2.1 INTRODUCTION:

The Chapter II presents a method for evaluating lens and liquid index which is based on interference phenomenon of light. To the best of our knowledge, A high resolution interferometer are used in our experiment, which is known as Lummer-Gehrcke plate, interferometer is developed by the Lummer and E. Gehrcke in the year 1901. This Interferometer is nearly a long plate of very perfect optical glass or quartz with parallel optically plane surface, From 10 to 20 cm. long, 1 to 2 cm. wide and a few millimeters thick. A small right angled prism of suitable angle is connected at one end of the plate.

The liquid immersion method of Kohlrausch 1968 has been adopted for the proposed experiment. In this method the test lens is placed on the vibration free optical bench in such a way that the parallel light passes through the glass cell and test lens. The parallel light beam passing after test lens and liquid converges to the back focal plane of the lens test lens. The focal length of the lens and liquid combination depends on the refractive index and concentration of the liquid. If liquid and test lens are of same refractive index, in this situation the focal length of the test lens

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Arvind Kumar Deshmukh and R.S. Kasana "Refractometry of lens by using Lummer Gehrcke plate interferometer" (Communicated).

becomes infinity and light passes through the lens liquid combination remains the parallel. Due to this situation the test lens becomes invisible and the combination of lens and liquid acts as a Plano parallel glass plate. A liquid immersion technique (1-3) having a mixture of liquids was used where parallel light passes through a test lens dipped in liquid.

Due to importance of refractive index of an optical material, many workers have centered their attention to designed and developed some new optical configuration depending upon the physical status of samples like solid, liquid and gas. The different aspect (4-9) estimating the value of refractive index of material, can be categorized as optical beam path variation measurement, total internal reflection studies, diffractometric fringe detection methods and interferometric fringe detection method.

The refractive index of liquid (10-13) has been measured by the spectroscopic method, but due to several instrumental components it has some limitations. A. C. Defranzo *et al.* (14) has measured the refractive index at low temperature. R. S. Kasana *et al.* (15-19) have reported a new liquid immersion techniques, they have replaced the telescope by Murty Shearing plate and low special frequency ruling grating. A new technique of capillary tube by Ghandoor *et al.* (20) has been developed for finding the refractive index of crude oils. Some significant piece of work has been reported in literature (21-25) for finding the refractive index of liquid and lens material by using interferometric and acousto–optic diffraction techniques.

Recently several scientists have centered their attention for determining the refractive index of lens material some of these are discussed below. Ming Deng *et al.* (26) have been proposed photonic crystal fiber based Fabry-Perot interferometer for measuring the refractive index of material. Ping Lu *et al.* (27) have suggested tapered fiber March–Zehnder interferometer for measurement of refractive index. Adam J. *et al.* (28) determined the refractive index by using interferometric methods. Severine Diziain *et al.* (29) have examined the variation of refractive index in thin films by heterodyne interferometric scanning method. Sergio De Nicola *et al.* (30) have discussed the March–Zehnder interferometer for measuring the refractive indices of uniaxial crystal, recently some piece of significant work has been discussed in literature (31-37).

An Innovative approach for finding the refractive index of lenses has been reported in this chapter. All the serious difficulties associated with refractive index measurement of glass material in the form of a lens have been set a side under the proposed techniques, the focal length displaying by the test lens and liquid is of main concern. The investigations of fringe pattern show that the separation between successive interference fringes depends on the focal length of the test lens.

2.2 THEORY:

The interference of light beam in case of Lummer-Gehreke plate has been considered. The Lummer plate is always employed with a monochromater. If e is the thickness of the plate and μ its refractive index for a wavelength λ, the optical path difference between successive emergent beams is

$$2\ \mu\ e\ \cos\theta$$

Therefore, the fundamental condition for their reinforcement is given by -

$$2\ \mu\ e\ \cos\theta = m\ \lambda \qquad \ldots [2.1]$$

Where, m is an integer called the order of interference or the order of fringe.

