( n=1 )</th>
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<tbody>
<tr>
<td>( b^0(x) = \begin{cases} 1 &amp;</td>
<td>x</td>
</tr>
<tr>
<td>( n=2 )</td>
<td>( n=3 )</td>
</tr>
<tr>
<td>( b^2(x) = \begin{cases} -2</td>
<td>x</td>
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</table>

Figure 2.5: Time domain response of B-splines of degree 0 to 3
From Table 2.1 and Figure 2.5, 2.6 & 2.7, it is found that out of four different degrees of spline functions, the best performance is of the cubic B-spline i.e. n=3 (30-31). Since the performance and computation efficiency of cubic B-spline is best out of four different spline functions so it is mostly used in practice (32).
2.3 Two dimensional (2-D) image interpolation

For processing digital images using the interpolation functions, the one-dimensional interpolating functions is transformed into two-dimensional functions by taking the product of one-dimensional functions in two axes. A two dimensional continuous signal \( s(x, y) \) is reconstructed from its discrete sample \( s(k, l) \) with \( s, x, y \in R \) and \( k, l \in N^* \). Thus, the amplitude at any position \((x, y)\) needs to be estimated from its discrete neighbors (33-34). This is equivalent to the convolution of the discrete image samples with the continuous two dimensional impulse response \( h(x, y) \), as:

\[
s(x, y) = \sum_{k} \sum_{l} s(k, l) h(x - k, y - l)
\] (2.6)

To improve the processing speed usually, symmetrical and separable interpolation functions are used which are also known as kernels. Depending upon the type of interpolation functions and the way these are used, they are categorized into two groups: adaptive and non-adaptive (35-37). Adaptive interpolation algorithms uses a pixel-by-pixel approach for interpolation and generate images of good visual quality whereas non adaptive interpolation algorithms interpolates by predetermined prototype for all pixels equally. The accuracy of output image depends on the number of pixels used in interpolation. But the computation time increases with the use of more number of pixels.

2.4 Key interpolation functions used in 2-D image processing

For two dimensional image processing, there are several interpolation functions which includes ideal filter or sinc function, nearest-neighbor, bilinear interpolation and bicubic interpolation. These are discussed below:

2.4.1 Ideal filter or sinc function

Figure 2.8 and Figure 2.9 shows the time domain and frequency domain representation of two-dimensional sinc function having constant one value in the pass band and zero value within the stop band in frequency domain (38-39).

† See Equation 2.4 & 2.5 for details.
2.4.2 Nearest-neighbor
This interpolation method has a rectangular shape in time domain, as shown in Figure 2.10 (40). This method has very less processing time but the major disadvantage is the poor visual quality of interpolated image as can be seen from its frequency domain response in Figure 2.11. It is because the representation, shown in Figure 2.11, involves the Fourier transformation of a square pulse, which is equivalent to a sinc function whose gain quickly falls off in the passband.

2.4.3 Bilinear interpolation
For interpolating the one dimensional function linear interpolation method is employed. But for two dimensional functions bilinear method is used. This bilinear method is the
extended version of linear interpolation method. In this technique the values of four nearest neighbor pixels are used for finding the value of unknown pixel (41-42). The average of the values of four nearest pixels is considered as the value of unknown pixel. In this way the generated images have visual quality better than the images generated by nearest neighbor method (41-44). The time and frequency domain representation of Bilinear interpolation is shown in Figure 2.12 & 2.13.

To better understand, consider the Figure 2.14, in which an intermediate pixel \( \hat{F}(p', q') \) is created by interpolating nearest four pixels \( F(p, q), F(p, q + 1), F(p + 1, q) \) and \( F(p + 1, q + 1) \). Initially two pixels in top row and bottom row are interpolated with linear interpolation; i.e., in horizontal direction. These interpolated pixels of top row and bottom row are then linearly interpolated in vertical direction (43-45).
Figure 2.12: Bilinear interpolation function in time domain.

Figure 2.13: Bilinear interpolation function in frequency domain.

Figure 2.14: Data distributions for Bilinear interpolation (43).
In particularly, an interpolation factor $a$, is used in X direction to interpolate $F(p, q)$ and $F(p, q + 1)$ as follows:

\[
F(p'') = (1.0 - a) \times F(p, q) + a \times F(p + 1, q)
\]

\[
F(p + 1'') = (1.0 - a) \times F(p, q + 1) + a \times F(p + 1, q + 1)
\]

An interpolation factor ‘$b$’, is used in Y direction to interpolate $F(p'')$ and $F(p + 1'')$ as follows:

\[
\hat{F}(p', q') = (1.0 - b) \times F(p'') + b \times F(p + 1'')
\]

2.4.4 Bicubic interpolation

The another technique used for two dimensional interpolation is Bicubic. The difference between the bilinear and bicubic technique is that bicubic technique uses the values of sixteen nearest neighbor pixels for finding the value of unknown pixel instead of four pixels used by bilinear method (46-47). Since this technique uses nearest sixteen pixels to generate an intermediary pixel $\hat{F}(p', q')$ as compared to four pixels used by bilinear method, the quality of output image generated is very good as compared to bilinear and nearest neighbor.

Figure 2.15: Data distributions for Bicubic interpolation (16)
In Figure 2.15, nearest 4 X 4 pixels are interpolated to create an intermediate pixel $\hat{F}(p', q')$ from $F(p - 1, q - 1)$ to $F(p + 2, q + 2)$, as:

$$
\hat{F}(p', q') = \sum_{m=-1}^{2} \sum_{n=-1}^{2} F(p+m, q+n)R_{c}(m-a)R_{c}(-(n-b))
$$

(2.9)

Where, $F(p + m, q + n)$ indicates pixel data at location $(p + m, q + n)$ and $R_{c}(\cdot)$ denotes a Bicubic interpolation function such as a B-Spline Bicubic, Triangular Bicubic or Bell Bicubic interpolation function.

The different Bicubic interpolation functions are briefly discussed below:

1. **Triangular Bicubic interpolation function:** It is a simple Bicubic function, as defined in the following equation:

$$
R_{1}(x) = \begin{cases} 
    x+1 & \text{for } -1 \leq x \leq 0 \\
    1-x & \text{for } 0 < x \leq 1 
\end{cases}
$$

(2.10)

The triangular function is shown in Figure 2.16 (46-47). It returns lower value for a high input and high value for a low input which causes blurriness.

2. **Bell Bicubic interpolation function:** The Bell Bicubic interpolation function is also shown in Figure 2.16 and is given by:

$$
R_{2}(x) = \begin{cases} 
    \frac{1}{2} \left( x + \frac{3}{2} \right)^2 & \text{for } -\frac{3}{2} \leq x \leq -\frac{1}{2} \\
    \frac{3}{4} (x)^2 & \text{for } -\frac{1}{2} < x \leq \frac{1}{2} \\
    \frac{1}{2} \left( x - \frac{3}{2} \right)^2 & \text{for } \frac{1}{2} < x \leq \frac{3}{2} 
\end{cases}
$$

(2.11)

The above equation indicates that when $x$ is high, Bell function is low and vice-versa (46-47). This will reduce the blurred effect and will produce smooth edges.
3. B-Spline Bicubic interpolation function: The mathematical expression for B-Spline bicubic interpolation function is:

\[
R_3(x) = \begin{cases} 
\frac{2}{3} + \frac{1}{2} |x|^3 - (x)^2 & \text{for } 0 \leq |x| \leq 1 \\
\frac{1}{6} (2-|x|)^3 & \text{for } 1 < |x| \leq 2
\end{cases}
\]  

(2.12)

This shows that value of Bicubic B-spline function is very low for high value of x i.e. the very low weight is assigned to the far intermediate pixels. Therefore, the final output image generated by Bicubic B-splines will be smoother than the image generated by Bell Bi-Cubic interpolated image (46-47).

Figure 2.16: Bi-Cubic interpolation functions.
The time and frequency domain response for two dimensional B-spline bicubic interpolation functions for different values of $n=0,1,2,3$ are shown in Figures 2.17-2.24. From the above discussion and the interpolation functions obtained in Figure 2.16, the degree of blurriness will be different for each function. Therefore, this thesis is focused on developing an algorithm to produce sharp and clear edges. The results obtained for these functions for different images are discussed in Chapter 3. The following section presents a review of the key related work done on traditional interpolation functions.

Figure 2.17: 2 dimensional Bi-cubic spline functions in time domain for $n=0$

Figure 2.18: 2 dimensional Bi-cubic spline functions in frequency domain for $n=0$
Figure 2.19: 2 dimensional Bi-cubic spline functions in time domain for $n=1$

Figure 2.20: 2 dimensional Bi-cubic spline functions in frequency domain for $n=1$
Figure 2.21: 2 dimensional Bi-cubic spline functions in time domain for $n=2$

Figure 2.22: 2 dimensional Bi-cubic spline functions in frequency domain for $n=2$
Figure 2.23: 2 dimensional Bi-cubic spline functions in time domain for $n=3$

Figure 2.24: 2 dimensional Bi-cubic spline functions in frequency domain for $n=3$
2.5 Key literature review of traditional interpolation methods

The above mentioned interpolation functions are also called the polynomial based interpolation, as they use a polynomial function to find the new pixel values to be used in the process of image processing operations such as resizing, rotation and scaling. The polynomial function can be both linear and/or non-linear and the process of using these functions in image processing is commonly termed polynomial based interpolation (48-49). Typical examples of most commonly used polynomial interpolation techniques/functions are: nearest-neighbor, bilinear, bicubic and Lagrange interpolation (50-53). These are in use for last several decades and several researchers, engineers, professionals and organizations/industry\(^5\) have used the same in various image processing operations and researches. It is not possible to present a review of all the literature; however, author is well-aware of related work done in various areas in wide perspective. This chapter has presented a review of some of the key work done in relation to the traditional polynomial based interpolation functions. Many researchers and engineers have proposed and implemented algorithms for improving the performance of these techniques, over the last two decades. This section presents a review of such related research done over the last decade.

During re-sampling between orthogonal and hexagonal lattices there is a loss of information. To minimize this loss Dimitri, V. et al. in 2002 (54) presented an approach which makes the use of an extension of 2D splines to hexagonal lattices in order to derive the reconstruction function. Authors implemented the mentioned re-sampling method for a gravure printing application and compared the results with the classical techniques.

Fractal-based image enlargement technique was presented by Chung K.H. et al. (55) in 2003 for improving the quality of reconstructed images. In the proposed technique, self-similarity property was used for encoding of an image. In the decoding process, an interpolated image of any desirable size can be reconstructed from an original image by

\(^5\) See references [10-69] for polynomial based interpolation used in various areas of image processing.
using precise contractive mappings. Generally during interpolation quality of the image is ruined. So, to enhance the image quality authors presented this technique. The projected technique maintains the smoothness in flat regions and preserves the details of edge regions. Authors reported that the proposed technique is superior to that of the traditional interpolation techniques such as cubic convolution and bilinear interpolation. Authors showed experimentally that the images enlarged by the proposed fractal based enlargement technique have higher PSNR values as compared to the images enlarged by the cubic convolution interpolation.

The modified adaptive warped distance method was recommended by Hadhoud et al. in (56) 2003 for image interpolation. Different adaptive methods are used in this technique for assigning the weights to the pixels for interpolation process. This adaptation method can be used for warped distance technique and also for various conventional methods such as bicubic, bilinear and cubic spline. Authors compared the results of proposed adaptive weighing of pixels method and traditional interpolation methods and claimed the better performance of proposed method over the traditional interpolation method.

For the color images, an interpolation technique was introduced by Malvar et al. in (57) 2004. The proposed technique was based on the simple linear filter and authors claimed the better performance of proposed method over the bilinear interpolation method.

Zhen et al. (58) in 2005 mentioned a down-sampling technique for reducing the image size. Authors applied their proposed method on the ultrasound breast phantom data and compared the performance of various interpolation techniques such as Lanczos, bicubic, bilinear and wavelet based. They used Polyline Distance Measure (PDM) and Hausdorff Distance Measure (HDM) parameters for performance measurement. Authors showed that Lanczos algorithm performs better than all other techniques.

For image resolution enhancement modified Laplacian Filter (MLF) and Intensity Correction (IC) techniques was proposed by Shen D.F. et al. (59) in 2006 based on the relationship between higher resolution images and lower-resolution images. For regenerating the frequency components that are lost during the down-sampling and
averaging a simple 3 X 3 modified Laplacian filter is designed. For increasing the resolution of the image the intensity correction technique iteratively refines the image quality. Authors claimed that both the intensity correction and modified Laplacian filter are simple in computations i.e. the processing time is less of the proposed method. They also claimed that the PSNR value and visual quality of image generated by proposed method is higher than as compared to traditional interpolation methods such as bicubic and bilinear.

An alternative to square pixel structure is a hexagonal structure which can also be used for image processing. To capture and display images, most of the existing hardware is based on square structure. So, for converting the square structure images into hexagonal structure smoothly proper software is required. Therefore, to obtain hexagonal structure image from square structure image, Xiangjian H. et al. (60) in 2007 projected a bilinear interpolation based algorithm. The proposed algorithm detects the edges of the image accurately and authors claimed the better performance of their proposed method over other edge detecting algorithms.

A spatial interpolation algorithm was presented by Gharavi et al. (61) in 2008 for Intra-Frame error concealment. The portion of the image which is affected by packet loss is interpolated by this method which connects the edges systematically. The missing areas are then divided into different regions by the connected edges for interpolation. Authors applied their proposed method on the Zelda and Lena images and showed by the simulation results that the missing areas are recovered with a greater accuracy. Authors also claimed the better PSNR values are obtained by the proposed technique as compared to the bilinear interpolation technique.

Image interpolation found applications in biomedical applications, image processing and in computer vision. During encoding process some of the pixels are discarded which needs to be regenerated during decoding process. These lost pixels are regenerated from the information available from the nearby pixels. Prasantha et al. (62) in 2009 compared the performance of different interpolation techniques such as bicubic,
bilinear and nearest neighbor. Authors used the MSE, PSNR and universal image quality index parameters for the performance measurement.

The resolution of ultrasound image is low which can be increased by interpolation. The quality of regenerated image depends on the precision of the algorithm. Most of the interpolation algorithms enhance both detailed region as well as the smooth region. This process degrades the quality of the image. Therefore, a new technique was presented by Bera D. et al. (63) in 2009. In this method the smooth portion of the image are preserved and detailed regions are interpolated. Thus the computational complexity of proposed method is reduced and hence can be used for real time systems like ultrasound.

Sheu et al. (64) in 2012 presented an omni-image interpolation technique. The catadioptric camera is used to capture the omni-images but the visual quality of these captured images by the camera is not good. So these images are interpolated to increase their resolution. Therefore to maintain the parameters of camera and to enhance the visual quality of images authors presented an algorithm. The equi-polar geometry constraint is utilized to preserve the camera properties. Authors applied their proposed algorithm on four different omni images and showed experimentally that the results of their projected method are better than the traditional interpolation methods.