Since,

$$\sin i = \mu \sin\theta$$

$$\mu = \sin i / \sin\theta$$

Rewrite the equation 2.1 in the form

$$2e\sqrt{(\mu^2 - \sin^2 i)} = m\lambda$$

$$4e\ (\mu^2 - \sin^2 i) = m^2\lambda^2 \qquad \ldots [2.2]$$

Refractive index formula for lens, from the figure 2.1 the fringe pattern is shown in the back focal plane of the converging lens. Hence triangle X X_0 X_1 one can find.

$$Sin\theta = \frac{R}{(F^2 + R^2)^{1/2}}$$

$$Cos\theta = \frac{F}{(F^2 + R^2)^{1/2}} \qquad \text{... [2.3]}$$

and

$$Tan\,\theta = \frac{R}{F}$$

Where,

F = Focal length of the test lens

R = Radius of the circular fringe

D = Diameter of the fringe, D = 2R.

Then, cos θ may be written as-

$$Cos\ \theta = \frac{F}{\left(F^2 + \frac{D^2}{4}\right)^{1/2}} \qquad \therefore\ D^2 = 4R^2 \qquad \text{... [2.4]}$$

The value of m can be represented as

$$m\lambda = 2e\ \left(1 - R^2/2F^2\right)$$

$$m = (2e/\lambda)\ \left(1 - R^2/2F^2\right) \qquad \text{... [2.5]}$$

$$m = (2e/\lambda)\ \left(1 - D_m{}^2/8F^2\right) \qquad \text{... [2.6]}$$

For (m+1)th fringe the equation becomes as

$$m + 1 = (2e/\lambda)\ (1 - D^2{}_{m+1}/8F^2) \qquad \text{... [2.7]}$$

By subtracting equation 2.6 from equation 2.7, then we get-

$$1 = -(2e/\lambda)\ (D^2{}_{m+1} - D^2{}_m)/8F^2$$

$$\frac{1}{F^2} = -(4\lambda/e)/\ (D^2{}_{m+1} - D^2{}_m)$$

$$\frac{1}{F} = -\frac{2(\lambda/e)^{1/2}}{(D^2{}_{m+1} - D^2{}_m)^{1/2}} \qquad \text{... [2.8]}$$

Where, the e, λ, and θ are remains constant for all point on a circular fringe. Than assuming that

$- \ 2(\lambda/e)^{1/2}$ = Constant = K_o

$(D^2{}_{m+1} - D^2{}_m)^{1/2}$ = G

In the liquid immersion technique the lens is immersed in to the liquid solution inside a glass cell. If the refractive index (n_L) of the liquid solution matches the lens, index n, the combined lens and liquid system behaves as a plano- parallel plate. The collimated incident light therefore remains unchanged. In this case the formula for focal length of a lens immersed in the liquid inside the glass cell, is represented by the (Kasana and Rosenbruch 1983 and 1984) the equation is

$$\frac{1}{F} = (n - n_L)(C_1 - C_2) + (n - n_L)^2 . t \quad c_1 \quad c_2 / n \qquad \text{... [2.9]}$$

Where,

n = Refractive index of the lens material

F = Focal length of the lens

t = Thickness of the lens

C_1 and C_2 = Curvature of the lens

n_L = Refractive index of liquid

For the focal length of the test lens corresponding to pth and qth liquid are given as. By using equation 2.8 and 2.9 for pth liquid we get

$$\frac{2(\lambda / e)^{1/2}}{(D^2{}_{m+1} - D^2{}_m)_p{}^{1/2}} = (n - n_P)(C_1 - C_2) + (n - n_p)^2. \;\; t\, C_1\, C_2 / n$$

For qth liquid

$$\frac{2(\lambda / e)^{1/2}}{(D^2{}_{m+1} - D^2{}_m)_q{}^{1/2}} = (n - n_q)(C_1 - C_2) + (n - n_q)^2. \;\; t\, C_1\, C_2 / n$$

Dividing and further soling both above equations we obtain

$$n = \frac{n_q (D^2{}_{m+1} - D^2{}_m)^{1/2}{}_q - n_p \;\; (D^2{}_{m+1} - D^2{}_m)^{1/2}{}_p}{(D^2{}_{m+1} - D^2{}_m)^{1/2}{}_q - (D^2{}_{m+1} - D^2{}_m)^{1/2}{}_p} \gamma \qquad \text{… [2.10]}$$

Where,

$$\gamma = 1 + \frac{-1 + \left\{ \begin{matrix} [(1 + (t\, C_1\, C_2 / C_1 - C_2)(1 + t\, C_1\, C_2 / C_1 - C_2))(G_p\; G_q) \\ .2(n_q\; G_q - n_p G_p)] / n_p\;\; G_q - n_q\;\; G_p \end{matrix} \right\}^{1/2}}{2(1 + t\;\, C_1\;\, C_2 / C_1 - C_2)}$$

The γ is real and complex quantity for the simplicity of the equation 2.10, the γ should be eliminating from the above equation

Where,

$$(D^2{}_{m+1} - D^2{}_m)_q{}^{1/2} = G_q$$

$$(D^2{}_{m+1} - D^2{}_m)_p{}^{1/2} = G_p$$

For the simplicity the equation 2.10, we can write as

$$n = \frac{n_q (D^2{}_{m+1} - D^2{}_m)^{1/2}{}_q - n_p \ (D^2{}_{m+1} - D^2{}_m)^{1/2}{}_p}{(D^2{}_{m+1} - D^2{}_m)^{1/2}{}_q - (D^2{}_{m+1} - D^2{}_m)_p{}^{1/2}} \qquad \ldots [2.11]$$

Equation 2.11 is the general formula for determining the refractive index of test lens which is used in the proposed techniques.

2.3 OPTICAL CONFIGURATION AND PROCEDURE:

The optical arrangement has been shown in figure (2.1). All the components have been named in the figure itself. The optical system consists of a glass cell filled with liquid. The test lens is immersed in liquid inside the glass cell. Lummer-Gehrcke plate is adjusted adjacent to the glass cell. The coherent parallel beam of light coming from the laser source insides a small right angled prism of suitable angle is cemented at one end of the Lummer-Gehrcke Plate. So that the angle of incidence at the inner surface of the plate is slightly less than the critical angle of total reflection. Light therefore, undergoes multiple internal reflections at angles near to the critical angle. The Lummer-Gehrcke Plate produces an interference fringe pattern. The test lens brings this pattern in its back focal plate as shown in figure.

The sets of narrow bright fringe, on an extended practically dark background are formed in the focal plane of the objective. The pattern X_0, X_1 is formed by the interference of waves emerging from the upper side, the two sets being separated by a dark zone. For each investigation, new liquid poured in to the glass cell. Thus, every time the circular fringe pattern is shifted to new focal plane corresponding to new liquid.

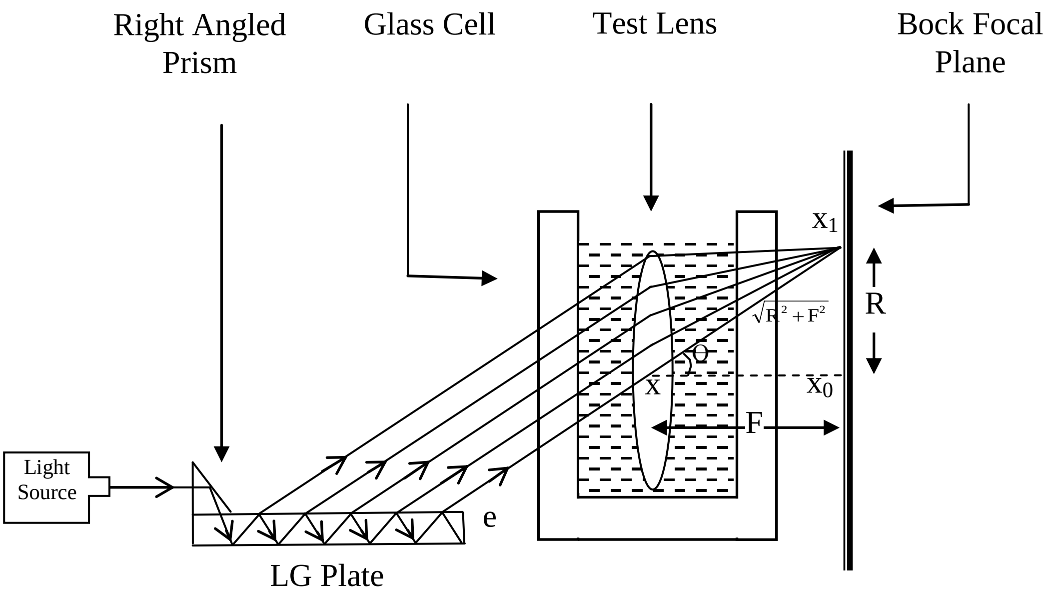

Figure 2.1: Optical Configuration of Lummer-Gehrcke Plate Interferometer with Liquid Immersion Techniques

The collimated light is made to inside with suitable angle of a small right angled prism, which is cemented at one end of the Lummer-Gehrcke Plate. The test lens immersed in liquid inside the glass cell, the parallel light after passing through the test lens, converges to its back focal plane. Thus, this pattern can be photographed and diameter of the fringes is measured by the comparator. The focal length of the test lens will be different in each liquid. The x-y recorder can also be used to find out the separation between consecutive fringes. The method and procedures are repeated for the each liquid.