Hung et al. (65) in 2012 proposed a technique for estimating the high resolution pixels using the bilateral filter which has less computation time along with the stable output as mentioned by the authors. A novel maximum posterior estimation is used for estimating the range of bilateral filter for both the horizontal-vertical and diagonal correlations. Experimentally authors proved that the images interpolated by the presented algorithm have better PSNR as compared to other methods.

To provide separate views for the assistant and surgeon Yuebing et al. (66) in 2012 used the term role-specific video imaging. The two different surgeons used this system and performed four operations successfully. Thus authors confirmed that zooming images
by a factor equal to or more than that used by bilinear interpolation can be achieved by high-quality role-specific imaging.

Image and video display, transmission and analysis are the basic operations that depend on image resizing. The traditional interpolation methods do not consider about the content of the image and so the visual quality of interpolated image is less than the original image. The seam carving based image resizing method is based on the content of image as it preserves the energy of the image during the resizing process and has better performance as compared to the traditional interpolation methods but still this technique has some drawbacks. So to overcome these drawbacks Zhi Wei et al. (67) in 2013 proposed an improved seam carving image resizing method. This technique utilizes the energy component for better describing the image and optimization methods are incorporated for reducing the computational complexity. Authors claimed the effectiveness of their proposed method by experimental results.

A real-time super-resolution method was presented by Wonseok et al. (68) in 2013 using image restoration and directionally adaptive image interpolation. In this technique the orientation of edges are estimated using steerable filters. Then along these estimated edge orientation, edges are refined. The degradation occurs in the images due to interpolation are effectively removed by the proposed method as claimed by the authors. The high resolution images generated by the proposed method are of better quality as compared to generated by super resolution methods as well as by advanced interpolation techniques. By the simulation results authors showed that the proposed method has high structural similarity (SSIM) values and PSNR values as compared to the conventional interpolation methods.

The images interpolated by traditional methods are suffered by blurring and fuzzy edges drawbacks. Therefore, Yihan et al. (69) in 2013 proposed an image zooming algorithm with hierarchical structure. This technique utilizes the retinex model to divide image into high and low frequency layer. To reconstruct the low frequency layer, low pass filters are used while high frequency layer is reconstructed by using the heat equation. These high frequency layers and low frequency layers are then merged to obtain the
zoomed images. Authors from their simulation results claimed that the proposed algorithm reduces the defects of nonlinear and linear interpolation algorithms.

In digital image processing the bilinear interpolation algorithm is widely used but the drawback of this technique is that it has low computation speed i.e. computation time is high. So, to improve its performance Yang Sa et al. (70) in 2014 presented a graphic processing unit acceleration-based bilinear interpolation parallel algorithm. This algorithm used the Wallis transforming method which is adaptable for GPU parallel processing structure. Authors showed experimentally that the processing speed of proposed algorithm is more than bilinear interpolation technique.

A directionally adaptive cubic-spline interpolation method was presented by Qiqin et al. (71) in 2014 for zooming the digital images captured by mobile cameras. Usually images exhibit blurring and jagging artifacts in the digitally zoomed images by traditional interpolation methods such as cubic-spline and linear. So, to overcome this problem authors presented a directionally adaptive cubic-spline interpolation method which interpolate the images according to the edge orientations. Authors showed experimentally that the images enlarged by the proposed method have low artifacts as compared to advanced interpolation and traditional interpolation methods.

An adaptive image amplification technique was presented by Xue-xia et al. (72) in 2014. In this technique the image is magnified by arbitrary integer multiples. Every low resolution pixel corresponds to a \( n \times n \) block of unknown high resolution pixels. Bicubic interpolation technique is used for zooming the low resolution image upto a given magnification factor \( n \). To calculate the orientation of low resolution edge pixels a weighted least-square estimation technique is adopted. Consequently, value of each pixel in the associated \( n \times n \) block is computed by using the obtained model parameters and a linear weighted summation using its eight neighborhood high resolution pixels. Author claimed that the proposed image zooming algorithm performs better than the conventional image zooming methods.
To enlarge the given image, interpolation method is employed to obtain the new unknown data points. Usually this interpolation method is used to reconstruct the image from its down sampled version. There are many interpolation algorithms for zooming the images but they suffer from many drawbacks such as jaggedness, blurriness and also the computation time. Therefore, for real time applications, Jha et al. (73) in 2014 presented an algorithm having low computation time. In this technique, depending upon the spatial position of unknown pixels they are classified into three groups. For different groups different prediction methods are used for reducing the computational time. The visual quality of the images reconstructed by the proposed technique is better than the traditional interpolation methods. Also, the computational complexity of proposed method is less than the traditional methods.

2.6 Introducing edge directed image interpolation

High resolution images with fine details are always required in many visual tasks. For this purpose, the interpolation functions are used in post-processing of images. The quality of generated high resolution image depends upon the selected interpolation function. The interpolated image quality is considered better if the edges are sharp and free from artifacts. Another important factor is the time required for interpolation.

2.7 Edge directed image interpolation

The rendering and correction are the two steps for interpolating low resolution images into higher resolution using edge direction (74-75). In this method a rectangular center-on-surround-off (COSO) filter is used for generating a HR edge map from the LR image and then the filter output is linearly interpolated. To prevent the interpolation across edges a bilinear interpolation technique is used. A sensor model is used for predicting the output and the mesh values are modified during the correction phase. This process is repeated many times to generate better quality of high resolution image and are discussed as:

The principle of basic edge-directed interpolation algorithm is shown in Figure 2.25 (77-77). In Figure 2.25, to obtain a HR edge map from the original image a sub-pixel
edge estimator is used. The final HR version is generated from low resolution image by interpolating according to the HR edge map.

Figure 2.26 shows the Edge-directed interpolation algorithm structure used in Figure 2.25. The rendering and correction are the two steps for interpolating low resolution images into higher resolution using edge direction as shown in Figure 2.26. Traditional bilinear interpolation technique is used for the rendering phase. To better understand the correction part, consider the Figure 2.26, in which the points on high and low resolution lattices are denoted by \( n \) and \( m \) respectively. \( x[m] \) and \( \hat{x}[m] \) are used to represent the true sensor data and the corrected sensor data by \( \hat{x}[m] \), the sensor model by the operator \( S \) and the edge-directed rendering step by the operator \( \mathcal{R} \), the estimated sensor data by \( \hat{x}[m] \) and the interpolated image by \( y[n] \). The term \( k \) represents the iteration index. So the following equations are described for the procedure depicted in Figure 2.26 with the mentioned notations:
The gain of the correction process is controlled by a constant $\lambda$ and the iteration starts with the initial condition $\bar{x}_0[m] = x[m]$. The equations 2.13 to 2.15 can also be represented as classical successive approximation procedure. The value of $\lambda$ depends on computing the probability of edges in a region.

2.8 New Edge Directed Interpolation (NEDI)

As discussed above, in natural images edges are very important features. So when images are interpolated there appears some jaggies along the edges due to which quality of image degraded. In the new edge directed interpolation the covariance based adaptation (CBA) method is used to adjust the predictor which finds the edge pixel values for better reconstruction of image. This is achieved by finding the geometric duality between the HR covariance and LR covariance which is used to connect the pair of pixels along the same orientation. In NEDI, the estimated HR covariance is used to derive the optimal MMSE interpolation by modelling the image as a locally stationary Gaussian process (76-77). The drawback of this method is that the processing time is about two times higher than that of processing time taken in linear interpolation. So the CBA interpolation is used only for the edge pixels. The bilinear interpolation technique is used for the non edge pixels in the smooth regions due to its simplicity. This hybrid approach is used in NEDI because the edge pixels are very less in comparison to the non edge pixels and the computation time of this hybrid approach in reconstructing the image will be less than the covariance-based adaptive interpolation for the whole image.