2.4 OBSERVATIONS:

Wave length of the light used λ = 632.8 nm

Room temperature T = 23°C

Radius of curvature C = 101.515 mm

Focal length of the lens F = 198.05

Lens index n = 1.5084

2.5 RESULTS AND DISCUSSION:

Table 2.1: Standard Refractive Index of Liquid and Air

Serial Number	Medium Name	Standard refractive index of Medium
1	Air	1.0000
2	Distilled Water	1.3331
3	Coconut oil	1.4455

Table 2.2 Calculated Value of G for the medium of Air

Number of circular fringe m	Diameter of circular fringe Dm (in mm)	Square of Diameter D^2m (in mm)	Calculated Value of $D_{m+1}^2-D^2_m$ (in mm)	Calculated Value of G (in mm)
First	3.845	14.784025	-	-
Second	5.813	33.790969	19.006944	4.3596540
Third	7.297	53.246209	19.455240	4.4108094
Fourth	8.648	74.787904	21.541695	4.6413032

Table 2.3: Calculated Value of G for the Distilled Water

Number of circular fringe m	Diameter of circular fringe Dm (in mm)	Square of Diameter D^2m (in mm)	Calculated Value of D_{m+1}^2-D^2_m (in mm)	Calculated Value of G (in mm)
First	11.220	125.888400	-	-
Second	16.885	285.103230	159.214830	12.618036
Third	22.03	449.567210	164.463980	12.824351
Fourth	25.105	630.261030	180.693820	13.442240

Table 2.4: Calculated Value of G for the Coconut Oil

Number of circular fringe m	Diameter of circular fringe Dm (in mm)	Square of Diameter D^2m (in mm)	Calculated Value of D_{m+1}^2-D^2_m (in mm)	Calculated Value of G (in mm)
First	42.885	2494.6329	-	-
Second	72.685	5283.1092	2788.4763	52.806025
Third	90.225	8140.5506	2857.4417	53.455041

Table 2.5: Calculated Refractive Index of Lens Material by using pair of medium as Water and Air

Serial Number	Pair Name of medium or Liquid	Predicated lens' Index n	Calculated Average Value of Lens Index n
1	Water - Air	1.5089	-
2	Water - Air	1.5077	1.5084
3	Water - Air	1.5087	-

Table 2.6: Calculated Refractive Index of Lens Material by using the pair of medium as Coconut oil and Air

Serial Number	Pair Name of medium or Liquid	Predicated lens' Index n	Calculated Average Value of Lens' Index n
1	Coconut oil – Air	1.5084	-
2	Coconut oil – Air	1.5085	1.5084
3	Coconut oil – Air	1.5083	-

The Lummer–Gehrcke plate interferometer with liquid immersion method is very simple and easy to get optical configuration. In the proposed interferometric technique, the difference between the diameters of the successive fringes is only measured parameter for determining the refractive index of lens. Table 2.1 shows the standard value of refractive index of medium. Table 2.2 - 2.4 shows the calculated value of G and the diameters difference of circular fringe corresponding to medium Air, Distilled Water and Coconut oil which is filled in glass cell. Table 2.5 – 2.6 shows the calculated average value of refractive index of test lens with respect to pair of medium as water – air and coconut oil – air. All the calculations for determining the refractive index of lens are made on the basis of equation 2.11.

In the proposed method we found that the calculated refractive index of lens correct up to fourth place of decimal. This method is independent from the focal length of lens and other parameters so it is clear that the error in calculation of refractive index of test lens is reduced.

2.6 CONCLUSION:

From the above discussed method it is clear that the measurement of the value refractive index of test lens is correct up to fourth place of decimals. The error in calculation can be reduced because the proposed method independent from some parameters like focal length, wave length of light source, radius of curvature of lens and so on. This method is very easy and economic for getting the optical configuration. But some limitations are associated with this method like a pairs of medium are required for determining the value of refractive index of test lens. The errors in calculation of refractive index due to instrumental components and change in temperature and other physical phenomenon are not taken in the account of calculation.

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