The analytical model of NEDI is presented below.

In NEDI, it is assumed that the LR image $X_{l,j}$ of size $H \times W$ directly comes from the size of $2H \times 2W$ image, i.e., $Y_{2l,2j} = X_{l,j}$. And, the interlacing lattice $Y_{2l+1,2j+1}$ is
interpolated from the lattice $Y_{2l,2j} = X_{l,j}$, using the fourth-order linear interpolation (Figure 2.27) as:

$$
\hat{Y}_{2l+1,2j+1} = \sum_{k=0}^{1} \sum_{l=0}^{1} \alpha_{2k+l} Y_{2(i+k),2(j+l)}
$$

(2.16)

Where, the interpolation is done along the diagonal directions for the four nearest neighbors and the optimal MMSE linear interpolation coefficient is given by:

$$
\tilde{\alpha} = R^{-1}\hat{r}
$$

(2.17)

Where, $R = [R_{kl}], (0 \leq k, l \leq 3)$ and $\hat{r} = [r_k], (0 \leq k \leq 3)$ are the local covariances at the high resolution (also called “high-resolution co-variances”). For example, $r_0$ is defined by $E[Y_{2l,2j}Y_{2l+1,2j+1}]$, as shown in Figure 2.27; where expectation operator ‘E’ denotes the average of the observation data.

Figure 2.27: Geometric duality when interpolating $Y_{2l+1,2j+1}$ from $Y_{2l,2j}$ (76-77).
In NEDI, based on the intrinsic geometric duality, the HR covariance is estimated from its LR counterpart. The geometric duality between the LR covariance $\hat{R}_{kl}, \hat{r}_k$ and the HR covariance $R_{kl}, r_k$ is shown in Figure 2.27 when the interlacing lattice $Y_{2i+1, 2j+1}$ is interpolated from $Y_{2i, 2j}$. Geometric duality is very useful for estimating the local covariance for two dimensional signals such as images without estimating the edge orientation. Similar geometric duality can also be observed in Figure 2.28.

The covariance method is used for estimating the LR covariance $\hat{R}_{kl}, \hat{r}_k$ from a local window of the LR image as:

$$\hat{R} = \frac{1}{M^2} C^T C, \hat{r} = \frac{1}{M^2} C^T \tilde{y}$$

(2.18)
Where, \( \vec{y} = [y_1 \cdots y_k \cdots y_{M^2}]^T \) is the data vector and \( C \) is a \( 4 \times M^2 \) data matrix. Combining equations 2.17 and 2.18 results to:

\[
\hat{\alpha} = (C^T C)^{-1}(C^T \vec{y})
\] (2.19)

Therefore, the interpolated value of \( Y_{2i+1,2j+1} \) can be obtained by substituting equation 2.19 into 2.16. The computational complexity is the major drawback of covariance-based adaptive interpolation. For computing the value of one pixel it requires about 1300 multiplications when the size of the local window is chosen to be \( M=8 \) as per equation 2.19. This shows that computational time taken by CBA interpolation is about two times higher than the computational time taken in linear interpolation. So the CBA interpolation is used only for the edge pixels due to its computational complexity and bilinear interpolation technique is used for the non edge pixels in the smooth regions due to its simplicity. This hybrid approach is used in NEDI because the edge pixels are very less in comparison to the non edge pixels and the computation time of this hybrid approach in reconstructing the image will be less than the covariance-based adaptive interpolation for the whole image.

2.9 Improved New Edge Directed Interpolation (iNEDI)

NEDI technique, as mentioned above, has several problems. So, it is modified to increase the interpolation accuracy, and the new technique is called improved New Edge Directed Interpolation (iNEDI) (78). The key problems, associated with NEDI are:

1. In the NEDI method, the four coefficients used to compute the interpolated value are computed assuming that in a square shaped window around the point, each pixel of the low resolution image is related through the same coefficients to its 4 closest neighbours. It is shown in Figure 2.29. This use of square shaped windows is not optimal (Figure 2.29) because it can introduce directional artifacts and makes the algorithm non-isotropic. These effects can be reduced by simply computing the parameters on approximately circular windows.
2. Secondly, when the four pixels are used to calculate the interpolated ones having a similar gray level, there is no need to compute the NEDI coefficients. If the covariance is stationary, a small error causes a bad conditioning of the solution. However, the use of linear interpolation instead changes slightly the results. In NEDI, a partial solution to this problem is provided using the bilinear interpolation; if local gray level variation is above a fixed threshold. Similar solution is adopted in iNEDI; however, the bicubic approximation is applied in low frequency regions. This choice is reported to not giving any improvements in image quality when threshold is low. However, it gives the possibility of obtaining a good tradeoff between edge direction preservation, accuracy and speed using higher values of the threshold, i.e., using iNEDI only for strong edges.

3. The third problem in the NEDI formulation is to ensure that the window used for the estimation belongs almost completely to the same edge. For each point $\tilde{x}$ in the enlarged grid to be fitted; the ideal window where points should be inserted in $C$ and $\tilde{y}$ should be a connected region including the 4 valued pixels closed to $\tilde{x}$ where local curvatures are smoothly changing. This is a condition that is stronger than the constant covariance constraint, and does not guarantee the absence of high component frequencies in the local fit.
To address a solution to the above problems, a region growing method is defined in the iNEDI method. In this method, the 4 valued neighboring pixels of the central point are taken and the neighbours (in the original grid) of these pixels are added iteratively with the following properties:

- The gray level between the maximum and minimum value of the 4 neighbours is not less than threshold (as in the central point).
- The gray level of each pixel is not larger than the maximum value of the gray level of the 4 neighbors of the central incremented by a threshold margin and not lower than the minimum of the 4 neighbours of the central point decremented by the same margin.
- The Euclidean distance between the pixel and central point is less than $r$.

The “edge” region is enlarged with the same rules by increasing $r$ up to a maximum value $R$. With this selective procedure and the control on the residual, the probability of obtaining a good interpolation is increased. But, there is still the possibility of having unwanted high frequencies (that are not excluded by the constant covariance condition and may occur in case of a small number of samples in the fit). For this reason, a further constraint is placed by replacing any interpolated value outside the intensity range of the four neighbours with closest of the values delimiting that range (i.e. maximum or minimum) by subtracting the average of the four neighbours intensities from the values inserted in $C$ and $\vec{y}$, i.e. replacing $C$ with:

$$C' = \begin{pmatrix}
  I_{h_1-1,k_1-1} - \bar{I}_{h_1,k_1} & I_{h_1-1,k_1-1} - \bar{I}_{h_1,k_1} & \cdots & \cdots \\
  I_{h_2-1,k_2-1} - \bar{I}_{h_2,k_2} & I_{h_2-1,k_2-1} - \bar{I}_{h_2,k_2} & \cdots & \cdots \\
  \cdots & \cdots & \cdots & \cdots \\
  I_{h_N-1,k_N-1} - \bar{I}_{h_N,k_N} & I_{h_N-1,k_N-1} - \bar{I}_{h_N,k_N} & \cdots & \cdots 
\end{pmatrix}$$  \hspace{1cm} (2.20)

Where, $h, k \in w(i, j)$ and changing $\vec{y}$ with

$$\vec{y}' = (I_{h_1,k_1} - \bar{I}_{h_1,k_1}, I_{h_2,k_2} - \bar{I}_{h_2,k_2}, \ldots, I_{h_N,k_N} - \bar{I}_{h_N,k_N})^T$$  \hspace{1cm} (2.21)

Where,
\[
\bar{I}_{h_1,k_1} = \frac{I_{h_1-1,k_1-1} + I_{h_1-1,k_1+1} + I_{h_1+1,k_1-1} + I_{h_1+1,k_1+1}}{4}
\]

(2.22)

\(I(i,j)\) is then clearly obtained as:

\[
I(i,j) = \alpha' \cdot (I_{i-1,j-1}, I_{i-1,j+1}, I_{i+1,j-1}, I_{i+1,j+1}) + \bar{I}_{i,j}
\]

(2.23)

This change clearly makes the matrix \(C'\) rank deficient. The fact that \(C\) is rank deficient means that the solution to the least squares problem is not unique, i.e. there are many vectors \(\bar{\alpha}\) that minimize \(|\|C \bar{\alpha} - \bar{y}\|\|^2\). A method that is often used to find a unique solution is to select the minimum norm solution that is obtained through the computation of the Moore-Penrose pseudo inverse. If it is assumed that the local four pixel configuration is the sum of a term exactly modelled by the constant covariance model plus an error term (i.e., for an odd point in the first step:

\[
\bar{I}_4 = (I_{2i+1,2j+1}, I_{2i+1,2j+1}, I_{2i+1,2j+1}, I_{2i+1,2j+1}) = \bar{I}_0 + \bar{I}_{\text{err}}
\]

, the squared error on the interpolated value \(I_{2i+1,2j+1} = \bar{\alpha}' \cdot \bar{I}_4\) is \((\bar{\alpha}' \cdot \bar{I}_{\text{err}})^2\) and it is in general lowered by choosing the minimum norm solution for \(\alpha'\). Therefore, the overconstrained system is solved using this method (79).

### 2.10 Iterative Curvature Based Interpolation (ICBI)

ICBI when applied on any image it doubles the size of image size every time like above discussed edge-directed methods (75). In this technique the low resolution pixels are put in a window and then holes are filled. This hole filling is a two step process. Firstly, to fill the hole, the closest points are linearly interpolated.

Consider Figure 2.30, to better understand the ICBI algorithm. As discussed above, darker pixel value is estimated from the weighted average of the four diagonal neighbor pixels. Similarly, the value of remaining holes (black pixels in Figure 2.30) is estimated from the weighted average of the four nearest neighbors in horizontal and vertical directions.
The interpolated value for the first step is usually computed as:

\[ I_{2l+1,2j+1} = \tilde{a} \cdot (I_{2l,2j}, I_{2l,2j+2}, I_{2l+2,2j}, I_{2l+2,2j+2}) \]  \hspace{1cm} (2.24)

The coefficients vector \( \tilde{a} = (\alpha_0, \alpha_1, \alpha_2, \alpha_3) \) is estimated from the neighbouring pixels in the grid. The way it is calculated depends upon the algorithm. At different scales in a large window, the local image covariance is assumed constant for computing the weights. The interpolation coefficients for an over-constrained system can be obtained by this assumption, as mentioned in previous sections. The step edge profile model is not ideal for NEDI method. The technique iNEDI attempts to provide a solution to this computational problem. In iNEDI the pseudo-inverse method is used to find the problem encountered in NEDI. However in ICBI, coefficients in opposite neighbours are assumed equal; which will give:

\[ I_{2l+1,2j+1} = \tilde{\beta} \cdot (I_{2l,2j} + I_{2l+2,2j+2}, I_{2l,2j+2} + I_{2l+2,2j}) \]  \hspace{1cm} (2.25)

At the coarser scale this relationship is assumed to be true with the same coefficients in a neighbourhood of the point (78). When the assumption is valid the interpolated pixel values are refined iteratively, using this approach. Similar to the NEDI algorithm, an over-constrained system is obtained in ICBI and is solved to find \( \beta_1 \) and \( \beta_2 \). The inverted matrix is full-ranked in this case. Thus, the above equation can be written as:
\begin{equation}
\beta_1(I_{2i,2j} - 2I_{2i+1,2j+1} + I_{2i+2,2j+2}) + \beta_2(I_{2i,2j+2} - 2I_{2i+1,2j+1} + I_{2i+2,2j}) = (1 - 2(\beta_1 + \beta_2))I_{2i+1,2j+1}
\end{equation}

To optimize the global energy an iterative greedy procedure is applied to modify the interpolated pixel values. Similarly after the second interpolation step, the same procedure is repeated to minimize the global energy. Thus in ICBI, two hole filling steps are used for interpolating the pixels. In the first step, to fill the hole, the closest points are linearly interpolated along the two diagonal directions as mentioned below:

\begin{equation}
I_{11}(2i + 1, 2j + 1) = I(2i - 2, 2j + 2) + I(2i, 2j) + I(2i + 2, 2j - 2) - 3I(2i, 2j + 2) - 3I(2i + 2, 2j) + I(2i, 2j + 4) + I(2i + 2, 2j + 2) + I(2i + 4, 2j)
\end{equation}

\begin{equation}
\tilde{I}_{22}(2i + 1, 2j + 1) = I(2i, 2j - 2) + I(2i + 2, 2j) + I(2i + 4, 2j + 2) - 3I(2i, 2j) - 3I(2i + 2, 2j + 2) + I(2i - 2, 2j) + I(2i, 2j + 2) + I(2i + 2, 2j + 4)
\end{equation}

Assigning to the point \((2i + 1, 2j + 1)\) the average of the two neighbors in the direction where the derivative is lower, as:

\begin{equation}
\begin{cases}
\frac{I(2i, 2j) + I(2i + 2, 2j + 2)}{2} & \text{if } \tilde{I}_{11}(2i + 1, 2j + 1) < \tilde{I}_{22}(2i + 1, 2j + 1) \\
\frac{I(2i + 2, 2j) + I(2i, 2j + 2)}{2} & \text{otherwise}
\end{cases}
\end{equation}

To preserve strong discontinuities an iterative refinement is then performed to minimize the energy function and update the values of the newly inserted pixels. This energy function is the sum of contribution of each interpolated pixel (78). A simple greedy minimization technique is finally implemented for adjusting the pixel values. Since in this technique the number of iterations is very high so the computational cost is high.

Analytically, the energy for each interpolated pixel is defined as:
\[ U_c(2i + 1,2j + 1) \]
\[ = w_1 \left( \left\lvert (I_{11}(2i + 1,2j + 1) - I_{11}(2i + 2,2j + 2)) \right\rvert \right. \]
\[ + \left. \left\lvert (I_{22}(2i + 1,2j + 1) - I_{22}(2i + 2,2j + 2)) \right\rvert \right) \]
\[ + w_2 \left( \left\lvert (I_{11}(2i + 1,2j + 1) - I_{11}(2i + 2,2j)) \right\rvert \right. \]
\[ + \left. \left\lvert (I_{22}(2i + 1,2j + 1) - I_{22}(2i + 2,2j)) \right\rvert \right) \]
\[ + w_3 \left( \left\lvert (I_{11}(2i + 1,2j + 1) - I_{11}(2i,2j + 2)) \right\rvert \right. \]
\[ + \left. \left\lvert (I_{22}(2i + 1,2j + 1) - I_{22}(2i,2j + 2)) \right\rvert \right) \]
\[ + w_4 \left( \left\lvert (I_{11}(2i + 1,2j + 1) - I_{11}(2i,2j)) \right\rvert \right. \]
\[ + \left. \left\lvert (I_{22}(2i + 1,2j + 1) - I_{22}(2i,2j)) \right\rvert \right) \]

(2.30)

Where, the \( I_{11} \) and \( I_{22} \) are estimated as:

\[ I_{11}(2i + 1,2j + 1) = I(2i - 1,2j - 1) + I(2i + 3,2j + 3) - 2I(2i + 1,2j + 1) \]  
(2.31)

\[ I_{22}(2i + 1,2j + 1) = I(2i - 1,2j + 3) + I(2i + 3,2j - 1) - 2I(2i + 1,2j + 1) \]  
(2.32)

This algorithm reconstructs images having low artifacts but it over smoothed the image. This drawback can be reduced by using the actual directional curvature instead of second order derivative estimation.

For this purpose, another energy term is added to enhance the absolute value of the second order derivatives:

\[ U_e(2i + 1,2j + 1) = -(\left\lvert I_{11}(2i + 1,2j + 1) \right\rvert + \left\lvert I_{22}(2i + 1,2j + 1) \right\rvert) \]  
(2.33)

This term although produce the sharper images but introduces the artifacts. So the weights assigned for interpolation should be limited. The artifacts can also be reduced by using the isolevel curves smoothing, and is defined as:

\[ f(I) = - \frac{I_1(i,j)^2 I_{22}(i,j) - 2I_1(i,j) I_2(i,j) I_{12}(i,j) + I_{11}(i,j)^2 I_2(i,j)}{I_1(i,j)^2 + I_2(i,j)^2} \]  
(2.34)
Where $I_1, I_2$ and $I_{11}, I_{22}, I_{12}$ are the local approximations of first order and second order directional derivatives respectively. The related energy term is given as:

$$U_i(2i + 1, 2j + 1) = f(I)|_{2i+1, 2j+1} I(2i + 1, 2j + 1)$$  \hspace{1cm} (2.35)

With $I_{11}, I_{22}$ computed as before, and:

$$I_{12}(2i + 1, 2j + 1) = 0.5(I(2i + 1, 2j - 1) + I(2i + 1, 2j + 3) - I(2i - 1, 2j + 1) \hspace{1cm} (2.36)$$

$$I_1(2i + 1, 2j + 1) = 0.5(I(2i, 2j) - I(2i + 2, 2j + 2)) \hspace{1cm} (2.37)$$

$$I_2(2i + 1, 2j + 1) = 0.5(I(2i, 2j + 2) - I(2i + 2, 2j)) \hspace{1cm} (2.38)$$

Thus for each pixel location $(2i + 1, 2j + 1)$, the complete energy function is defined as the sum of the “curvature continuity”, “curvature enhancement” and “isophote smoothing” terms, as:

$$U(2i + 1, 2j + 1)$$

$$= aU_c(2i + 1, 2j + 1) + bU_e(2i + 1, 2j + 1) \hspace{1cm} (2.39)$$

$$+ cU_l(2i + 1, 2j + 1)$$

Using this pixel energy, a simple greedy minimization technique is finally implemented for adjusting the pixel values as follows: the original pixels are placed at locations $(2i, 2j)$ and the rough interpolated pixel is inserted at location $(2i + 1, 2j + 1)$. For each new pixel, the energy function $U(2i + 1, 2j + 1)$ and the two modified energies $U^+(2i + 1, 2j + 1)$ and $U^-(2i + 1, 2j + 1)$ i.e. the energy values obtained by adding or subtracting a small value $\delta$ to the local image value $I(2i + 1, 2j + 1)$ are computed. The intensity of interpolated pixel is replaced with the intensity value of pixel having lower energy. This procedure is repeated again and again till the intensity value of interpolated pixel is lower than the defined threshold value or the number of iteration reached to the maximum value.
2.11 Key literature review of Edge directed interpolation methods

This section presents review of related research done by many researchers and engineers for improving the performance of the above mentioned techniques.

Xin et al. (76) in 2001 proposed an EDI algorithm for natural images. Authors first estimate the local covariance coefficients from a LR image which are used for interpolating low resolution image into higher resolution. Authors mentioned that the interpolation coefficients are tuned to match an arbitrarily oriented step edge by the edge-directed property of covariance-based interpolation. They applied the Bilinear interpolation technique for the non edge pixels and CBA interpolation for edge pixels. This hybrid approach was used to reduce the processing time. Authors from their simulation results claimed that the performance of their projected technique is superior to the traditional interpolation techniques.

For image interpolation, Chen M. J. et al. (77) in 2005 introduced a fast algorithm. The digital images are segmented into edge areas and homogeneous areas in this algorithm. Different areas are interpolated by different algorithms for the better performance of interpolating images. Authors apply their proposed algorithm to interpolate still images and compared objective and subjective qualities of Bicubic, NEDI, Zero-order, Bilinear and their proposed algorithm. Bicubic interpolation method takes more time to reconstruct the image as compared to bilinear interpolation method. So, in order to reduce computational time, bilinear interpolation was used by the authors. The subjective quality of the projected technique is similar to that of new edge directed interpolation but better than the other three methods. Authors showed that the PSNR value of their proposed algorithm for Lena image is better than NEDI, bicubic and bilinear algorithms.

A method for interpolating images with repetitive structures was presented by Luong H. et al. (78) in 2006. In this technique the value of interpolated pixel is anticipated based on the whole image instead of local surrounding neighborhood pixels as in conventional interpolation methods. They exploited the repetitive character of the image. This technique reconstructs good quality of interpolated image as more information is
available instead of only neighborhood pixels value. Authors showed that the proposed method reconstruct image with very less artifacts and better edges. So, authors claimed the superiority of their technique over other interpolation methods.

Wang Q. et al. (79) in 2007 proposed an isophote orientation-adaptive interpolation method that reduces zig-zagging in edges as well as in ridges. Authors focused mainly on the interpolation behavior along ridges, which is not addressed in many other methods. They showed experimentally and analytically that the visual quality of the projected method is better than the traditional bilinear and bicubic methods. Authors showed experimentally that as threshold gradually increases, the edge pixels occupy a higher percentage in the average, and better matches the visual impression. They estimated the SNR values for three different images using Bi-Linear, Bi-cubic, EDI, and by projected methods and compared their results.

A modified new edge-directed interpolation method was presented by Wing-Shan et al. (80) in 2009. In this technique, a modified training window was used by the authors for eliminating the prediction error accumulation problem. For suppressing the covariance miss-match problem, authors extend the covariance matching into multiple directions. The performance of the projected method is better than the other interpolation methods.

For efficiently preserving the natural appearance and the edge features of images, Xinfeng et al. (81) in 2009 proposed a NEDI algorithm. In the proposed scheme, by exploiting autoregressive model (AR) and the geometric duality between the HR and LR images, authors compute the interpolation coefficients when the mean square error is minimum. According to authors, their proposed technique preserved the sharpness of edges better than the other interpolation methods. Authors compared the performance of projected method with the existing methods and claimed that their algorithm can not only improve the subjective quality significantly, but also improve the objective quality of the interpolated images. However, it is associated with high complexity.

The MEDI was presented by Tam W. S. et al. (82) in 2009. This technique is the modified version of NEDI method in which authors proposed a different training window. The enlarged training window increases the error because of covariance
mismatch problem. The interpolated image obtained by improved edge directed interpolation was of lower quality than that of the NEDI method. To reduce the error, authors proposed to apply multiple training windows. Authors compared the simulation results of the projected method with the other EDI methods and claimed the better performance of proposed technique over other edge directed interpolation methods.

In comparison to non-adaptive traditional interpolation methods, Zhenhua et al. (83) in 2010 verified that EDI methods produce better results, both quantitatively and visually for scalar images. Therefore, authors have modified the edge-directed concept. The proposed technique was based on the improved edge-directed scalar image interpolation. The proposed technique was tested on many images and authors claimed the better performance of their technique as compared to other methods.

A context-based image resolution up-conversion technique was proposed by Guangming S. et al. (84) in 2010. In this technique image interpolation and de-convolution jointly are performed in a single estimation framework. For estimating a missing high resolution pixel, the technique uses two one dimensional context-based interpolators in two orthogonal directions. When the missing high resolution pixels are recovered than to restore the observed low resolution pixels a spatial de-convolution is performed. For further improving the estimated HR image, the restored LR image is used in an iterative process. Authors showed experimentally that their technique can produce sharper and smoother edges than existing techniques such as the edge guided interpolation (EGI), NEDI technique and the bicubic interpolation technique.

To improve the rate-distortion relation of video sequences and compressed images, Dung T. V. et al. (85) in 2010 proposed a selective data pruning based compression scheme. Before compression the original frames are pruned to a smaller size. Then EDI method is used to interpolate the decoded image to its original size. To adapt the interpolation in different directions authors proposed a novel high-order interpolation. The edge-directed method is used only up to fourth-order but the proposed method uses more pixels in the same window, so the proposed method is more robust. Then this proposed algorithm is applied on the multi-frame. Authors showed by the simulation
results that the proposed algorithm reduces the artifacts as well as jaggies along the strong edges.

A novel edge-adaptive image interpolation was presented by Mishiba K. et al. (86) in 2010. Based on an observation model an enlarged image was estimated from the original image. This estimated image has many edge-directed smooth pixels. These edge-directed pixels were measured using the proposed edge-directed smoothness filter. Authors performed experiments on six different images and showed that PSNR value of proposed method for Lena image is calculated to be 35.46; whereas, it is 34.07, 33.63, 34.65 & 31.46 for bicubic, NEDI, SAI and NLBP respectively. Authors results showed that proposed method has higher PSNR value; but computational time is calculated as 2.03 sec more than bicubic (0.012 sec) and less than all other techniques NEDI (20.28 sec), SAI (2.44 sec), NLBP (5.74 sec).

A fast edge-directed interpolation algorithm was presented by Qichong et al. (87) in 2012. LR image is interpolated to obtain a HR image. To obtain a better quality of interpolated image having low processing time authors presented this fast edge-directed interpolation. In this technique the first step is to determine the edge pixels and non edge pixels. For non edge pixels the conventional bilinear interpolation technique is applied and for edge pixels the edge-adaptive interpolation is used. Authors experimental results showed that their presented technique outperforms the other interpolation algorithms in terms of processing speed and image quality.

Shaode et al. (88) in 2013 compared the performance of four EDI methods with two traditional methods on two groups of images. These methods include new edge-directed interpolation, edge-guided image interpolation, iterative curvature-based interpolation, directional cubic convolution interpolation and two traditional approaches, bi-cubic and bi-linear. Authors also proposed two parameters to measure edge-preserving ability of edge-adaptive interpolation approaches. One evaluated accuracy and the other measured robustness of edge-preservation ability. Performance evaluation is based on six parameters. Objective assessment and visual analysis were illustrated and conclusions were drawn from theoretical backgrounds and practical results. Authors found that all
edge directed interpolation methods were able to obtain high resolution images having enriched details, even unnatural textures were induced. Among these four EDIs, ICBI restricts artefacts and recover crisper and de-blurred images. Authors finally concluded that the comparison presented in this paper is not exhaustive and deep investigation can be extent to robustness to noisy images, images of different categories.

MRI of fetal spine are LR images. So in order to detect any abnormality clearly these LR images are interpolated to HR images. But during interpolation there is a loss of edge structures. Since these edges are very important as they carry very valuable information, so the preservation of edges is necessary. These high resolution images help the doctors to make accurate judgment. Therefore, Shaode et al. (89) in 2013 proposed an edge-directed interpolation method. They applied the proposed method on the group of fetal spine magnetic resonance images. In this technique to modify the edge pixel value Canny edge detector is used to obtain the edge messages. Bilinear method was used by the authors to obtain HR image from its LR counterpart. Then to soften or sharpen edge structures, the edge information obtained from the HR and LR are put into a twofold strategy. Finally a high resolution image is generated having well defined edges. The results obtained by simulation are compared with the results of four different edge directed interpolation methods. Authors claimed that the quantitative results of projected method are better than the other methods.

Shaode Y. et al. (90) in 2013 compared the performance of EDI methods and traditional methods. Authors performed experiments on digital images and MR images. Performance evaluation is based on seven parameters. Authors showed that EDI methods are better than traditional methods except time cost. In EDI methods, SNR and PSNR are increased from 1.67dB to 2.60dB, SSIM is from 0.016 to 0.023, FSIM is from 0.022 to 0.026, MI is promoted from 0.60 to 0.76, EPRa is from 0.178 to 0.234 and EPRr is promoted from 0.077 to 0.101, while the time cost increases from 139 times to 932 times. Time consumptions are cautiously concerned in real-time applications. In MR images again SNR and PSNR are increased from 6.313dB to 8.884dB, SSIM and FSIM are promoted from 0.031 to 0.046, MI is from 1.517 to
1.728, EPRa is increased from 0.343 to 0.439, and ERPr is from 0.131 to 0.163, while the time cost increases from 92 times to 775 times. So, the main drawback in EDI methods is time cost which can be improved by hardware and fast programming.

### 2.12 Introducing wavelet based image enhancement

As discussed above, the edges of an image are high-frequency component which shows jaggies when they are interpolated by various techniques i.e. the major defects in image are occurred at the edges. The main reason for these defects is the smoothing effect. So, preserving the edges is essential for getting the good quality of an image. Therefore, an interpolation technique which considers only the HF components of an image is used. This technique is based on DWT which when applied on the original image decomposes it into different subband images; namely, high-high (HH), high-low (HL), low-high (LH) and low-low (LL) as shown in Figure 2.31. Out of these four subbands, three subbands contain the HF components of the input image (91-92). After decomposing the image, the interpolation is applied to input image as well as on the subbands having HF components of the input image. Finally, the IDWT of the interpolated input image and interpolated subband images generate the final HR output image. In this technique, sane interpolation technique is applied on input image and high frequency subbands. The quality of the super-resolved images by this technique depends on the interpolation and wavelet functions. Interpolation has already been discussed in previous chapters; whereas, the continuous wavelet transform (CWT) is defined as:

\[
CWT_x^\psi(\tau, s) = \psi_x^\psi(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t) \psi^* \left( \frac{t - \tau}{s} \right) \, dt
\]  

(2.40)

Where, the basis function is defined as:

\[
\psi_{t,s}(t) = \frac{1}{\sqrt{|s|}} \psi \left( \frac{t - \tau}{s} \right)
\]  

(2.41)

and, \( \psi(t) \) is the mother wavelet, \( s \) is the scale parameter and \( \tau \) is the translation parameter.
In Figure 2.31, an input image is divided into four subbands using DWT. The three subbands contain the high frequency components and fourth contains only LF component of the input image. The input image and HF subbands are then interpolated. Finally, the IDWT of the interpolated input image and interpolated subband images generate the final HR output image.

Following the DWT technique, Gholamreza Anbarjafari and Hasan Demirel (97) proposed another technique to increase the resolution of satellite images. In this method, DT-CWT method is used to divide an input image into LF and HF subbands. Then the LR input image and the HF subband images are interpolated same as done in DWT based method. Finally these interpolated images are combined and IDT-CWT is applied on them to produce HR images, as shown in Figure 2.32. The DT-CWT-SR method
shown in Figure 2.32 has the disadvantage of having poor sharpness in edges. To reduce these defects an intermediate stage using SWT was proposed. The subbands obtained from the SWT technique and the interpolated high frequency subbands based on DWT are of same size and so they are combined to modify their values. Then inverse discrete wavelet transform (IDWT) is applied on these combined subbands to generate a new high resolution image. In Figure 2.33, SWT is used to decrease the information loss that occurs due to down-sampling in each of the DWT subbands. The output of stationary wavelet transform contains equal number of samples as there in the input image. So there are N redundancies in the wavelet coefficients for decomposition upto N levels. The added high frequency subbands are further interpolated for getting high resolution image of better quality. The quality of the images produced is discussed in next Chapter. The key related literature review is reported below.

Figure 2.33: Block diagram of DWT-SWT SR method (97).
2.13 Key literature review of wavelet interpolation methods

Based on a contrast measure, Ahmed et al. (93) in 2007 proposed an image enhancement technique based on DWT. The ratio of high spatial-frequency content of an image and low spatial-frequency content of an image is defined as contrast measure. Authors mentioned that the qualitative properties of the image obtained after applying the proposed technique of contrast measure is better than the DCT based enhancement technique.

For getting high-resolution image from uncompressed video a spatial domain based super-resolution reconstruction technique was used. But this technique is not suitable for compressed videos having quantization errors. So, Zhu et al. (94) in 2007 presented a DWT based reconstruction approach. For reducing the blurring and ringing noise artefacts, they used a projection operator and maximum-a-posteriori (MAP) evaluation techniques. Authors showed experimentally that their proposed approach recover the edges of an object and edge details that become blurred during compression and expansion.

The visual quality of digital images can be improved by using an algorithm proposed by Numan et al. (95) in 2009. Authors mentioned that image interpretation for defence and security tasks, the proposed algorithm preserves the local contrast. According to authors, their proposed algorithm can also be applied to video streaming for aviation safety. The authors’ applied their proposed algorithm on the aerial images and claimed that their proposed algorithm gives better image quality and has robustness.

For the estimation of wavelet coefficients, Turgay et al. (96) in 2009 proposed a new algorithm. In this technique, the HR image is reconstructed from the given LR image by using the inverse and forward DT-CWT method. The image is decomposed by using DT-CWT technique and set of wavelet coefficients is estimated for the high resolution image. Authors apply their proposed method on different high resolution images and compared their results with other image resolution enhancement methods.
A new super-resolution technique was proposed by Gholamreza et al. (97) in 2010 in which an input image is divided into four subbands using DWT. The three subbands contain the high frequency components and fourth contains only low frequency component of the input image. The input image and HF subbands are then interpolated. Finally, the IDWT of the interpolated input image and interpolated subband images produce the final HR output image. Authors tested their technique on various images such as Baboon, Elaine, Lena and Pepper. Authors showed experimentally that the visual quality and PSNR value of their proposed method is better than the traditional interpolation techniques.

Hasan et al. (98) in 2010 proposed another technique to increase the resolution of satellite images. In this technique, DT-CWT is used to divide an input image into low and high frequency subbands. Then the LR input image and the HF subband images are interpolated same as done in DWT based method. Finally these interpolated images are combined and IDT-CWT is applied on them to produce HR images. Authors claimed that the visual results and the PSNR of proposed technique show the superiority over the wavelet zero padding based image resolution enhancement technique and traditional bicubic interpolation.

Based on the DWT and singular value decomposition, Hasan et al. (99) in 2010 proposed a contrast enhancement technique for satellite images. In this technique the singular value matrix is estimated for LL subband image which is obtained by applying DWT on the input image. Then, the HR image is reconstructed by applying IDWT. The authors compared their proposed technique with the techniques such as; singular value equalization, brightness preserving dynamic histogram equalization. The experimental results showed the better performance of author’s method over traditional methods.

Image resolution enhancement techniques find many applications such as geographical information systems, astronomy and geosciences studies. Image interpolation is used to increase the resolution of an image but when interpolation is applied on the image there is a loss of clarity and jaggies are appeared. So, a dual-tree complex wavelet transforms (DTCWT) based interpolation method was proposed by Pilla et al. (100) in 2011. By
applying the proposed interpolation method high frequency components are recovered which provides better visual quality and good quality of SR images.

Harikrishna et al. (101) in 2012 projected a technique to enhance the resolution of satellite images. In this technique, the target image is divided into low and high frequency subbands using DWT technique. These subbands are then interpolated to increase the resolution. Finally these interpolated images are combined and IDWT is applied on them to generate high-resolution images. An intermediate stage has been proposed by the authors for estimating the high-frequency sub bands. This technique was applied by the authors on many satellite images and claimed that the visual results, RMSE and the PSNR of projected method show the superiority over the traditional interpolation techniques.

Ultrasonic devices are frequently used for medical imaging and the images produced by these devices often have to be converted to a form that is better suited for image analysis and understanding, which are referred as image enhancement techniques. Karthikeyan et al. (102) in 2012 projected three techniques for edge enhancement, image enlargement and image fusion. All the algorithms have the common goal of improving the visual quality of ultrasonic images and are based on wavelets and other image processing techniques. Author’s proved experimentally that the projected models have better performance than the traditional systems.

The resolution enhancement (RE) schemes suffer show blurring in the enhanced image. The DWT based resolution enhancement scheme generates artefacts because of the shift-variant property of DWT. So, for the resolution enhancement of the satellite images, Muhammad et al. (103) in 2013 presented a wavelet-domain. In this approach to obtain high-frequency (HF) subbands, an input image is decomposed into different subbands by using DT-CWT technique. The Lanczos interpolator interpolates the input image and the HF subbands. To overcome the drawbacks of DT-CWT, the HF subbands were passed through a nonlocal means filter. The filtered low resolution and high resolution subbands were combined and then inverse DT-CWT method is applied to
obtain a HR image. Authors’ claimed that the subjective and objective analyses showed better performance of their proposed technique.

Satellite images are used in various applications like geographical information systems, astronomy and geoscientific studies which require high resolution images. There are many conventional image interpolation techniques such as Lanczos, Bicubic, Bilinear and Nearest Neighbor. However in compared to all traditional methods, Ramdas et al. (104) in 2013 mentioned that the wavelet methods give better result. Authors mentioned that the wavelet transform retains high frequency components and provides time and frequency representation simultaneously. In this paper, authors used SWT and DWT to enhance image resolution. The intermediate subbands of image produced by SWT and DWT are then interpolated using Lanczos interpolation. Finally, authors combined all subbands using IDWT.

Sangeetha et al. (105) in 2013 proposed a stationary wavelet transform (SWT) based method. In this technique, DWT is used to divide the image into four subbands. The SWT and DWT techniques produce the subbands of equal size, so they are combined to modify their values. Then inverse discrete wavelet transform (IDWT) is applied on these combined subbands. Authors showed that the visual and quantitative results of presented technique have better performance as compared to the traditional techniques.

### 2.14 Summary

The discussion made in this chapter summarized as:

1) In traditional interpolation method, the processing time of nearest neighbour method is least but the quality of interpolated image is poor. The Bell Bi-cubic technique produces better visual quality images but the processing time is higher.

2) The quality of interpolated image generated by ICBI method is very good but its computation time is very high as compared to all other methods.

3) Wavelet interpolation methods also produce good quality images but processing time is more than traditional interpolation methods.